

Teaching of Ordinary Differential Equations Using the Assumptions of the PBL Method

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Abstract—Mathematics is of fundamental importance to any natural sciences program (because it provides analytical and approximate results that can be simulated and modeled) and perhaps to other areas of human knowledge. Ordinary differential equations are especially important to engineering programs because the modeling of all phenomena of interest for these programs involves Ordinary differential equations solutions; at the same time, students experience difficulties when learning Ordinary differential equations and about their applications to real physical scenarios. In this study, the problem-based learning method was used to study a group of mechanical engineering students at the SENAI CIMATEC University Center. The study analyzes the effectiveness of Ordinary differential equations instruction given through this program using the specified methodology. To consolidate the study, Conceptual Field Theory is used along with an evaluation of the results to observe student behaviors and attitudes in relation to actions arising from the method. At the end of the study, a survey was completed by the students, through which they evaluated the effectiveness of the teaching methodology employed.

Keywords—Ordinary differential equation, PBL, computational simulation

1 Introduction

Mathematics and its concepts are present in the diverse relationships of all processes in many areas of engineering, in theoretical constructs applied to descriptions and phenomenological representations, and in the structuring of physical space in which professionals of these areas operate. This mediation of mathematics has increasingly extended to social and economic relations over the last century, enhancing its importance and significance [1].

This paper proposes the use of a hybrid teaching and learning strategy disseminated by teaching ordinary differential equations (ODEs) through problem-based learning (PBL), and computational simulations based on conceptual field theory of Vergnaud,

focused on information and communication technologies (ICTs), specifically the WINPLOT software, a tool for graphic modeling. Thus, the specific problem examined is linked to the following questions often posed in research on equations: Why must they be studied? Where can they be applied? Answers to such questions (and others related) are intended to be used to consolidate our research proposal.

Related works justify the gains in the use of PBL, currently recognized as an active methodology and present in several engineering studies. In this sense, we highlight the work of [2], which presents the study of students who reached out for PBL to develop satellites using project management. Another relevant work is that of [3], who applied PBL to get students involved throughout each class and the use of the Student Agent Object Portfolio (LAO) course. In addition, PBL was used to improve innovative teaching-learning practices in higher education [4] and also as a pedagogical tool for engineering concerning kinematic analysis of planar connections, in a Machine Mechanics course [5].

2 Literature Review

2.1 Information technology and math teaching

According to [6], ICTs' integration with the equally increasing use of mathematics - both in the formalization of theories and in applications of the most diverse areas of knowledge, especially those that have processes and natural phenomena as the main object has required the transformation of students' cognitive structures. Computational modeling resulting from applications of ICT as part of the analysis of these processes and of natural phenomena constitutes an opportunity to deepen and expand research and applications of various areas of engineering. To meet demands for qualified professionals who are necessary for economic and social development require students to master knowledge and applications of mathematics [7].

Paradoxically, in recent years, students have experienced more difficulty with learning mathematics both at the secondary and postsecondary levels [8]. Various authors have conducted studies on difficulties associated with learning and teaching mathematics. In "Educação Matemática e a construção do número pela criança" [Mathematical education and the construction of the number by the child] by [9], supported by the Piagetian theory, the author traces students' relationships with school life to the social environment. The author verifies that despite being one of the exact sciences, the meaning of mathematics is shaped based on a series of social factors [10]. Such research leads us to a single point: relating mathematics taught in the classroom with the mathematics of daily life facilitates student comprehension of the applications of this subject.

With the expansion of higher education in Brazil, which began with the implementation of the National Educational Guidelines and Framework Law (Lei de Diretrizes e Bases da Educação Nacional - LDB) or Law 9394 (December 20, 1996), new engineering programs have been implemented throughout the country to stimulate the training of professionals in basic sciences (chemistry, physics, mathematics, etc.) and, more

specifically the training of engineers to meet the demand for qualified professionals instrumental to economic and social development.

Within this context, the use of ODEs as a powerful tool for the analysis of problems and for proposing engineering solutions is highlighted. The incorporation of ICT resources through the manipulation of large loads of data and using powerful computational processors has significantly potentiated the use of mathematics [7].

2.2 Vergnaud's proposal

Although Vergnaud's theory is not explicitly a didactic one, it has important implications, for it signals the need, when it comes to the teacher, to watch student's learning from the perspective of complexity, diversity, evolution, and the apprentice's repertoire of schemes, often slow and tortuous, full of ups and downs. From this perspective, it is possible to base new approaches to teaching, with fundamental propositions to the curriculum and assessment. The application of research in a way that emphasizes that Vergnaud's proposition encompasses a theory serves as a way of systematizing certain area of knowledge and, therefore, involves adopting a learning theory that seeks the systematic interpretation of the learning/hermeneutics of an author or of authors on the subject of learning [11].

