Interactive Computer Aided Learning and Teaching of Analytical Mechanics

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Abstract—**The aim of this paper is to introduce a new possibility for learning Analytical Mechanics via internet. This subject belongs to the field of engineering, the theoretical parts of lecture materials are available online in PDF format and have special links to switch on the interactive examples on the worksheets of the Computer Package Mathcad.**

Index Terms—**computer aided engineering, interactivity, attractiveness, internet, analytical mechanics, Computer Package Mathcad.**

I. INTRODUCTION

The digital age has transformed education, both formally in the classroom and permanently in any other environment. Although there are many types of interactive materials that have been used in learning for decades, in most cases interactive learning materials are thought of as those that are done on a computer. Many of these materials are made available over the internet, although some are distributed in the form of software [1].

Interactive learning is most commonly defined into three distinct types. These learning types are grouped based on the types of interaction in which they engage students. The first, learner-content, is a type of teaching where the student interacts solely with the facts, readings and information. In the second style, learner-instructor, the primary interaction is with the instructor. The third style is called learner-learner, and this is where the student primarily interacts with his peers to learn new material [2, 3].

U.S. Department of Education report 2009 showed that the most effective interactive learning occurs when all three types of interaction are used [2].

Interactive learning is the counterpart of lecture-type learning. It is the active learning process as opposite to the passive system of a lecture-type. In interactive learning, the students use tangible objects to acquire information. Teachers do not directly pass the information to them. Students acquire the information themselves by allowing them to discover it. Of course, guidance by teachers is necessary. They still provide initial information to the students [4].

In interactive environment students are engaged with the materials rather than simply memorizing the information, the learning is deeper and more lasting. With more engagement student spent more time on the learning task [2]. In interactive learning materials student is in control of the progress of the lesson to some degree. This means in individual learning the learner can make his own pace and studying style of him. In many schools and universities the teachers have lack of time and are unable to give dedicated attention to individual students for very long.

Interactive materials can help provide learning experience without the need for continuous teacher intervention [1].

Engineering teaching material has to be attractive and interactive to make students research technical problems. Attractive learning material means that it is visualized by using many schemes, figures, and animations. Students will understand learning materials better if they can actively engage in the examples of problem-solving via computer screen and change relevant parameters, equations, and simulate the motions. These are hands-on learning materials on the computer. The students can approach to attain information through hands-on interactive experience by using the computer.

There are articles available about interactive learning opportunities in other subjects. For example, Maxima Software – a tool for developing online applications intended for interactive solutions of the examples in Control Theory by K. Zakova [5]. The advantages of the Maxima software include its online availability via internet for all interested users any time, and it can be used locally on the desktop computer or in the notebook. For all technical universities devoted to electrical engineering can use Maxima for computations like useful instrument in an educational process mainly in online and distance forms of education [5]. Also in mathematics we can find interactive learning materials. Heinloo [6] and Talman [7] have made e-courses of mathematics that are interactive and visualized with figures and animations. Heinloo [6] used interactive Mathcad worksheets for lecture notes, examples and exercises. Also there are the links to the video clips, which provide a student with an opportunity to watch and review the example problem solutions at home.

From internet we can find lot of interactive learning materials for different subjects [8, 9, 10].

This paper introduces interactive learning materials on Analytical Mechanics, composed in the Institute of Technology of Estonian University of Life Sciences in Estonian language [11]. These interactive materials are mostly learner-instructor type, but on the computer and over the internet they are also considered learner-content learning material.

II. MATERIALS AND METHODS

Main text of interactive learning materials have been composed by the program Microsoft Word [12] and converted to PDF (Portable Document Format) [13] format. In PDF format all the linked citations work normally as defined. The figures have been composed by using program Adobe Illustrator [14] with occasional addition of pictures. Interactive examples have been composed on the worksheets of the Computer Package Mathcad. Main text

in PDF and Mathcad files are connected with links and are available in Estonian e-Learning Development Centre repository [15].

III. ON VISUALIZATION OF LEARNING MATERIAL

Colorful and, where possible, lifelike figures are used to make the material more attractive for students (Fig. 1). Some of examples in the learning materials are visualized by video clips, for example [16].

