ICTs and Geometry

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Abstract—This paper is a literature review of researches from 2014 to 2019 on how to teach students Geometry through ICTs methods. The articles were searched through Google Scholar. At first, the focus of the article is on the teaching of geometry with software available on the Internet for PCs. Secondly, educational games for geometry are presented that a student can install on any device, such as a smartphone or tablet and are available to the user at any time. Almost all of the described researches have tested the effectiveness of ICTs teaching methods for geometry in relation to classical teaching methods such as paper-and-pencil. Based on findings, the teaching of geometry with ICTs had better educational results than the paper-and-pencil method of teaching.

Keywords—ICTs, teaching geometry, apps, education games

1 Introduction

In recent years, ICTs have been used widely in classrooms because, as it is shown in the article below, children learn more easily and more effectively in this way in relation to classical teaching methods [36]. In this article, the term "ICTs" (information and communication technologies) is used for all types of technological means that are used in teaching methods, including computers, tablets, smartphones, interactive whiteboards, etc. Consequently, teachers from all scientific disciplines have broadly embedded ICTs to improve their way of teaching [3, 9, 28]. ICTs are often used by teachers as a motivation to students, to diversify power classes and to focus on an individual way to teach [29].

ICTs can positively affect the performance in mathematics, provided pedagogical exploration is used as opposed to traditional pedagogical teaching method [30,31]. As far as geometry is concerned studies have shown that Geometry is considered a difficult subject and this difficulty is mainly found in solving geometry problems [32, 33]. The Van Hiele theory (five levels) was intended to explain and identify students' weaknesses in geometry [34], and nowadays is the best model used to determine students' level of thought in geometry [1].

In this research we will try to answer the following research questions:

- Is teaching geometry via ICTs more efficient than the traditional methods (paper and pencil)?
- Is the use of ICTs for teaching geometry more efficient to certain age groups?
- What areas of geometry can be taught with the help of ICTs?
- Do students find the teaching of geometry through ICTs more alluring than traditional methods?

2 Softwares

Nocar and Zdrahal (2016) made an experiment for learning geometry with three methods. Firstly they used paper-and-pencil graphing technology (by hand), secondly, they used enhanced methods of Computer Algebra System (CAS) Maxima (with the teacher) and thirdly they used Geogebra Dynamic Geometry Software (DGS) (without the help of a teacher). The experiment took place in the Czech Republic, in which 150 students of grade 7 from the Elementary Schools of three regions participated. Students were evaluated by the Mathematics Assessment Project (http://map.mathshell.org/index.php) after they were taught with the three previous methods of teaching. Results showed that there where unimportant differences between the three used methods of teaching. One of the reasons is that the graphing of linear is very easy with paper-and-pencil method (it only needs two points). Results may have been different if graphing needed more complicated functions [2].

Hwang et al (2015) developed a Ubiquitous geometry (UG) system that taught students geometry. In this study, 20 sixth-grade students participated (7 males and 13 females). UG system and computer can be used to measure the length of actual objects that having 2D geometry shapes (rectangle, triangle, parallelogram, rhombus, and trapezoid). According to the results of the analysis, students loved to study geometry with computer and UG system [4].

Three years later, Hwang et al presented a new study, in which 82 fifth-grade elementary students participated. There were three groups: the experimental group (EG), consisting of 27 students, the ruler measurement group (RG), consisting of 27 students, and the control group (CG) that experienced paper-and-pencil method, consisting of 28 students. First and second groups (EG- RG) measured objects in authentic environments, while the CG received classic classroom education. The three groups learned 2D geometric shapes (the same material) by the same mathematics teacher. The results showed that when students measured in authentic contexts in their daily life with UG system they consolidated their learning of geometry [5].

Seloraji and Kwan Eu (2017) tried to teach 24 children (12 boys and 12 girls) with the use of GeoGebra software. Their ages were 5-6 years old. The class divided into three groups: Extension group contained 6 students that were very good in Math, Core group is included 12 student's medium performance and last one Support group consisted of 6 students that usually needed help to complete their tasks. At the end of the study, results showed that the use of GeoGebra improved students' mathematical skills [7].

In 2017, Fiallo and Gutierrez evaluated the effect of the dynamic geometry software environment in the learning of trigonometry. In the experimental study, 17 students of grade 10 from Floridablanca participated. The students worked in groups of two or three with a Computer (Cabri). As the results showed, the dynamic geometry software (DGS) environment helped students learning and improved their knowledge of trigonometry [8].