Such an organized complex allows us to affirm that cognitive learning focuses on cognitive processes themselves and on the act of knowing, whereas affective learning refers to experiences such as pain, satisfaction, discontent, joy, anxiety, and psychomotor learning intrinsic to muscular responses acquired through training and practice.

2.3 Problem-Based Learning (PBL)

Through the present work, we seek to develop a methodology that applies PBL to support teaching on derivatives, the rate of change, integrals, and algebraic problem modeling and specifically in introductory teaching of ODEs for Calculus A.

PBL uses real-life or simulated problems to motivate the development of critical thinking and problem-solving skills, as well as the learning of fundamental concepts of the field in question. This goes against the traditional education paradigm by presenting problems based on real scenarios without simultaneously providing suggestions or mathematical arguments, and by presenting problems (like a mathematical problem) without neglecting what is real on account of the correlation between what is real and existing mathematical elements or entities for solving them.

Thus, while conventional teaching is centered on transferring knowledge from teacher to students, besides promoting the acquisition of knowledge, PBL seeks to develop skills and attitudes essential to effective professional practice. In applying this method, the student is constantly encouraged to learn and to act as part of the learning process [12]; that is, students are responsible for understanding their situation, for perceiving the given context, and for critically evaluating their learning.

PBL adoption is justified by its creators - as a response to teachers' perceptions that students complete programs understanding many concepts but with little capacity to use them - as a means to contextualize concepts through everyday experience [13].

2.4 Problem-based actions

Furthermore, problem-based action proposals do not challenge learning theories and the ways in which they attempt to explain how people learn along with their phases, evolution, and results. With this, the use of the methodology - with the ballast that a learning theory is advocated - becomes feasible, e.g., Vergnaud's proposal and its developments. From such learning process, [13] identified educational objectives of PBL for teaching medicine that can be applied to other areas of knowledge:

1. Learning from an integrated knowledge base.
2. Learning from a knowledge base structured around real problems found in the professional field in question.
3. Learning from a knowledge base linked to ways of solving these problems and the development of an effective and efficient problem-solving process.
4. The development of effective autonomous learning skills and group work.

PBL is widely used in various medical and health sciences schools. Internationally, we can cite the University of Maastrich (Netherlands), McMaster University (Canada), the University of Liège (Belgium), and the University of Sherbrooke (Canada). In Brazil, a significant number of universities use PBL, including the State University of Londrina, the Marília School of Medicine, and the School of Public Health of Ceará, which pioneered the implementation of curricular proposals based on such methodology. In Bahia, Southwest Bahia State University, the Federal University of Bahia, the Bahia School of Medicine and Public Health, the State University of Santa Cruz, and the College of Technology and Science have either fully or partially adopted PBL [14].

3 Materials and Methods

The amount of differential equations that withstand analytical methods have led to the investigation of numerical approximation methods. Effective numerical integration methods were developed in approximately 1900, but their implementation was hampered by a need to perform calculations by hand or with very primitive computational equipment. Over the past 60 years, the development of increasingly powerful and versatile computers has greatly expanded the range of problems that can be effectively investigated by numerical methods. At the same time, extremely refined, robust and easily available numerical integrators were developed. Appropriate versions for personal computers have made it possible for students to solve many significant problems.

The same authors of this methodology proposed means of consolidating the theoretical load and day-to-day problems of a professional, because while created for the teaching of medicine, PBL has already been used at other educational levels, e.g., in elementary, high school, and higher education. In the case of higher education, there are feasible application to various areas of technology to engineering in particular.

Starting from the premise that the calculus curriculum of the higher education program at CIMATEC requires more knowledge regarding ODE instruction, the present study aimed to introduce content to students without using traditional methods. Our

study applied qualitative approach (available at appendix a.4 in [15]) and quantitative data processing. Quantitative data were systematized and organized into tables for subsequent interpretation and applicable inference.

We studied the private SENAI CIMATEC University Center in the city of Salvador, which offers engineering classes and a higher technological course (Curso Superior Tecnológico - CST). We focused on a daytime mechanical engineering undergraduate class with roughly 47 students. All students volunteered to participate in the study and signed our informed consent form.

