Enhancement of Mathematical Conceptual Understanding in a Cloud Learning Environment for Undergraduate Students

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Benjamas Chimmalee, Anuchit Anupan(✉)
Sisaket Rajabhat University, Sisaket, Thailand
anuchit.a@sskru.ac.th

Abstract—Mathematical conceptual understanding is fundamental ability that students should have for learning practice in mathematics. This study presents a learning approach based on a development strategy for mathematical understanding in the cloud learning environment, to enhance undergraduate students' comprehension of mathematical concepts. A quantitative research design with a post-test only control group was deployed. All 56 undergraduate students enrolled on the Number Theory Course were involved in this study. The 28 students in the experimental group were taught using a mathematical understanding development strategy in a cloud learning environment, while the 28 students in the control group were taught using a conventional approach. The research instruments were an instructional plan and a mathematical conceptual understanding test. Data were analyzed by mean, standard deviation, and t-test. The results showed that the mean score of mathematical conceptual understanding of students in the experimental group was higher than the 60 percent criterion. There were statistically significant differences between the mean scores of the students in the experimental and the control groups in their mathematical conceptual understanding in the post-test in favor of the experimental group at a significance level of .05. The results of this study demonstrate the positive impact of a learning approach based on a mathematical understanding development strategy in a cloud learning environment for undergraduate students.

Keywords—mathematical conceptual understanding, cloud learning environment, undergraduate student

1 Introduction

Mathematics is one of the core subjects taught in higher education institutions. It plays a role as an educational tool in the study of science and technology. It is, therefore, essential to technological, academic, and social expansion and change [1]. Mathematics involves the use of thinking, which is the cognitive process of finding sensible solutions using related mathematical principles and algorithms [2]. In addition, mathematics has its own language, which is represented by symbols. The use of mathematical thinking enables students to manipulate complex points and expand their understanding.

Therefore, it can be said that mathematics is not only the study of basic concepts and skills, but that it also requires thinking skills, understanding, and the application of knowledge to problem-solving [3]. Mathematics problem-solving is a difficult skill for most learners to develop, due to the complexity of the problem-solving process [4]. Mathematics courses in higher education usually include a substantial quantity of content compared to the number of study hours [5]. The characteristic teaching and learning techniques at the higher education level are generally lectures or other conventional methods. When a problem or question has been given, the instructor usually explains or demonstrates solutions for solving it, rather than focusing on students practicing their understanding of mathematics [6].

Learning mathematics with understanding is recognized as an important factor that is significant for students in universities to develop [7]. Conceptual understanding is the comprehension not only of what to do, but also why it should be done. Learning mathematics with comprehension is important to enable students to apply principles in solving the future problems that will inevitably arise [8]. In addition, the approach to learning and teaching mathematics has shifted to an emphasis on conceptual understanding, which is consistent with the objectives of mathematics education [9, 10]. Therefore, the main goal of organizing mathematics learning activities is to focus on developing students' understanding of mathematical concepts, their ability to communicate, and their reasoning and problem-solving in order to gain knowledge [11]. As stated in [12], it is the development of a person's ability to use a mathematical concept in a variety of situations, both within and outside the general mathematics context. In other words, in teaching mathematics in the classroom, students should develop the ability to use mathematical concepts and mathematical reasoning, which are considered to be essential competencies. In addition, mathematical conceptual understanding results in better learning performance, in part due to the strategies applied in the classroom [13].

Mathematical understanding arises when people are able to define information clearly through mathematical thinking [14]. It is a response developed through problem-solving and reasoning by writing, explaining with text, images, and symbols, and the construction of mathematical concepts [15]. It is also an important basis for self-improvement using strategies to achieve the concepts of self-explanation, review, analysis, and self-assessment of learning ability [16]. Mathematics learning which focuses on understanding is more important than learning that focuses on memorizing [17–19]. If students have conceptual understanding, they will be able to remember facts and mathematical principles that can be used to build their knowledge. In addition, they will be able to connect existing knowledge and previous solutions to new methods of solving mathematical problems [20]. However, the development of mathematical understanding is a changing, dynamic process [21]. A great deal of research in this area has one common goal—to enhance learning with understanding. The organization of mathematics activities enables students to have more durable academic performance than other teaching models [12]. Likewise, as mentioned in [22], the importance of mathematical conceptual understanding as a reflection of academic performance is a key factor affecting mathematics achievement. However, there are several factors affecting students' mathematical understanding, such as learning strategies, learning materials, the organization of learning approaches, the learning process, and the learning environment [23].