IV. INTERACTIVE LEARNING MATERIAL

If the learning materials are on the computer, student can use program features like search, zoom, copy, etc. that accentuate their interactivity.

In terms of collaboration with other computer program features, it is possible to make learning material more interactive by using inside the text links to citations or equation numbers. This gives an opportunity to move inside the text and switch between linked parts on different pages.

Fig. 2 shows a part of the contents of learning material [18] where clicking on the word "Näide 2.6" leads to the page with the solution of the example 2.6
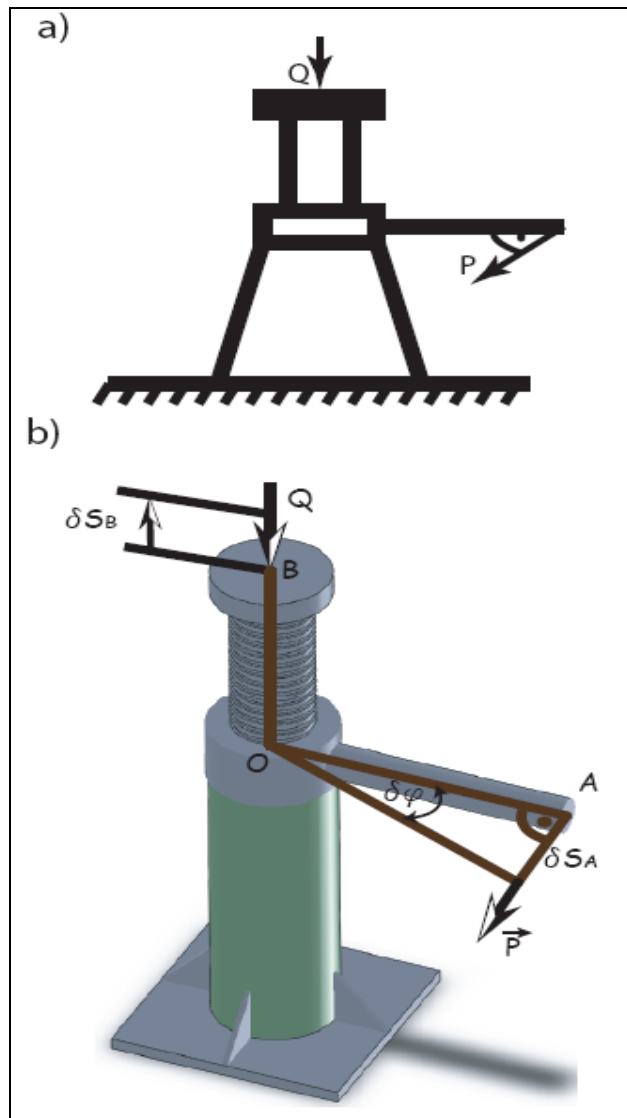

Figure 1.   Comparison of traditional (a) [17] and lifelike (b) [18] screw jack calculation figure

In Fig. 3 we can see an example (*Näide 2.6*), where mass A is connected with cylinder B over wheel O by rope. Cylinder B is moving down and pulling mass A downwards. At the same time cylinder B is spinning off the rope. The question is - what is the acceleration of point C.

In the problem solution (Fig. 4) we can see that one of the equations that has been used is Lagrange equations (*Lagrange`i võrrandid*) and in the main text it is equation 2.40. Clicking on equation number leads to the explanation (Fig. 5) of the Lagrange equations (*Lagrange võrrandid*) in the main text.

Figure 2.   Links in the contents

Näide 2.6. (*example*)

Ühtlasele silindrile *B*, mille kaal on P_1, on keritud üle ploki *O* minev nöör (joonis 2.7). Koormis *A*, mille kaal on P_2, saab libiseda röhtsal tasapinnal. Koormise ja tasapinna vaheline höördetegur on *f*. Leida koormise ja silindri keskpunkti *C* kiirendused süsteemi liikumisel, jättes arvestamata ploki *O* ja nööri massid [5].