Our studies were based on traditional teaching models, from which updated information was conveyed while the same means of attaining knowledge were applied. Thus, the students accumulated scientific knowledge by being given answers. According to the above cited authors, regarding calculus instruction given through engineering programs, students experience great difficulty when studying ODEs both in terms of the use of techniques for resolution and in graphical interpretation and understanding of concepts. These difficulties are mainly observed when ODEs are studied in reference to contextualized problems.

Seeking initially a better understanding on the part of the students regarding the teaching and learning of ODEs, the PBL method and WINPLOT software - as a PBL implementation tool - were applied. We applied Conceptual Field Theory while salvaging concepts studied initially through the Calculus A course, which served as a foundation for our studies of ODEs to understand related concepts, the interpretation and solution of proposed problems, their graphic representation, and discussions of their meaning.

Proposals and theoretical frameworks concerning differential equations and teaching and learning theories - especially [14] conceptual field theory and problem-based or project-based learning, while seeking results from modeling methods proposed for teaching differential equations to previously chosen groups of students - were used in our literature review to keep our selection and search of results free and reliable and to analyze them after collecting information from specific surveys (available at appendix A.7 in [15]) and through subsequent analysis of the results. Figure 1 illustrates the relationship between PBL, the computational tool, and theory of the discipline [16].

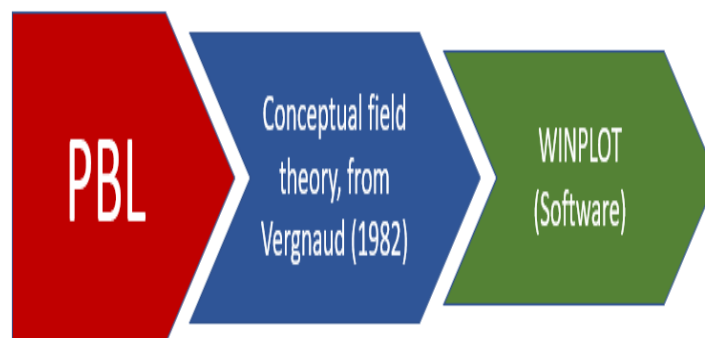


Fig. 1. Methodology for research on ODEs. Source: Authors.

Thus, after the literature review was conducted with a specific focus, theories studied led the modeling of a teaching and learning strategy allied to Vergnaud's cognitive theory in conjunction with a focus group of students for the study and development of the construction of computationally instrumentalized conceptual fields of mathematics.

For this process, tests were administered, and the results are not defined as part of a correct or incorrect response model, as we sought to observe, record, and interpret behavioral models, including students' manifestations (e.g., speech, writing, drawings/graphs, and actions).

Usage of this research type is justified for mathematics education, as researchers often interact with the subject to design tasks that will facilitate interactions between internal structures of the subject and mathematical structures [17],[18] and [19].

Finally, we were interested in studying differential calculus mobilized through teaching under different contexts and representation systems and the system's applicability through verbal recording to register the form of writing used from lists or questionnaires.

The methodology and procedures used are associated with a pedagogical behavior model constructed based on the application of Conceptual Field Theory and based on measurement of teaching/learning process throughout its application in the classroom. According to Vergnaud, individual skill can be defined based on the following three criteria:

1. What students are able to do from a class or a number of classes?
2. If students can apply a more efficient, economical, and effective procedure or method that enables superior performance.
3. If students have acquired a repertoire of alternative procedures or methods that allow them to adapt to the different situations that they face in a more refined way depending on their evaluation of different features of situations.

Thus, we sought to identify and interpret whether students use - and in which form - conceptual fields that form part of their cognitive structure for solving problems, as they were unable to establish a relationship between the questions administered and the theory and its applicability in solving problems.

Once a few exercises were completed, over a progressive scale of interpretation and difficulty (determined with a group of participants subjected to the same learning conditions and resources in a randomly chosen class), students were given questions that they could not solve.

After determining that the students occupied the zone of proximal development, the professor re-explained the subject previously addressed with interpretations that forced the students to relate the elements of a certain conceptual field using, for example, stories, correlations to practical life addressing the respective subject, and suggestions that the students themselves made with and extracted from their everyday experiences.

4 Results and Discussion

During tutorial meetings in class, students gave their testimonials, which were of great contribution for the making this research. Expressing their feelings and actions concerning PBL and how the method contributed to their pursuit of knowledge, group work, and colleagues' acceptance on how they could perform tasks contributed greatly to their professional growth. The statements of some students from the class are reported below. For the researcher, students' statements were filled with enthusiasm and content.

