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REPORT

Assessing Difficulty Levels of Mathematical Tasks through Subjective and Behavioral Criteria

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ABSTRACT

This paper aims to assess the difficulty levels of mathematical tasks in relation to the appearance of unconscious patterns of thinking in students' cognitive processes that play a role in learning. The data analyzed using a mixed-methods approach was collected from a population of undergraduate engineering and science students enrolled in Calculus courses at the university while answering three questionnaires as part of their online class activities. Two criteria were used to find categories of difficulty levels: one subjective, given by an evaluation carried out by the subjects, and another behavioral, related to obtaining the correct answers. The relationships of these criteria with the appearance of these unconscious patterns of thinking were identified: a significant and strong correlation was noticed between the number of erroneous unconscious patterns detected and task difficulty levels determined by the percentage of correct answers, as well as a significant and strong correlation between task difficulty levels determined by the subjective evaluation and the number of these patterns recognized. Based on the results obtained, it can be stated that these unconscious erroneous patterns in students' reasoning about a mathematical concept are related to the index of the difficulty of a task and could be considered indicators of mental effort according to the cognitive load theory. The analysis showed the recognition of these unconscious patterns in students' cognitive mechanisms is relevant when solving mathematical tasks that require information processing at a higher level and could play a role in assessing the difficulty levels of a task related to the study of mathematical concepts in Calculus courses, which constitutes the main novelty of this study.

KEYWORDS

cognitive load theory, implicit learning, difficulty assessment, advanced engineering mathematics, Calculus

1 INTRODUCTION

Although the role of conscious and unconscious cognitive mechanisms that develop in the learning processes of any subject has been studied in psychology, it

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remains one of the most challenging problems in educational studies [1]. Several studies show that unconscious cognitive mechanisms can lead to unconscious knowledge acquisition [2–6]. Furthermore, these unconscious cognitive mechanisms could be structurally and functionally more complex than conscious ones and could freely affect conscious learning processes [2]. Thus, many of the difficulties engineering and science students encounter while learning different mathematical concepts at university are due to unconscious models that act without awareness and control in their reasoning processes [5].

Some researchers state that in the context of mathematics in higher education, specifically in Calculus courses for engineering and science, students are not prepared to handle concepts not derived from experience [7, 8]. Consequently, when trying to understand an abstract mathematical concept, students may unconsciously produce schemas or models derived from their experiences, giving some practical and unifying meaning to this concept, which later tends to substitute the original one in their reasoning processes unconsciously. In most cases, those models are suggested by the initial schemas formed through the empirical reality from which the mathematical concept has been abstracted and which are automatically and unconsciously relied upon for subsequent abstract mathematical constructions, long after they should have been replaced for more complex, elaborated, and adequate schemas [6].

Related to the above and based on Polanyi's tacit knowledge [9], Fischbein [6] claims that some types of these intuitive, unconscious patterns of thinking (that he calls *tacit models*) appear when students deal with too abstract, difficult, or complex mathematical concepts. In these cases, he argues, they automatically resort to these elementary mental models or schemes that help them represent these complex concepts in a simplified way. According to him, these schemas formed at the beginning of the learning process sometimes become implicit or tacit, unconsciously affecting the learner's subsequent comprehension and resolution strategies, most of the time in misleading or erroneous ways.

In the present paper, we consider *tacit models* defined by Fischbein related to the study of infinity and the limit concept in the context of undergraduate Calculus courses for engineering and science students. More explicitly, we consider some unconscious representations of these mathematical abstract notions that constitute obstacles to their proper understanding [6].

On the other hand, some authors affirm that the study of these unconscious patterns of reasoning is a key to understanding difficulties that arise in this case in terms of the cognitive effort required [10]. In this setting, the difficulty levels of a task, which correspond to the mental effort exerted to solve it, can be measured using subjective criteria that consider the subjects' perception of the complexity of the activity performed and behavioral criteria related to the objective performance in that activity [11]. In this work, these criteria are used to assess the difficulty levels of the tasks being solved in relation to the *tacit models* appearing in engineering and science students' cognitive processes related to the study of infinity and the limit concept in the context of Calculus courses.