Developments in recent decades have changed the traditional social and educational structures, and technology has played a significant role in the educational context [24]. With the change in social education and the advance of information technology, people working in the educational field have adapted and adjusted their conventional teaching and learning approaches [25, 26]. Thus, for the younger generation, studying mathematics should promote meaningful learning to students, with an emphasis on mathematics comprehension to develop the skills that are essential in the twenty-first century to be competitive internationally [10, 16].

Due to the spread of COVID-19, traditional learning methods may not meet the needs and inquisitiveness of undergraduate students [27, 28]. A learning environment with abundant resources such as social media, educational applications, and an appropriately designed learning process can foster students' curiosity [28–30]. It was necessary to use an instructional process, and advances in technology enabled educators to apply online learning environments in this situation [31]. Therefore, it has become essential to have a learning management system that can support learners to have access and learn at all times. Cloud technology has been widely applied to support the process [32]. Cloud learning environments (CLE) are among the significant types of technology in use, because of their functions for remote learning and education [2]. The possibilities of cloud technology provide service models and tools based on the students' unique requirements and needs [33, 34]. Also, applications that utilize cloud technological platforms are accessible from mobile devices or computers [35, 36].

It has been suggested that teaching mathematics with learning theory, but without any technological materials, does not reflect the learning outcomes of the students [37, 29]. According to [38], a combination of information and communication tools- GeoGebra, Google SketchUp and Microsoft Mathematics - to organize activities for teaching mathematics has a positive effect on students' mathematics achievement. A significant point for the pedagogy of mathematics instructors in digital times during the COVID-19 pandemic is promoting students to understand mathematical concepts in a CLE [39, 34]. Another challenge facing teachers from the impact of distance learning is the lack of technological equipment and insufficient training in new technologies [40]. Moreover, instructors are uncertain whether the course content is sufficient to adapt to digital media, leading to a need for research into proper learning objectives [41]. This involves some difficulties, such as the time spent, the anxiety that these tools are not suitable for the course, and so on [42]. Therefore, the key research problem in this study is whether cloud technologies, including the cloud services used in this approach to mathematics strategies, contribute to improving students' mathematical understanding. Thus, this study proposed an instructional approach that combined the strategy framework of mathematical understanding development with the technological tools displayed in the cloud.

The objectives of this study were (1) to study the mathematical conceptual understanding of undergraduate students using a learning approach based on a mathematical understanding development strategy in a CLE compared to a criterion of 60 percent, and (2) to compare the mathematical conceptual understanding of undergraduate students between groups using learning approaches based on a CLE and a conventional approach.

2 Related work

This section of the study gives a review of the literature to develop the instructional strategies used, and brief backgrounds on the importance of a cloud learning environment and mathematical conceptual understanding.

2.1 Overview of previous work on instructional strategy for the development of mathematical understanding

In general, mathematical content is often abstract [3]. The majority of mathematics students in our university had grades below the fair level in the past academic year, which is partly due to the teaching and learning process. Students' deficiencies are caused by trying to remember the algorithm, rather than trying to understand the meaning and mathematical principle [43]. If instructors focus on the steps in a calculation for finding a solution, or memorizing rather than teaching how to understand, students will not be able to think for themselves, because they do not understand the content and have no conceptual knowledge of the subject [19]. In our opinion, to improve achievement, the learning processes should provide students with a meaningful understanding, because mathematical understanding is an integral part of math achievement [10]. It is therefore important to discuss the techniques or strategies used to achieve such results. This subsection reviews the conceptual frameworks and strategies related to the development of mathematical understanding, in order to analyze the approaches used for learning activities to develop mathematical understanding. The main points are summarized as follows.