Joonis 2.7. Mehaanikaline süsteem

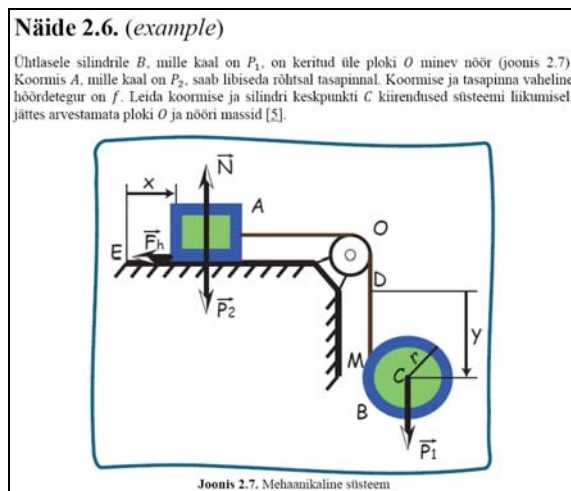Figure 3.   The mechanical system in the example

Lahendus (*solution*)

Süsteemil on kaks vabadusastet. Valime üldistatud koordinaatideks koormise *A* kauguse *x* tasapinna mingist punktist *E* ja silindri tsentri kauguse *y* nööri mingist punktist *D* ($q_1 = x, q_2 = y$). Antud süsteemi Lagrange´i võrranditeks (*equation* 2.40) on

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_1}\right) - \frac{\partial T}{\partial q_1} = Q_1,$$

. . . .

Figure 4.   A part of solution of the example problem

If a student wishes to learn more about Lagrange equations, he or she will be directed to corresponding literature by clicking on the citation (Fig. 6).

If the literature is available on the internet, the student can continue from there by using relevant internet addresses. The name of the book can be connected with links to library internet catalog. In Estonia there is library e-catalog Ester that shows the quantity and availability of books in library [19].

Let us consider an interactive example about physical pendulum on Mathcad [20] worksheet to which leads a special link in the main text (Fig. 7).

Fig. 8 shows the first possible derivation of the equations of motion of physical pendulum in the main text.

Fig. 9 shows the second possible derivation of the equations of motion of physical pendulum, and the link leading to the interactive example on the worksheet of Mathcad.

On Mathcad worksheet a learner can visualize the motion of pendulum by graphs (Fig. 10). A learner can also change the values of inclination angle φ and angular velocity φ' at the initial moment $t = 0$ and observe what happens on figures.

On the worksheet of Mathcad a learner can use controls like slider to define the parameters of pendulum. Fig. 11 shows the slider, defining the length of the pendulum.

Lagrange teist tüüpi võrrandeid on k tükki, st sama palju kui on süsteemil vabadusastmeid [3, 5, 6]

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_1}\right) - \frac{\partial T}{\partial q_1} = Q_1$$
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_2}\right) - \frac{\partial T}{\partial q_2} = Q_2 \cdot \qquad (2.40)$$
$$\cdots\cdots\cdots$$
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial q_k} = Q_k$$

Lagrange teist tüüpi võrrandite arv on minimaalne, mis on vajalik süsteemi liikumise määramiseks. Lagrange teist tüüpi . . . .

Figure 5.   A part in main text with Lagrange equations

KIRJANDUS (*literature*)

1. TTÜ Eesti Inseneripedagoogika Keskus. Kättesaadav: http://www.ttu.ee/ttu-eesti-inseneripedagoogika-keskus/ (04.05.2011).
2. Topnik, E. Teoreetiline mehaanika ülesannetest IV. Analüütiline mehaanika. – Tallinn: TPI, 1987. – 160 lk.
3. Lepik, Ü., Roots, L. Teoreetiline mehaanika. – Tallinn: Valgus, 1971. – 484 lk.
4. Topnik, E. Teoreetiline mehaanika IV. Analüütilise mehaanika metoodiline materjal. – Tallinn: TPI, 1985. – 68 lk.
5. Butenin, N., Lunts, J., Merkin, D. Teoreetiline mehaanika. Dünaamika. – Tallinn: Valgus, 1986 – 552 lk.
. . . .