Although *tacit models* have been examined before in different contexts and from diverse perspectives [6, 12, 13], they have never been considered for the assessment of difficulty levels of mathematical tasks relevant to Calculus courses in undergraduate engineering and science programs. Thus, to address this research gap, the following research questions arise:

1. What correlations can be found between the *tacit models* observed, the subjective evaluation of the tasks' difficulty levels, and the percentage of the correct answers given by undergraduate engineering and science students?
2. Can these models be used to assess the difficulty levels present in a task related to the study of these concepts?

2 BACKGROUND

Mathematics at the undergraduate level for engineering and science students is highly abstract; in this context, substantial mental activity is required to solve mathematical tasks [14]. Many factors influence task-solving processes and their difficulty levels in mathematics, and some of these factors are related to cognitive load theory [15]. This theory encompasses all widely accepted theories about how the human brain processes and stores information [16].

In general, the difficulty of a task could be assessed in terms of the cognitive effort needed to solve it. More precisely, the concept of cognitive load is related to the cognitive mechanisms that develop and could indicate difficulty levels during the resolution of a task. Cognitive load theory states that knowledge accumulated through learning processes is stored in long-term memory through schemas that organize it in a way that allows it to be remembered and used later in future learning processes [15]. According to this theory, learning develops through transforming and combining different schemas, leading to increasingly complex schemas. These processes of transformation into higher-level schemas occur continuously during the development of the cognitive processes. Automation becomes an important process during the construction of these increasingly complicated schemes, through which information is processed automatically with minimal conscious effort [17]. Thus, according to this theory, automatic processing of information is mostly unconscious, which is verified by the empirical evidence indicating that we are unaware of how the brain organizes information [1].

3 MATERIALS AND METHODS

3.1 Sampling

The data analyzed was collected in a time interval of approximately six months during the academic year 2021, from different samples of a population of 304 undergraduate science and engineering students enrolled in eight different Calculus courses at the university who gave their written consent to participate in the study. Students were aged between 18 and 25 years old. These participants were attained randomly based on the convenience sampling technique [18]. This form of sampling allows researchers to distribute and collect questionnaires according to both parties' convenience.

3.2 Survey instruments

The three questionnaires used to collect the data (see Figures 1, 2, and 3) were designed considering *tacit models* according to Belmonte and Sierra [12] and validated according to the usual criteria set by the current bibliography [19]. For this analysis, we consider *tacit models* associated with infinity and the limit concepts [20]. Table 1 shows the models considered.

Table 1. *Tacit models* considered in the analysis

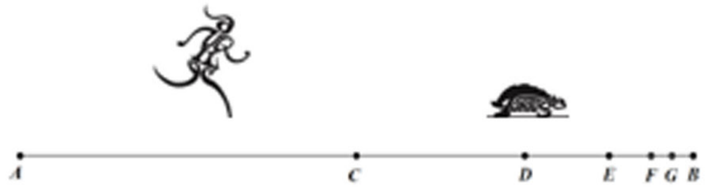
Tacit Model	Erroneous Assumption
Undefined	An infinite sum cannot be calculated due to the undefined number of terms.
Divergence	The result of the infinite sum of finite quantities cannot be finite.
Unreachable	The limit is a value that cannot be reached.
Inexhaustible	An infinite sum cannot be calculated because it is always possible to continue adding terms.
Point-mark	Points can be identified with marks in the geometric line.
Dependency	Convergence depends on numerical distances that can be identified with segments viewed as geometric spaces.
Squeezing	All infinite sets are of the same size.
Inclusion	A part of an infinite set must be smaller than the whole of the set.
Slipping	There is no one-to-one correspondence between an infinite set and its proper subset.
Infinite-unbounded	An infinite set must be unbounded.
Bounded-finite	A bounded set must have a finite number of elements.
Bounded-unbounded	An unbounded set must have more elements than a bounded set.