The strategies for the development of mathematical understanding concerning symbols and operations suggested by [44] include writing and expressing the meaning of symbols, the link between symbols and objects, learning to use the mathematical rules, methods and processes, and verifying solutions. As stated in [45], the strategies for the development of mathematical understanding are based on the link between mathematical knowledge and procedure. This follows [46]. The processes of understanding mathematical concepts proposed by Pirie and Kieren [47] consist of the following eight levels: primitive knowing, image making, image having, property noticing, formalizing, observing, structuring, and invention. Utilizing learning activities based on such approaches, the students' conceptual understanding of mathematics arises from building knowledge, understanding, and classifying and organizing information by themselves. The learning management guidelines to enhance mathematical understanding proposed by [10] consist of concrete and semi-concrete symbols and methods. Meanwhile, [48] suggested that a good mathematics teacher should pay close attention to students' mathematical thinking, by explaining how to construct an understanding of mathematics, focusing on the derivation of mathematical properties and structures, and addressing any misconceptions about mathematics.

Following Katz's framework [48], mathematical understanding depends on other factors, such as profound knowledge, reasoning ability, and personal features including learning style and anxiety about studying mathematics. Furthermore, encouraging students' interactions in the classroom is one of the key methods of supporting mathematics learning with understanding [49, 15]. The comprehensive mathematics instruction (CMI) framework introduced by [50], which emphasizes thinking skills and the inquiry process, consists of three main stages: preparation, learning activity, and summary. Research on the implementation of this framework found that undergraduate mathematics students using the CMI framework had higher mathematical skills than the criterion of 60 percent. They also had statistically significant higher mathematical skills after learning than prior to its use [51]. According to [52], the researchers proposed a model for the development of mathematical understanding based on the consideration of overall comprehension, which is level, dynamic, and non-linear, in which they applied the model in fractions.

However, the following gaps emerged from these previous studies, which have placed the emphasis on the theoretical teaching of mathematics, according to its structure, language, symbols and mathematical operations. Few studies have focused on the use of technological tools to support the learning process. This study explores cloud technology in relation to mathematics instruction, based on understanding development strategy.

2.2 Cloud learning environment and mathematics education

In this part, we discuss the reasons for, and the importance of, incorporating cloud technology, including the cloud environment, into the process of mathematics learning.

The cloud environment is the features of software operating on the Internet, based on cloud operating systems [53]. A CLE is a learning facility enabled by cloud learning services. The users are academics or students with the same privileges, which include controlling, choosing and sharing of content on the platform [54]. It is the integration between traditional and new teaching modes using the cloud as the center of learning management which allows instructors to manage their courses more easily, as well as to collaborate on learning activities between distant students in the same environment [30]. Due to the COVID-19 situation, which requires social distancing, the enhancement of students' learning experiences in a new digital environment is an unprecedented requirement for mathematics [55, 39]. CLE is essential to the educational process, as it is a learning environment where students do not have physically to meet with their peers and teachers [56]. The CLE has emerged as a new way of learning that gives learners the cloud-based tools to create their own learning environments, based on their needs and specific desires. It also offers flexibility in terms of time and place [57].

One of the most important advantages of CLE, as compared to traditional IT systems, is that the data are not restricted to any specific storage location. Applications and data are stored remotely in the cloud server, which can be accessed at anytime, anywhere, from any digital technology device [58]. Also, in CLE, the users do not need to set up a system, or to install additional functions, unlike other online platforms [57]. It relies on the cloud's features to work through the Internet [59]. CLE can be effectively employed in the teaching and learning process, whether with blended learning [60], a 5E inquiry model [31], or through integration with pedagogical content and knowledge [61]. Some studies have found that incorporating CLE into the learning process can effectively enhance students' academic performance, such as their creative skills [62], their critical thinking skills [63], and also their higher order thinking skills [31].