Figure 6.   Literature from main text

Näide 2.5. (*example*)

Kasutades Lagrange'i võrrandeid koostada füüsikalise pendli võnkumiste võrrandid (joonis 2.6.). Kus P on pendli kaal ja $2l$ pendli pikkus [6].

Joonis 2.6. Füüsikaline pendel

Figure 7.   A physical pendulum in the example

Lahendus (*solution*)

Pendlil on üks vabadusaste ja asendi määrab nurk φ. Järelikult $q_1 = \varphi$. Anname nurgale φ positiivse juurdekasvu $\delta\varphi$, et sellel nihkel teeb tööd ainult raskusjõud P, siis $\delta A_1 = (-P \cdot l \cdot \sin \varphi) \delta\varphi$. Seega $Q_1 = -P \cdot l \cdot \sin q_1$. Pendli kineetilise energia saame valemist (vt. lk. 47)

$$T = \frac{I \cdot \omega^2}{2} \text{ ehk } T = \frac{I_o \cdot \dot{q}_1^2}{2}.$$

Lagrange'i võrrand omab antud juhul kuju

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_1}\right) - \frac{\partial T}{\partial q_1} = Q_1.$$

Kuna kineetiline energia T ei sõltu nurgast φ, siis arvutus annab, et

$$\frac{\partial T}{\partial q_1} = 0, \qquad \frac{\partial T}{\partial \dot{q}_1} = I_o \cdot \dot{q}_1, \qquad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_1}\right) = I_o \cdot \ddot{q}_1.$$

Asendades saadud suurused Lagrange'i võrrandisse saame

$$I_o \cdot \ddot{q}_1 = -P \cdot l \cdot \sin q_1.$$

. . . .

Figure 8.   Derivation of equation of motion for a physical pendulum

. . . .

Kuna jõud P on potentsiaalne, võib kasutada ka võrrandeid (2.44). Suunates z-telje vertikaalselt üles, saame potentsiaalseks energiaks $\Pi = P_z = -P \cdot l \cdot \cos \varphi$. Kasutades Lagrange'i funktsiooni (2.42)

$$L = \frac{I_o \cdot \dot{q}_1^2}{2} - P \cdot l \cdot \cos q_1.$$

Leiame osatuletised

$$\frac{\partial L}{\partial q_1} = -P \cdot l \cdot \sin q_1, \qquad \frac{\partial L}{\partial \dot{q}_1} = I_o \cdot \dot{q}_1, \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) = I_o \cdot \ddot{q}_1.$$

Asetades tulemused Lagrange'i võrrandisse, saame sama tulemuse, mis eelnevalt

$$I_o \cdot \ddot{q}_1 + P \cdot l \cdot \sin q_1 = 0.$$

Mathcadi interaktiivne näide. (*Mathcad interactive example*)

Figure 9.   Link to Mathcad interactive example [21]

Pendli nurga muutumise graafik
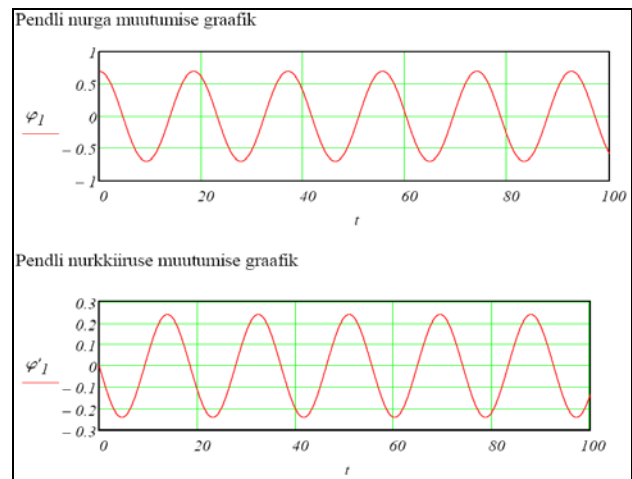
Pendli nurkkiiruse muutumise graafik

Figure 10. Pendulum angle (upper) and angular velocity (lower) change

Frames in Fig. 12 from [22] visualize the motion of a double pendulum determined by their equations of motion.