Questionnaires were individually answered as part of the online classes' activities in the context of the COVID-19 health emergency.

To gather more information that would allow us to observe these models in students' cognitive mechanisms, in each question they were asked to explain their answers (see Figures 1, 2, and 3).

This famous paradox, credited to the Greek mathematician Zeno of Elea (490-430 BC), can be stated, for this study, as follows: Achilles, the Greek hero of the Trojan War, decides to compete in a race with the slow Tortoise, along a straight path of non-zero length from point A to point B. As Achilles doubles the Tortoise's speed, he, in a gesture of generosity, allows her to start the race in the middle of the path, that is, at point C as seen in Figure 8, so that AC = CB = AB/2. Given this situation, consider:

1. When Achilles reaches point C, the Tortoise has already moved to point D, but when Achilles reaches point D, the Tortoise has already moved to point E. In turn, when Achilles reaches point E, the Tortoise has moved to point F, and when Achilles reaches point F, the Tortoise has already moved to point G.



The following table shows the total distance traveled by Achilles from steps 1 to 5.

Step	Total distance traveled by Achilles
1	$\frac{\overline{AB}}{2}$
2	$\frac{\overline{AB}}{2}+\frac{\overline{AB}}{4}$
3	$\frac{\overline{AB}}{2}+\frac{\overline{AB}}{4}+\frac{\overline{AB}}{8}$
4	$\frac{\overline{AB}}{2}+\frac{\overline{AB}}{4}+\frac{\overline{AB}}{8}+\frac{\overline{AB}}{16}$
5	$\frac{\overline{AB}}{2}+\frac{\overline{AB}}{4}+\frac{\overline{AB}}{8}+\frac{\overline{AB}}{16}+\frac{\overline{AB}}{32}$

Continuing this iterative process, what will be the total distance traveled by Achilles in the *nth* step?

2. What will be the total distance traveled by the Tortoise in the n^{th} step?

3. What will be the distance that separates them in the n^{th} step?

4. Will Achilles ever catch up with the tortoise? Explain your answer.

5. Is it possible to calculate the infinite sum $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots+\frac{1}{2^n}+\ldots$? Explain your answer.

6. Is it possible to verify the sum $\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots+\frac{1}{2^n}+\ldots=\frac{1}{2}$? Explain your answer.

7. Given your answers to the questions above, will it happen that $\frac{1}{2^n}=0$ at some point? Explain your answer.

8. Finally, who will win the race? Explain your answer.

Fig. 1. Questionnaire on *Achilles and the tortoise*

The Cantor set is an outstanding fractal subset of the real interval [0, 1], which admits a geometric definition, of a recursive nature. It can be constructed by eliminating at each step the open interval corresponding to the central third of each interval; that is, this set is built recursively following the steps:

1- Remove from the interval [0,1] the open interval $(\frac{1}{3}, \frac{2}{3})$.
2- To the remaining two intervals $[0, \frac{1}{3}]$ y $[\frac{2}{3}, 1]$, remove the open intervals $(\frac{1}{9}, \frac{2}{9})$ and $(\frac{7}{9}, \frac{8}{9})$.
3- To the four remaining intervals, remove the four open intervals that constitute their respective central thirds, and so on, in the *nth* step, to the remaining 2^{n-1} intervals, their respective interior thirds formed by their central open intervals, will be removed.