"By using PBL, we become researchers instead of mere users of methods provided by our professors" (STUDENT in Group A)

"When you described the study and proposed the use of PBL and WINPLOT software for graphical representation and analysis to the class, I thought this would be difficult to achieve by the end of the trimester, but over the course of the project I really began to like this way of working" (STUDENT in Group C)."

Other students from the mechanical engineering class stated the following.

"This method should be applied to interdisciplinary work; for example, professors could design an interdisciplinary project, present it to the class at the beginning of the trimester, and then follow up through weekly meetings and ask for reports from the groups and a call for presentations at the end of the learning unit (STUDENT in Group I)."

"This experience was very rewarding because rather than acting as mere users of observations with the single goal of obtaining enough scores to pass the course, we were elevated to the status of researchers. It was quite a challenge, but from the discussions onwards, learning became very enjoyable " (STUDENT in Group L).

In the first trimester, the traditional teaching method was used as per Figure 1 and in the second trimester, the same method was used for the first two evaluations formalized as Quantitative Assessment (QTA). For the first two assessments (QTA1 and QTA2), students were assessed according to the traditional teaching model while for QTA3, the PBL method was used as per Figure 2.

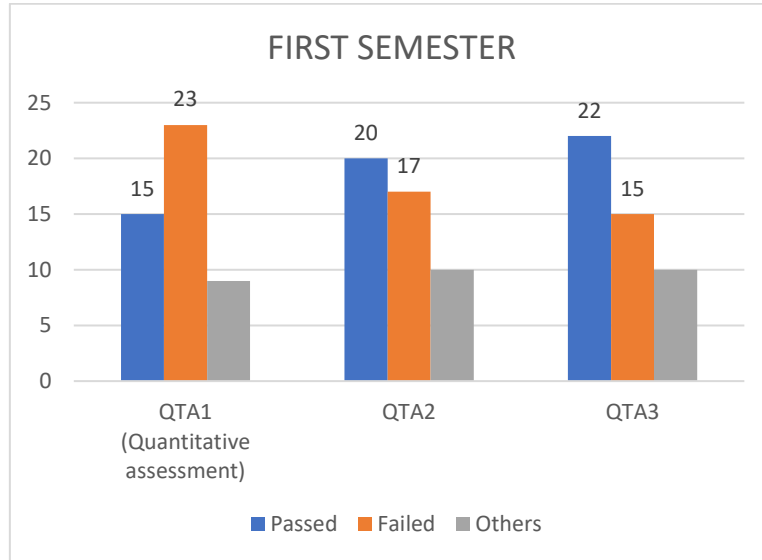


Fig. 2. Number of assessments given in the first semester. Source: Author.

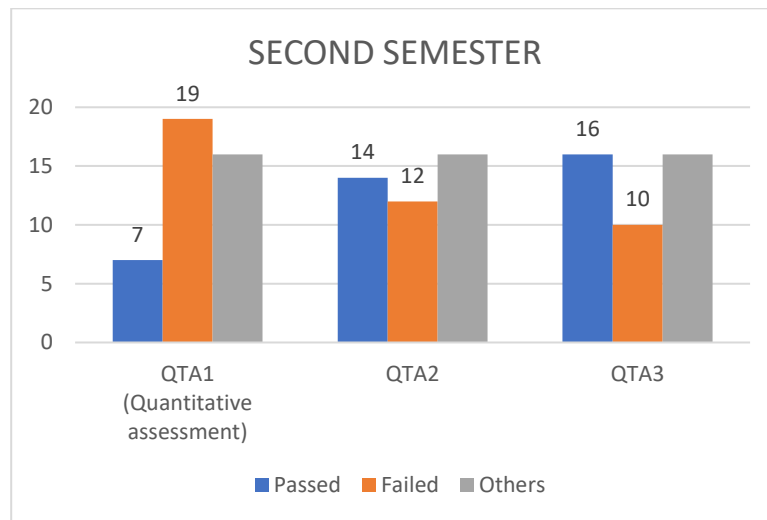


Fig. 3. Number of assessments given in the second semester. Source: Author.

The graphs shown in Figures 3 and 4 denote behavioral changes and improvements in assessment results.