Azzizul and Din (2016) developed mathematics teaching materials for geometry with the help of GeoGebra software for Form Four students. The Form Four mathematics Teaching Module included Linear, Circle III and Trigonometry II subjects. In the experiment, 23 students of 4A Class from Malaysia participated, who with computer received the activity via OpenLearning platform. Researchers found out that students were able to use this software easily and understand the concept of geometry [10].

Boo and Leong (2016) presented experience with teaching and learning activities. In the study 22 female and 28 male from a primary school participated. The experience included six main GeoGebra activities in which students should measure and construct angles. At the end of this study, students pointed out that they liked the use of the application GeoGebra to learn and understand angles in geometry [11].

Dhayanti et al (2018) carried out a survey in which they applied the Realistic Mathematics Education (RME) method using the Geometry Sketchpad software. In the survey, 25 students of eighth grade from Aceh participated. The results showed that students with the use of geometry sketchpad improved their critical and creative thinking. The software also helped students to develop and master the knowledge they possessed [12].

3 Apps for Smartphones and Tablets

Maia-Lima et al (2015) conducted a survey in which 76 students of geometry subject of the 2nd Grade of Basic Education Bachelor in Portugal participated. The survey consisted of 2 activities. At first task, students studied polygons, the symmetry figures and the nets of an open cubic box with the help of smartphones and QD codes. At the second task, students played the TetrisLeb (the name comes from the combination of the classic Tetris game and the Basic Bachelor Degree). Students preferred the second task and they pointed out that after the game (on Geogebra's applet) they:

- Understood the meaning of isometrics
- Discovered the combination of across parallel and perpendicular axis
- Knew which vectors, reflection axis and rotation centers to choose
- made faster moves and corrected more easily a wrong transformation [13].

In 2017 Buchori et al developed a mobile application using the Waterfall Model development method. In the search, 32 students of University PGRI Semarang on their first semester participated. The course of geometry required students to have high cognitive and spatial abilities. The design and understanding of 3D graphs is a

difficult process. This research ascertained that with the help of mobile augmented reality media design the learning of geometry in college became simpler [14].

Prabowo et el (2018) developed an application called "Math Learn" for the teaching of trigonometry and features such as angle, phi, sinus, and cosines using the Android operating system. The application was given to 25 senior high school students in Yogyakarta, Indonesia, for testing. "Math Learn" could be installed on smartphones with the Android operation system and has the ability of audio, text, image and video [15].

The smartphone application called "MathDroid" for Android was created from Juariah J. et al (2017) to teach three-dimensional geometry. The application was tried to eight grade students at Negeri. With the help of the application, students could learn the attributes of the elements, properties, information about each shape and ways of defining diagonal planes and space diagonal. As we expected at the end of the survey, students improved their mathematical skills [16].

In 2016 Gutiérrez de Ravé et al developed the mobile augmented reality application named" Diedric AR" that helps students to learn Descriptive Geometry. This application has several advantages:

- It could easily be installed on a mobile or tablet
- It is compatible with Android and iOS operating mobile systems
- It has positive effects on spatial perception skills because students interact with geometric elements

Researchers, in order to test the effectiveness of the application, carried out a survey. In the survey, 50 students of Descriptive Geometry object were divided into two groups. The first group was taught with the conventional method while the second received the "Diedric AR" application alongside the lectures. Based on the findings of this study, the students of the second group reduced the time needed to solve specific exercises [17].

Leitão and collogues (2018) developed a mobile educational game based on Augmented Reality (AR) for students in primary school. The game named "Vertice" was focused on improving student's visualization of geometry shapes (2D and 3D) and in the understanding of the properties of the geometric solids. Researchers in order to test the effectiveness of the game presented a survey in Portugal. The survey was conducted in a primary school. Pupils were divided into two groups (first group of 27 pupils and the second 26 students). The first group carried out the traditional method of teaching, while the second group was also given the education game "Vertice". At the end of the experiment, the children of the second group answered more questions correctly in relation to the first group [18, 19].

In 2016 Chang, Wu, Lai & Sung designed the geometry system HOLD (Hands-On Learning by Doing) based on Duval's four critical elements of geometric learning. In their research, 58 high school students from the city of Taipei participated. They were divided into two groups: the experimental (31 students) and the control group (27 students participated). The same teacher and the same educational material were used in both groups. The only difference was that the experimental group used "HOLD" to perform the activities in geometry. The aim of the research was to investigate whether

"HOLD" was effective on perceptual, operative and sequential apprehension, and overall spatial geometry scores. The results of the study showed that "HOLD" improved operative and sequential apprehension. However, perceptual apprehension wasn't identified between the two methods [20].