For mathematics education, cloud technologies can be utilized as tools to view mathematics, such as learning mathematical concepts [61], text-based theory and computer-aided visualization [41]. Digital technology has become part of a mathematics educator's supply of tools, knowledge and processes for making the teaching and learning process easier and smoother [64, 65]. [29] found that the learning environment is a key aspect affecting the conceptual learning behavior of students. Similarly, [66, 65] discussed how conceptual learning efficiency can be expedited if educational technology is used properly. In our view, this implies that technology and instructors have become indispensable for teaching mathematics in the digital age.

2.3 Mathematical conceptual understanding

Concepts are the foundation of human knowledge [22]. A mathematical concept is fundamental to the acquisition of mathematical knowledge, included in which there are other elements of mathematical knowledge based on generalization and skills in building, assimilating, and learning [9]. Conceptual understanding is one of the indicators of mathematical ability [67]. According to [46], mathematical understanding can be classified into two categories: conceptual understanding and procedural understanding. Conceptual understanding is the understanding of each process, whereas procedural understanding is the knowledge of the process of performance of the mathematical computational steps, without understanding the coherence or relationship of different processes and the connections between them [68]. It is important and affects the development of problem-solving skills [2]. It enables students to use mathematical terms, equations, and symbols in class and in daily life [69]. Additionally, it helps students avoid making numerous mistakes in solving mathematical problems. This enables them to become reasonably proficient in an unfamiliar problem-solving scenario [20]. Since mathematical processes are those that promote advanced thinking and problem solving, it is necessary to evaluate them with an emphasis on the growth of conceptual understanding [29]. From our perspective, the conceptual understanding of mathematics is therefore a reflection of the students' reasoning ability and understanding of mathematical concepts, relations, and operations. Several studies have emphasized the significance of conceptual understanding as well as deep conceptual learning in mathematics, as ways in which instruction and learning outcomes are developed [70, 29]. As supported by [71], conceptual skills are required for the modern mathematical profession. It is recommended to use various strategies to develop students' abilities, focusing on the growth of conceptual understanding [72, 73].

Mathematical conceptual understanding is the foundation of cognitive ability and manifested by the following behaviors: translation, interpretation, and explanation [2]. According to [50], students perform mathematical understanding when they can identify, label, and give relevant examples of concepts via reasoning, and use diagrams, symbols, or other mathematical representations to explore mathematical concepts. Following [74], there are three components of mathematical conceptual understanding: (1) the expressive ability to explain mathematical definitions, relationships, and the connections of mathematical concepts, (2) the interpretation of mathematical principles, theories, and rules, and (3) the ability to explain the reasoning of mathematical procedures/methods. Students who understand mathematical concepts will be able to explain mathematical definitions in different ways [20, 71]. Moreover, both concept and principle learning are based on Gagne's theory of learning. Similarly, [68] found that students study better when they are allowed to explore the relationship of concept groups in mathematical structure, in accordance with the approach of constructivist-oriented mathematics.

There are some dimensions of conceptual understanding in mathematics. Following [75], the measurement of mathematical conceptual understanding is a measure of the cognitive behavior of explanation, interpretation, application, perspective, empathy, and self-knowledge. The three levels related to understanding processes that are critical for the development of mathematical understanding [76] include an initial level of understanding, knowledge application and skill level. According to [77], the competences that indicate a conceptual understanding of mathematics consist of the ability to interpret vocabulary and mathematical symbols, propose the properties of predictable rules, and prove the relevant theorem and propose an effective solution to the problem. Furthermore, students will have gained a conceptual understanding of mathematics if they can present a wide range of concepts; identify and apply principles, facts and definitions, including comparing and integrating relevant concepts and principles; and recognize, interpret, and use symbols to represent concepts [8]. Based on the above theoretical background, the learning behavior that demonstrates a conceptual understanding of mathematics in this study consists of (1) the ability to describe and summarize the meaning of mathematical texts in various situations in the students' own words, (2) the ability to relate mathematical situations to mathematical operations, and (3) the ability reasonably to infer mathematical rules or principles.