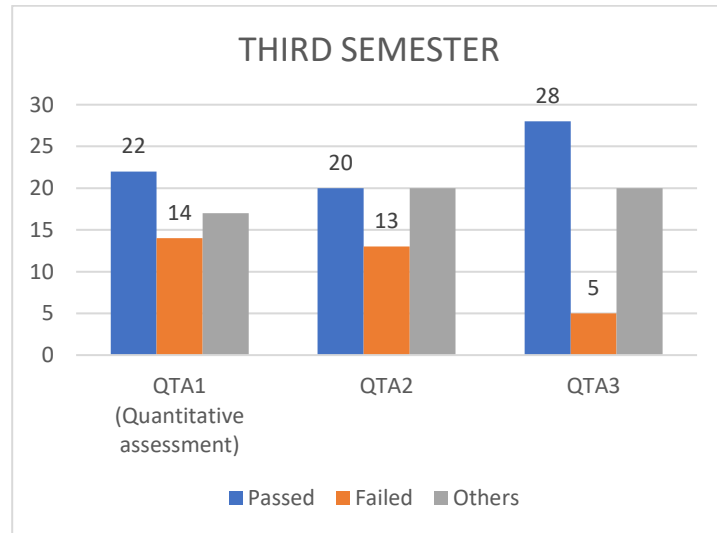


Fig. 4. Number of evaluations given in the third semester. Source: Author.

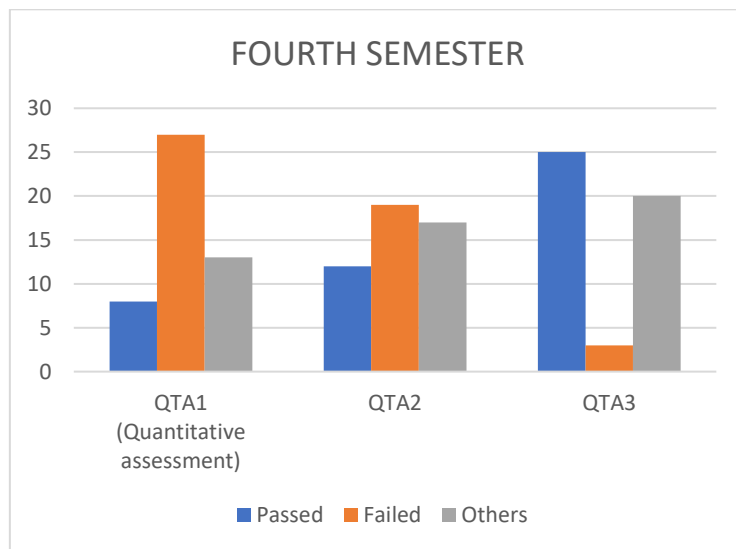


Fig. 5. Number of assessments given in the second semester. Source: Author.

The assessment was performed after applying the method, and in QTA3, we observed an increase in the number of passing grades. Only 7 students passed in QTA1, whereas 16 passed in QTA3.

As shown in Figures 3 and 4, the traditional teaching method was used for the first two evaluations, QTA1 and QTA2. After applying the PBL method upon administering QTA3, the number of students who passed was more significant than that observed for other assessments.

The number drop out students was relatively high, but the value falls within the average range for this course at the institution under study.

Figures 5, 6, and 7 show that for the two semesters of applying the PBL method, there was a significant decrease in the number of failed grades. While this may not prove the effectiveness of the method, it draws attention to certain attributes of the approach.

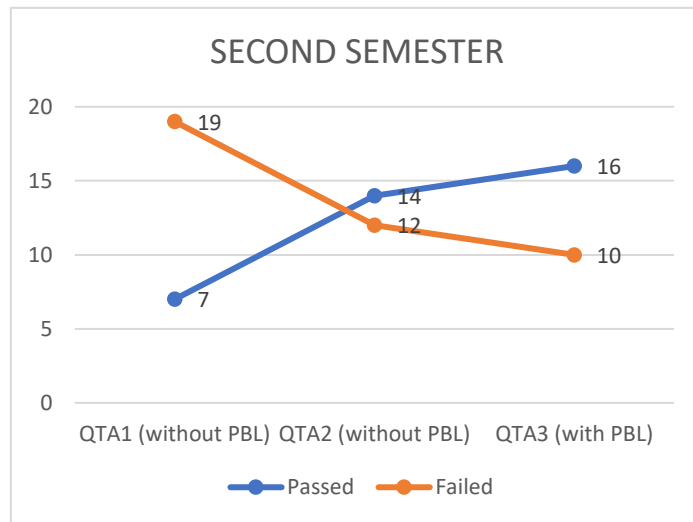


Fig. 6. Comparisons of PBL use in the second semester. Source: Author.