Figure above shows the first six steps of the iterative process, starting from the closed interval [0, 1]. The Cantor set is the set of all remaining intervals. Considering this construction, answer the following questions:

1	Does the iterative process by which the Cantor set is built come to an end? Explain your answer.
2	How many intervals will there be in the interval [0, 1] after *n* steps? Explain your answer.
3	Based on the above, how many elements will remain in the set after *n* steps? Explain your answer.
4	Following a similar reasoning and considering intervals as sets of points, in which of these intervals will there be more points? Explain your answer. 
5	**On which of these curves will there be more points? Explain your answer.** 

Fig. 2. Questionnaire on *Cantor set*

Below are presented the first six iterations of the process by which the fractal known as the Sierpinski triangle is constructed. Observe that, starting from an initial equilateral triangle, a process similar to the construction of the Cantor set is followed, i.e., this triangle is constructed by eliminating the central equilateral triangles that are formed by taking the midpoints of each of the sides of the triangles which are gradually formed in the iterative process.

Based on the above, answer the following questions:

1	Will this iterative process come to an end? Explain your answer.
2	If the initial triangle has an area equal to $1u^2$, what would be the remaining area after *n* iterations? Explain your answer.
3	Continuing with the iterative process, what will the area of the "final" triangle be equal to? Explain your answer.
4	Following the same reasoning, what will the perimeter of the "final" triangle be equal to? Explain your answer.

Fig. 3. Questionnaire on *Sierpinski triangle*

3.3 Data analysis

A mixed research approach was used [21]. For each questionnaire, the data was analyzed using two criteria: one subjective, given by the evaluation of the difficulty level of the questions given by the students; and a behavioral one, given by the percentage of correct solutions.

Finally, relevant correlations were sought between the difficulty levels given by the subjective evaluation of students for each question, the percentage of correct solutions obtained, and the number of these *tacit models* observed in their arguments by the qualitative analysis.

3.4 Difficulty category levels: A subjective evaluation

After answering each question, students evaluated the difficulty level encountered by answering a survey with the following statement: "Evaluate on a scale from 0 to 10 (where 0 – very easy, 10 – very difficult) to what extent the question was easy – difficult." The difficulty level was measured using the eleven-point Likert scale, and on its basis, the categories were determined (refer to Table 2).

Table 2. Categories for difficulty levels (subjective criterion)

Range of Points	Difficulty Levels
0–2	very easy (1)
3–4	easy (2)
5–6	moderately difficult (3)
7–8	difficult (4)
9–10	very difficult (5)

3.5 Difficulty category levels: A behavioral criterion

To objectively determine the difficulty levels of the questions, the percentage of correct answers was used (behavioral criterion). Table 3 shows the scale used in this case.

Table 3. Categories for difficulty levels (objective criterion)

% Correct Answers	Difficulty Levels
0–19	very difficult (5)
20–39	difficult (4)
40–59	moderately difficult (3)
60–79	easy (2)
80–100	very easy (1)

4 RESULTS

This section shows the results of the analyses carried out for each of the three questionnaires considering the two criteria mentioned above.

4.1 Questionnaire *Achilles and the tortoise*

The subjective evaluation of difficulty levels given by students in this case is shown in Table 4. Among the questions presented in this questionnaire, none were categorized as "very easy" or "very difficult." Note that question No. 7 was the only one considered "difficult" and that it was perceived as having a higher level of difficulty than the rest.

Table 4. Difficulty levels given by students (N = 156)

Question	Mean	S.D.	Difficulty Levels
No. 1	2,84	2,00	easy (2)
No. 2	3,41	1,88	easy (2)
No. 3	3,8	2,2	easy (2)
No. 4	4,35	2,00	moderately difficult (3)
No. 5	3,97	2,15	easy (2)
No. 6	3,95	2,16	easy (2)
No. 7	6,51	2,1	difficult (4)
No. 8	5,93	2,41	moderately difficult (3)

The percentage of correct answers obtained (behavioral criterion) is shown in Table 5. The lowest percentage of correct answers was obtained for questions No. 8 (16.02%) and No. 7 (16.67%), while the highest percentage of correct answers was found in question No. 1 (65.38%).