It is possible to hide equations and arguments on the worksheet of Mathcad. This gives a possibility to provide interactive sheets with controls only. In Fig. 13 there are the sliders to change the pendulum's mass and length. To change the initial angles of the pendulums we can use radio buttons on the worksheet of Mathcad (Fig. 13) and then visualize the motion of a double pendulum on the figure.

V. CONCLUSION

This paper presented opportunities to make learning materials more attractive and interactive using examples from subject analytical mechanics in Estonian University of Life Sciences.

The learning material has been made attractive and interactive by using different computer programs and internet.

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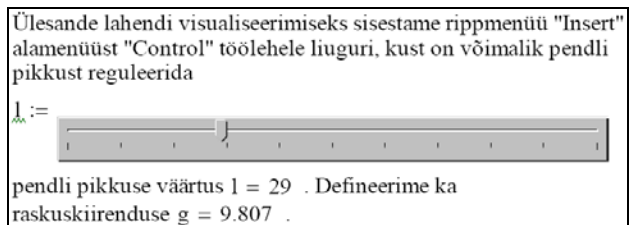
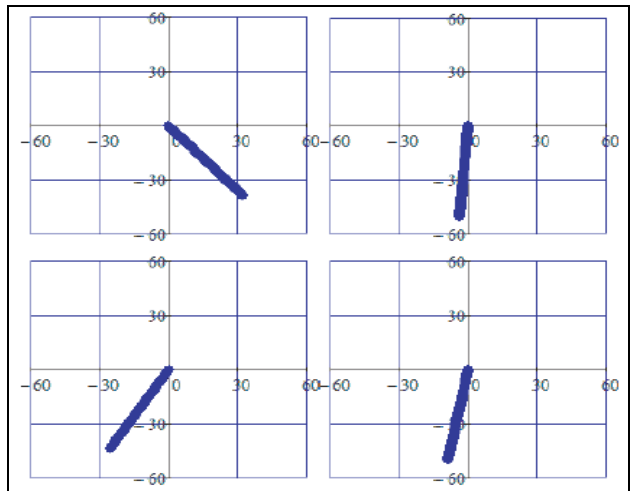Figure 11. A Slider on the worksheet of Mathcad, defining a pendulum length
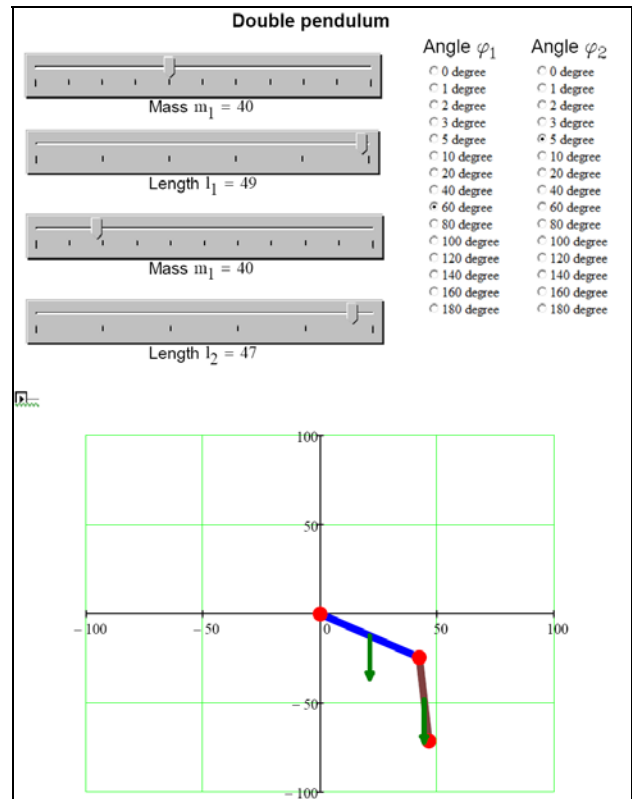

Figure 12. Frames from animated pendulum move

Figure 13. Slider and radio button controlled double pendulum [23]

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