According to the previous literature, this study is significant as it utilizes the teaching strategies based on emphasizing the development of mathematical understanding using a CLE to enhance undergraduate students' mathematical conceptual understanding. The premise of this current study is the combination of the approach with the cloud-based tools consistent with the current situation in the teaching and learning context.

3 Materials and methods

3.1 Research design

This study adopted a quasi-experimental design using a post-test only control group [78]. The subjects were randomly selected either to be given an intervention or not (experimental or control groups). The experimental group used a mathematical understanding development strategy in a CLE, whereas the control group were taught using a conventional approach. The mathematical conceptual understanding is determined after the intervention in order to determine its effects.

3.2 Subjects

The purposive subjects consisted of 56 undergraduate students who were enrolled on the Number Theory course, Faculty of Education, Sisaket Rajabhat University. Both the experimental class and the control class comprised 28 students.

3.3 Research procedure

The development of the learning approach involved the following steps. (1) The study of mathematics education theory, and research on mathematical understanding, plus teaching approaches for the development of mathematical understanding. (2) Analysis of appropriate keywords that served as a conceptual framework for the learning behavior expressing conceptual mathematical understanding. (3) Analysis of suitable frameworks to synthesize the learning management process to enhance mathematical understanding.

The theoretical perspectives for synthesizing the process of learning to support our approach were developed from [44], Pirie–Kieren's theory of the growth of mathematical understanding [47], Katz [48], Hilton et al. [49], the definition of acting and expressing in eight levels of understanding in the Pirie–Kieren model [77], and Sabry [14], as shown in Table 1.

Table 1. The analysis and synthesis of approaches for developing conceptual understanding in mathematics

Strategy	References						Learning Phase
	[44]	[47]	[48]	[49]	[14]	Expressing Mathematical Understanding [77]	
Review of primitive knowledge		✓		✓	✓	Using primitive knowledge for new learning	1. Review of primitive knowledge of mathematics symbols
Connection of mathematical symbols and thinking	✓				✓		
Image making		✓				Create new images from doing, explaining properties of the picture by noticing and linking concepts	2. Observation and discussion of properties, definitions, rules and theorems
Image having		✓					
Property, rule noticing	✓	✓		✓			
Mentioning misconceptions			✓	✓	✓		
Formalizing		✓					
Observing, reflecting on the learning outcomes		✓	✓	✓	✓		
Structuring		✓	✓			Able to prove and apply theorems	3. Application of mathematical rules and theorems
Inventing		✓		✓			
Computation from mathematical definition and formula	✓			✓	✓		
Summarize lesson				✓	✓		

The adoption of the learning approach synthesized in Table 1 with CLE involves the main dimensions following: (a) during class preparation, the instructor made the mathematics lessons available in Google Classroom, including for asynchronous activity; (b) the learning stage, in which students follow the steps to organize learning activities. Table 2 describes how the learning approach is implemented in CLE and Google Meet, as video conferencing is utilized for virtual face-to-face learning activities (in other words, it is digital face-to-face learning in the form of synchronous mode). These allow students and instructors in the classroom to see each other, share documents and visualizations, communicate through audio, or chat and engage in individual or in group interactive activities.

Tabel 2. Syntax of learning approach based on a mathematical understanding development strategy in a cloud learning environment

Learning Phase	Learning Activities	Indicator of Development of Mathematical Understanding
1. Review of primitive knowledge of mathematical symbols	Fundamental mathematics symbols are introduced. An exploration of facts and mathematical vocabulary. Students are encouraged by questions requiring basic knowledge related to the lesson. Students review and explore mathematics materials via Google Classroom and YouTube.	Describe basic related knowledge for learning new concepts, list and explain relevant facts and data.
2. Observation and discussion of properties related to definitions, rules, and theorems	The stimulation of students to attempt to predict properties of definitions, rules, and theorems or the problem that needs to be answered. The practice of connecting images or symbols that represent concepts together or considering the differences between the images. Students were able to interact about observable conclusions and properties. Students were asked to prepare Google slides and present these in the class, share them via the CLE, and then store them in Google Drive.	Explain the derivation of mathematical principles, express predictable properties of mathematical rules and principles that are fundamentally related to the theorems and that need to be proved.
3. Application of rules and theorems	Inferring the theorem obtained from the mutual relation of the observed results. Once the students had undertaken a reflective review, the instructor guided them in the practice of proving mathematical rules and theorems. Mathematical statements and theorems were given for the students to prove in this phase. Students were provided with the opportunity to share their ideas and reasons for the proof with their peers. Here, the students engaged the cloud tool facilities—Google Jamboard prepared in the learning environment.	Prove theorems and apply relevant theorems to solve problems reasonably.