Table 5. Percentage of correct answers as a criterion of difficulty level (N = 156)

Question	% Correct Answers	Difficulty Level
No. 8	16,02	very difficult (5)
No. 7	16,67	
No. 6	30,77	difficult (4)
No. 4	28,21	
No. 5	41,03	moderately difficult (3)
No. 3	44,23	
No. 2	43,59	
No. 1	65,38	easy (2)

The qualitative analysis of students' arguments and explanations shows the *tacit models* adopted: *inexhaustible* as an infinite number "cannot be calculated"; *undefined* due to the lack of definition that an infinite number supposes; *divergence* because "it always adds up" or "it keeps on adding up." In close relation to these three previous models, the model *unreachable* is observed too, as well as *dependency*, when the segment is associated as a geometric space with a numerical distance. In the same way, due to geometric limitations, the *bounded-finite* model appears. The distribution of *tacit models* throughout the questionnaire is shown in Table 6.

Table 6. *Tacit models* (*Achilles and the tortoise*) (N = 156)

Question	*Tacit Models*
No. 1	–
No. 2	–
No. 3	–
No. 4	*inexhaustible, dependency, divergence, undefined*
No. 5	*inexhaustible, divergence, undefined, unreachable*
No. 6	*inexhaustible, divergence, undefined, unreachable*
No. 7	*inexhaustible, divergence, undefined, unreachable*
No. 8	*inexhaustible, dependency, divergence, undefined, bounded-finite*

Next, a comparison of difficulty levels (according to the two criteria) and the number of *tacit models* observed is presented in Figure 4.

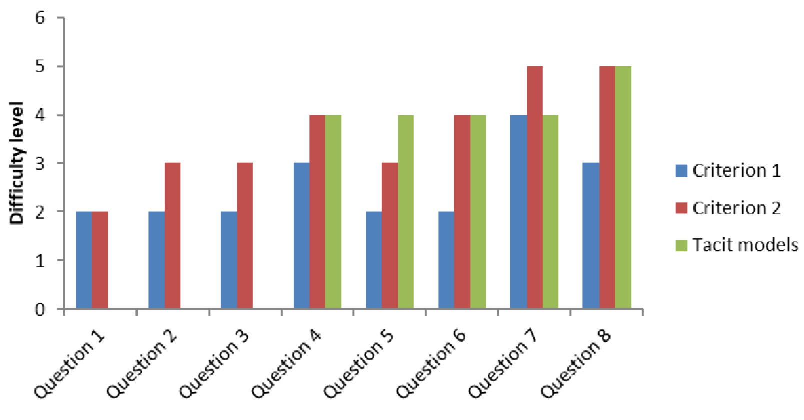


Fig. 4. Difficulty levels according to the two criteria, in comparison to the difficulty level indicated by the number of *tacit models* (*Achilles and the tortoise*) (N = 156)

Let us recall that criterion 1 refers to the subjective evaluation of difficulty levels given by the students, while criterion 2 refers to difficulty levels according to the percentage of correct answers obtained in each question.

The results obtained based on the percentage of correct answers consistently show that students perceive the questions as easier than they are, according to the percentage of correct answers obtained. The biggest differences occurred for questions No. 6 and No. 8. Question No. 6 was "easy" according to the perception of the students, although it obtained a low indicator of correct answers, according to which it is categorized as "difficult." A similar contradiction occurred in the case of question No. 8, which according to the percentage of correct answers was considered "very difficult," but according to the perception of the students, it is only a "moderately difficult" question.

Question No. 7, evaluated as the most "difficult" according to the subjective criterion, presents four *tacit models* showing that students perceive the complexity of the question. Something similar occurs with the questions evaluated as "moderately difficult" (No. 4 and No. 8), where four and five (respectively) of these models are observed. Let us also note that in this case the "easy" questions (No. 1, No. 2, and No. 3) are not conducive to the appearance of any model.

Thus, it is observed that the correlation between the mean difficulty level given by the students and the number of *tacit models* present in their reasoning for each question is strong and significant (r = 0.70, $p < 0.05$), showing that students actually perceived the complexity of the question in each case, which is precisely what makes them resort to these simplified models of reasoning.