Lai et al (2016) developed a virtual reality (VR) system that taught students how to calculate the volume and manipulate the dimensions of 3D shapes. The game was designed for the Samsung Gear VR HMD, which was named "Geometry Explorer" and allowed students to view and manipulate shapes [21].

Andrea el at (2019) carried out a survey in elementary school students. The target of the research was to teach students how to calculate the volume and surface of shapes. Students were divided into 2 groups (experimental and control group). The first group would use the "Magic Boosed" application, while the second group would use the classical method of teaching. The application can be installed on a smartphone and is based on AR (Augmented Reality) technology. On the basis of the results, the performance of the pupils of the first group improved more in relation to the second group [22].

Srinivas at el (2016) have created a game based on geometry, called "Police Quad". The game was experimentally given to students in Mumbai and Mizoram of the eighth, ninth and tenth grades. Through the game, students were able among others to understand the attributes of the shapes and learn the relationships of specific quadrilateral [23].

Ozcakir at el (2015) tried to find the effects of using DGS (dynamic geometry software) on 63 eighth grade students. The research took place in Turkey and its target was students' conceptual knowledge in the domain of triangles. The first group (experimental) was conceived of 32 students and it was received DGS supported instruction. The second group (comparison) was conceived of 31 students and it received traditional instruction, by the same mathematics teacher. Students through the activities observed, explored and constructed triangles. Before the study students had similar academic knowledge in triangles. The results showed that students of the first group achieved higher levels of academic and estimation performance than students in the second group [24].

An augmented reality math application named Cyberchase Shape Quest for students of elementary- school was developed from PBS KIDS, Curious Media and THIRTEEN that taught geometry. The apps consisted of three games Patch the Path, Feed the Critters and Hide & Seek and it was compatible with iOS, Android and Kindle tablets (Radu at el, 2015) [25].

A research was carried out from Rahman and Puteh (2016) about how effective it was to learn student's trigonometry using GeoGebra Learning Module (GML). In this study, 21 students participated and at the end of two weeks sessions, students said that GML helped them maintain their attention and they learn unexpected things [26].

Rehm et al (2015) presented a game named "Towards Smart City Learning" that was developed in Unity3D. An experiment took place in Denmark, in which 12 third grade students participated. This mobile game was carried out in an outdoor environment (eld, town) and students had to find geometric shapes around them such as in the

buildings, at home. The mobile learning research had positive results in geometry learning to children [27].

4 Robot

STEM education is actively involved in teaching mathematics to help students not only in the cognitive domain but also in the affective domain. At times, several researchers have suggested that robots facilitate mathematics teaching (Resnick, 2000; Mauch, 2001; Bers, 2002; Bobis, 2011; Benitti, 2011; Oritiz, 2015) [35,37].

In 2017 Briggs and Benedict tried to use a programmable robot Coder MiP in eighth-grade students. Firstly, students used programmed motions for Mip to sketch a graph between distance and time, secondly, they used triangles and tried to calculate triangle sides and learn Pythagorean Theorem with Mip help [6].

Kim and Lee (2016) examined the effects of teaching geometry to fourth grade students from South Korea by a robot. In the survey 121 students, aged 15- 18 years, participated. They were divided into two groups, the "Robot" group (58 students) and the "Ruler & Protractor" group (63 students). The main target of the research was to find out whether a robot with Skemp's has a positive effect on the teaching of geometry. The results showed that both the Robot and Ruler & Protractor groups didn't have significant differences in performance. However, students of the Robot group were more interested in teaching and student participation was higher than the Ruler & Protractor group [35].

5 Conclusion

In this paper, we have presented researches that prove how useful ICTs are for teaching and learning geometry compared to traditional teaching methods (paper-and-pencil). These methods can be applied in a wide representative range of geometry such as Pythagorean Theorem, trigonometry, solid geometry, etc. Furthermore, we listed very interesting ways of teaching geometry through simulations, virtual reality, video games that have proven to be effective to children of all ages. These methods do not need some specialized knowledge from students to use and are very easy for someone to access them by simply having a mobile device or tablet. As we have concluded, a lot of effort is being made by researchers to simplify the teaching of geometry and make it easy for anyone to handle. Generally, students showed more interest in Geometry when they were taught through ICTs.

6 References

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