3.4 Research instruments

The experimental materials constructed by the authors were instructional plans using a learning approach based on a mathematical understanding strategy in a CLE. The research instrument used for data collection was a test of mathematical conceptual understanding. This test was constructed by the authors based on the concepts related to Number Theory in three topics—divisibility, prime numbers, and congruence. There were a total of six questions and a total score of 100 (this accounts for 40 percent contributed to their final grades). It was a subjective test based on the indicators to examine conceptual understanding in this study, two questions to test students' ability to give a definition related to such a domain in their own words, two questions to test students' ability to describe mathematical situations in mathematical operations, and two questions to test students' ability to reason a mathematical proof.

Regarding the validity of the test, the contents and scoring scale were reviewed and approved by three experts selected for their academic qualifications and expertise in teaching mathematics. The revisions were made according to the experts' recommendations before actually being applied to the subjects. The difficulty of the test items ranged from 0.21 to 0.71, which were considered appropriate. The discriminant value ranged between 0.28 and 0.80, which were suitable values, and the reliability of the test, calculated by Cronbach's alpha, was 0.71.

3.5 Research implementation

The study was implemented on 56 undergraduate students and was employed in the context of the Number Theory Course.

To verify there was no difference in the two classes of subjects before the experiment, in the initial stage of conducting the study, they were given a pre-test to determine their mathematical conceptual understanding. The scores were tested for variance using the F-test. This found that the variances between the two groups were not statistically significant. The difference in mean scores of the students from both the experimental and control groups was investigated using a t-test (as shown in Table 3).

Table 3. The analysis of mean scores of mathematical conceptual understanding for the pre-test of two independent subjects

Subject Group	N	Mean	S.D.	t	Sig.
Experimental	28	36.10	1.66	1.797	.078
Control	28	37.00	2.03		

Table 3 shows the analysis using an independent sample t-test for comparison of the mean scores in the pre-test of mathematical conceptual understanding for both the experimental group and the control group. The t value was 1.797 and the significance value was .078; accordingly, the mean of the students in both groups was not significantly different at the .05 level. The result signified that there was no difference in the mathematical conceptual understanding of the two groups before conducting the experiment. The experimental group was then randomly nominated as class 1 and the control group as class 2.

The students in the experimental class followed a learning approach based on a mathematical understanding development strategy in a CLE. Regarding the experimental group, the process of organizing mathematics classes for each lesson included three phases, as given in Table 2. The students in the control class followed a conventional approach. After completing the course, all subjects in both groups were tested again with the same test to assess their mathematical conceptual understanding (post-test).

3.6 Data analysis

Data were obtained through the test of mathematical conceptual understanding, which involved a ratio scale. The quantitative data of the scores were analyzed using quantitative methods. To ascertain the significance of the findings, normality tests had been performed previously, which found that the data were approximately normally distributed. The descriptive analysis of the mean and standard deviation (S.D.) for post-test scores of students in the experimental group was reported and was then compared with the 60 percent criterion using a one-sample t-test at the statistical significance level of .05. The difference in mathematical conceptual understanding scores of the post-tests between the experimental group and the control group was analyzed using mean and standard deviation and determined by independent sample t-test at the statistical significance level of .05.

However, the quantitative approach uses the test with the scope of the course that the study has conducted. The duration of this study was one semester. The analysis of data expresses the overall ability to understand concepts in the three domains of number theory, which is the study's limitation in terms of content.