On the other hand, the distribution of the results obtained according to the percentage of correct answers also reflects difficulty levels of the questions by the number of *tacit models* observed. Indeed, the correlation between the percentage of correct answers and the number of these models present in each question is strong and significant (r = –0.80, $p < 0.05$). Thus, the greater the number of *tacit models* that appear in the reasoning of the students in a question, the lower the percentage of correct answers obtained, corroborating in this way that the use of these models represents obstacles in the proper understanding of the mathematical concept under study.

The correlation between the mean difficulty levels given by the students and the percentage of correct answers is strong and significant (r = –0.90, $p < 0.05$), which implies that students actually find these questions more difficult when they make the most mistakes.

4.2 Questionnaire *Cantor set*

The subjective evaluation of difficulty levels (mean values) given by the students for this questionnaire is shown in Table 7. Based on the values shown, it can be affirmed that, among the questions presented, none were categorized into the two extreme categories: "very easy" or "very difficult," nor in the "easy" category. Note that students generally found this questionnaire more difficult than the first.

Table 7. Difficulty levels given by students (N = 131)

Question	Mean	S.D.	Difficulty Levels
No. 1	4,94	2,56	moderately difficult (3)
No. 2	6,3	2,32	difficult (4)
No. 3	6,3	2,43	difficult (4)
No. 4	5,24	2,76	moderately difficult (3)
No. 5	6,05	2,91	difficult (4)

According to the results presented in Table 8, question No. 2 showed the lowest percentage of correct answers (24.43%), while the highest percentage was found for question No. 1 (77.1%).

Table 8. Percentage of correct answers as a criterion of difficulty level (N = 131)

Question	% Correct Answers	Difficulty Level
No. 1	77,1	easy (2)
No. 2	24,43	difficult (4)
No. 3	45,04	moderately difficult (3)
No. 4	35,11	difficult (4)
No. 5	34,35	difficult (4)

Below, in Table 9, are the *tacit models* observed in the student's reasoning in each question posed. Let us note that this questionnaire turned out to be unusually rich in terms of the number of models that appeared during the analysis.

Table 9. *Tacit models* (*Cantor set*) (N = 131)

Question	*Tacit Models*
No. 1	*inexhaustible, divergence, undefined, unreachable*
No. 2	*dependency, bounded-finite, undefined, point-mark, inexhaustible*
No. 3	*dependency, infinite-unbounded, undefined, point-mark, inexhaustible*
No. 4	*squeezing, dependency, inclusion, undefined, point-mark*
No. 5	*squeezing, dependency, inclusion, undefined, slipping*

Next, in the following Figure 5, a comparison of difficulty levels (according to the two criteria) with the number of *tacit models* observed for each question is presented, according to the qualitative analysis carried out.

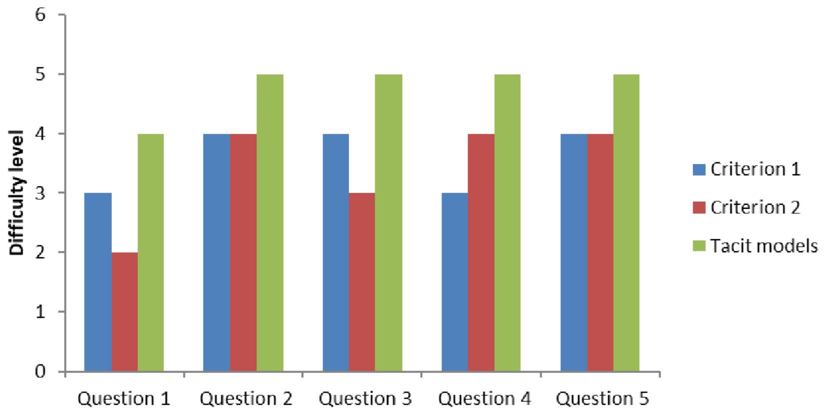


Fig. 5. Difficulty levels according to the two criteria, in comparison to the difficulty level indicated by the number of *tacit models* (*Cantor set*) (N = 131)

Let us recall again that criterion 1 refers to the subjective evaluation of the difficulty level given by the students, while criterion 2 refers to the difficulty level according to the percentage of correct answers obtained in each question.