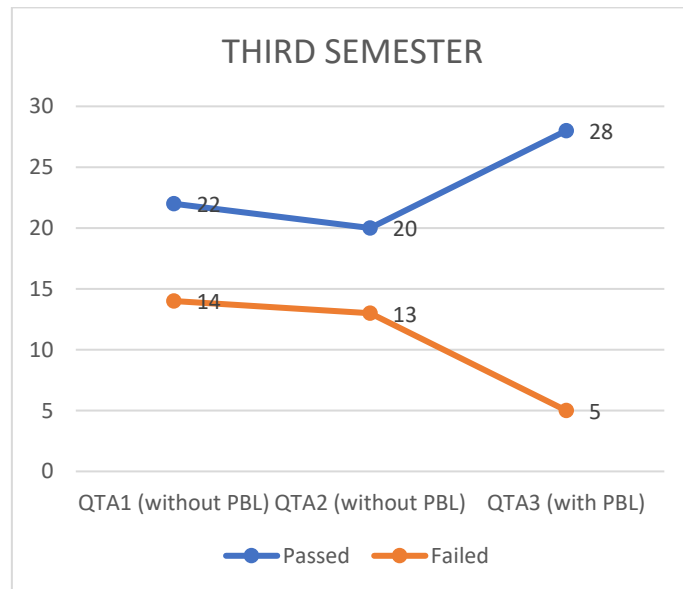


Fig. 7. Comparisons of PBL use in the third semester. Source: Author.

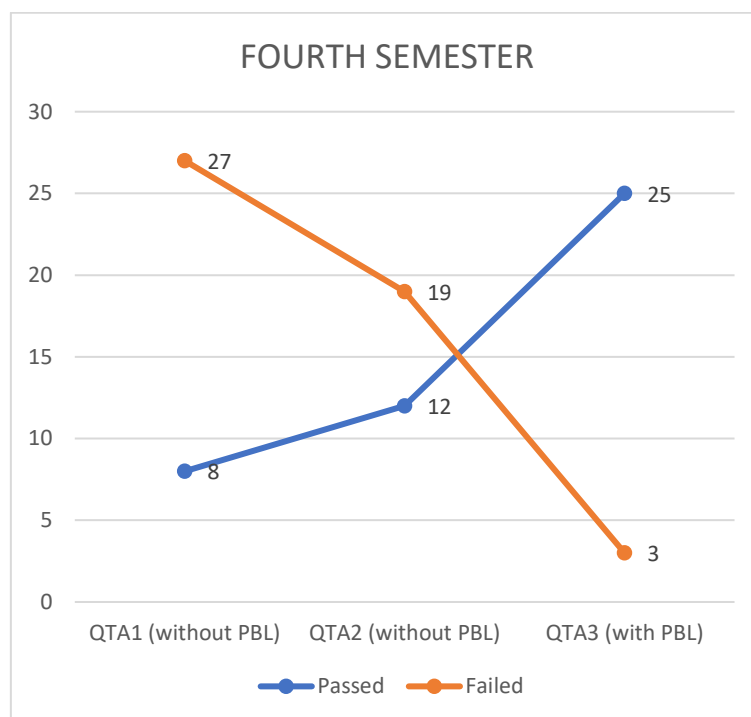


Fig. 8. Comparisons of PBL use in the fourth semester. Source: Author.

5 Final Considerations

Within the scope of our research, a methodology was developed inspired by PBL approach for ODE instruction. Students were evaluated through surveys based on contents that consolidate goals of our research objectives and in a way that consolidates them in their entirety. Thus, the ODEs studied are truly aligned with the professional lives of students.

As our research problem deals with the veracity and effectiveness of ODE instruction, for the tutorial sections, it was necessary for the tutor to carefully listen to the students and consider the following: collocations identified, posture, interventions, richness of the content, arguments, and relationships with future professionals - reflecting an unusual approach in relation to other teaching methodologies, but which did not abandon the guiding, formative, and conciliatory nature of tutorial meetings, so that all problems studied were addressed.

Through this study we confirmed that, as advocated by Vergnaud, it is through situations and problem solving that a concept acquires meaning for the subject; therefore, it is possible to infer from this study that the concepts studied became significant through situations with which students could interact. This interaction occurred with the proposed problems (see Appendix A) along with the use of the WINPLOT software for graphical analysis.

Thus, it is clear that cognitive development depends on situations and conceptualizations and, from the PBL approach, the conceptual field theory, and knowledge already acquired, it was possible to research, understand, and define ODEs and to illustrate and solve them.

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