4 Main results

This study presents a learning approach as displayed in Table 2. The study of the conceptual understanding of mathematics of students in the experimental group was higher than the threshold of 60% or not, given in Table 4. The result of the comparison of the two research groups is showed in Table 5.

Table 4. Analysis of post-tests of mathematical conceptual understanding compared with the required criterion

Group	N	Mean	S.D.	Std. Error Mean	t	Sig.
Experimental	28	72.77	5.54	1.04	12.19	.000*

Note: *$p < .05$, test value is 60.

Table 4 shows the descriptive statistics of mathematical conceptual understanding that the undergraduate students achieved after using the development strategy in a CLE. The post-test of the mean mathematical conceptual understanding score of undergraduate students in the experimental group was 72.77 with a standard deviation of 5.54. By using a one sample t-test, it was found that the students had a statistical significance 60 percent higher than the threshold at the .05 level.

To examine the effect of the learning approach based on the students' development of their mathematical understanding using a CLE, the comparative results between the experimental and control groups are presented in Table 5.

Table 5. The mean scores of mathematical conceptual understanding for post-tests according to learning approach

Group	N	Mean	S.D.	Std. Error Mean	t	Sig.
Experimental	28	72.77	5.54	1.04	6.67	.000*
Control	28	57.64	10.64	2.01		

*Note:**$p < .05$.

Table 5 displays t-test analysis for post-testing of mathematical conceptual understanding. The mean score for the experimental group was 72.77, which was higher than that for the control group, which was 57.64. The mean difference was 15.13. Using independent sample t-test, the t value was 6.67, with the significance value of .000. There was a statistically significant difference between the experimental group and the control group for the post-tests at $p < .05$. As a result, the learning approach based on a mathematical understanding development strategy in a CLE had a statistically significant positive influence toward mathematical conceptual understanding.

5 Discussion

In this study, the integration of strategies of mathematical understanding development into a CLE has been considered as an instructional approach to study undergraduate students' mathematical conceptual understanding. This quantitative study attempts to investigate the undergraduate students' mathematical conceptual understanding using a learning approach based on a mathematical understanding development strategy in CLE, and compared students to that of a conventional approach. Based on the findings, after application of the CLE teaching approach to the experimental group, they had a mathematical conceptual understanding significantly higher than 60 percent. This accords with [18, 21, 51]. The results of the study show that the proposed learning approach is effective in enhancing mathematical conceptual understanding. The authors attribute these findings to the following reasons. The pedagogical framework used in this study emphasized that students constructed meaningful knowledge themselves. Students recognized the stimulus experienced and used their prior knowledge to understand it, such as mathematical definitions and symbols. Some learning activities in this approach – such as observations, considering mathematical properties, differences, relationships of images, diagrams, symbols, and equations – also corresponded to educational theory in the context of learning the rules and principles of Gagne's theory of learning. Moreover, students played a role in practicing finding explanations and observing the prediction of properties based on mathematical problems. The connection of students' new knowledge with prior knowledge with the support of cloud tools was a key component that helped to develop their conceptual understanding [43, 46, 65].

The findings indicate that there was a significant difference between the experimental group and the control group in the post-test. This implies that learning using the proposed approach was better at developing the students' mathematical conceptual understanding than the conventional approach. This is consistent with the study by [2], who developed conceptual understanding by deploying a prediction, discussion, explain, observe, discuss, explain strategy (PDEODE) supported by an e-learning environment. According to [12, 68], students have to be exposed to building their representation of the conceptual structure of a given situation via concept mapping. Also, this follows [51], who found that undergraduate students who were taught using a CMI framework had better mathematical skills. This strategy incorporates cloud-based tools and an environment that provides students with deep, independent learning. In terms of the CLE, this conforms with [31], who deployed Moodle as a learning environment with a 5E inquiry model, which found that cognitive ability can be improved by using appropriate technologies. Furthermore, students can research using tools to explore their knowledge via cloud learning utilizing various resources. Previous works found similar findings [38, 39, 53, 65]. This finding suggests that the learning environment affects students' learning performance [23, 29, 55].

This study is intended to employ the approaches of developing understanding in teaching and learning mathematics through CLE based on a framework from the literature, our mathematics teaching experience, and platform conditions on the use of information and communication technologies within the university. According to the proposed learning approach, students have to communicate and explain their thinking. Cloud-based learning environments provide privacy while learning [28, 41] and give students constructivist learning [68]. These may affect their confidence in learning and encourage their mathematical conceptual understanding. From the practical implementation, we have deduced that cognitive abilities – e.g., mathematical conceptual understanding – can be enhanced by combining the strategy of a learning approach with appropriate cloud technologies, tools and services. The exploration and understanding of mathematical concepts, which ultimately affects learning performance, is made easier with the assistance of technology, including CLE [38, 60]. This may also make students more inquisitive about mathematics.

6 Conclusions

This study analyzes the learning models, strategies, and modules involved in developing mathematical understanding in order to analyze and synthesize them as guidelines for learning phases. As more learning techniques and the modern cloud-based platform for the adaptation of learning activities get incorporated into each learning phase, such an approach should complement other educational theories. This was a learning approach utilized in a CLE, developed for this study to measure conceptual understanding in mathematics among undergraduate students. The study was guided by the following two research questions: whether the post-test scores of mathematical conceptual understanding of the undergraduate students exceed the criterion of 60%, and whether there are differences in the post-test scores of mathematical conceptual understanding for the two subjects groups. The quasi-experimental approach was conducted.

The purposes of this study were: to study the understanding of the mathematical concepts of students using the proposed learning against a 60% threshold, and to compare the mathematical conceptual understanding of students between an experimental and a control group.

In this study, we answered the research questions. The results revealed that the experimental group significantly outperformed the 60% threshold, and there was a significant difference between the mean scores of students in the experimental group and their peers in the control group. The experimental group significantly outperformed the control group on the post-test. The above experimental findings have demonstrated that the learning approach based on a mathematical understanding development strategy in a CLE can be significantly effective in enhancing undergraduate students' performance in mathematics courses. Students learn better when using this approach.

As education had to be moved online because of COVID-19, our findings will contribute to the employment of cloud technologies as tools to promote the learning environment and to benefit students' conceptual understanding in the mathematics class. In this situation, cloud technology is interpolated in mathematical instruction for significant improvement in the understanding of mathematical concepts. It could be used as a practical teaching tool in mathematics education. Also, it integrates theoretical point of views to support how our approach affects the development of mathematical understanding.

The important recommendations put forward by this study are that instructions for developing mathematical conceptual understanding through mathematics activities supported by a CLE should focus on both teaching content and application, and not rely on any specific algorithm. There should be further study of what type of mathematical content is suitable for developing the conceptual understanding of mathematics. Different methods of teaching and learning should be studied and compared to ascertain which are more effective in developing students' mathematical conceptual understanding.

In future work, the authors suggest conducting research in the following areas: enhancing other variables, such as learning outcomes; critical thinking using the proposed approach; the study of the integration of some modern mathematical technological tools together with teaching strategies in cognitive domains; and, the study of cloud-based learning materials to promote conceptual understanding among students at other educational stages.

7 References

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8 Authors

Benjamas Chimmalee is an assistant professor at Department of Mathematics Faculty of Education, Sisaket Rajabhat University, Sisaket, Thailand. She received her Ph.D. in Mathematics from Mahidol University, Thailand in 2017. Her research interests include mathematics education, using technology to support teaching and learning mathematics (email: benjamas.c@sskru.ac.th).

Anuchit Anupan is a lecturer at Department of Computer Education, Faculty of Education, Sisaket Rajabhat University, Sisaket, Thailand. He received his Ph.D. in Information and Communication Technology for Education, King Mongkut's University of Technology North Bangkok, Thailand in 2016. His research interests include cloud technology, information and communication technology for education (email: anuchit.a@sskru.ac.th).

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