In this case, the results obtained based on the percentage of correct answers are generally consistent with those obtained according to the subjective evaluation criteria. Except in the case of questions No. 1 and No. 3, the difficulty level of the questions is perceived as equal to/or less than the level of difficulty indicated by the percentage of correct answers. Indeed, the correlation between the difficulty level given by the students and the percentage of correct answers is strong (r = –0.67).

In this case, we find that students, in general, detect the complexity of the questions, which supports the appearance of these simplified models in their arguments. The correlation between the mean difficulty level given by the students and the number of *tacit models* observed in their reasoning is strong and significant (r = 0.80, $p < 0.05$), showing that indeed students perceive the complexity of the question in each case, which is precisely what induced them to resort to these simplified models.

On the other hand, the correlation between the percentage of correct answers and the number of these models observed in the reasoning of students for each question is strong and significant (r = –0.93, $p < 0.05$). In general, low percentages of correct answers were observed in all the questions (except perhaps in question No. 1), which also shows the high level of difficulty of this questionnaire.

4.3 Questionnaire *Sierpinski triangle*

The following Table 10 shows the subjective evaluation of difficulty levels (mean values) given by students in this case. In this questionnaire, no question was categorized into the two extreme categories, nor in the "easy" category. Questions No. 2 (mean = 6.35), No. 3 (mean = 6.43), No. 4 (mean = 4.88), and No. 1 (mean = 4.03) were categorized as "moderately difficult."

Table 10. Difficulty levels given by students (N = 77)

Question	Mean	S.D.	Difficulty Levels
No. 1	4,03	2,74	moderately difficult (3)
No. 2	6,35	2,04	difficult (4)
No. 3	6,43	2,08	difficult (4)
No. 4	4,92	3,02	moderately difficult (3)

The difficulty levels of the questions given by the percentage of correct answers for this questionnaire are shown in Table 11. The lowest percentage of correct answers was obtained for question No. 3 (32.47%), while questions No. 2 (53.25%) and No. 4 (54.55%) followed him in order. The highest percentage of correct answers was found for question No. 1 (93.51%).

Table 11. Percentage of correct answers as a criterion of difficulty level (N = 77)

Question	% Correct Answers	Difficulty Level
No. 1	93,51	very easy (1)
No. 2	53,25	moderately difficult (3)
No. 3	32,47	difficult (4)
No. 4	54,55	moderately difficult (3)

It is observed that the percentage of correct answers in question No. 1 is much higher than all the others, even though it was perceived by the students as having the same level of difficulty as the rest (as can be seen in Table 10). Table 12 presented below shows the *tacit models* detected for each of the questions posed in this case.

Table 12. *Tacit models* (*Sierpinski triangle*) (N = 77)

Question	*Tacit Models*
No. 1	*inexhaustible, divergence, undefined, unreachable*
No. 2	*dependency, bounded-finite, divergence, point-mark*
No. 3	*unreachable, infinite-unbounded, undefined, point-mark dependency, inexhaustible, squeezing, divergence*
No. 4	*point-mark, dependency, undefined, bounded-unbounded divergence, inexhaustible*

In Figure 6, a comparison of the difficulty levels of each question is presented (according to the two criteria) with the number of *tacit models* detected. In this case, the results obtained based on the percentage of correct answers are generally consistent with those obtained according to the subjective evaluation criteria. Their correlation is strong (r = –0.69) but it is not significant. The higher the level of difficulty of the question is evaluated, the lower the percentage of correct answers obtained, which is consistent with the other questionnaires. The greatest differences occurred in questions No. 1 and No. 2, where the level of difficulty of the questions is perceived as greater than the level of difficulty indicated by the objective measurement of the percentage of correct answers.

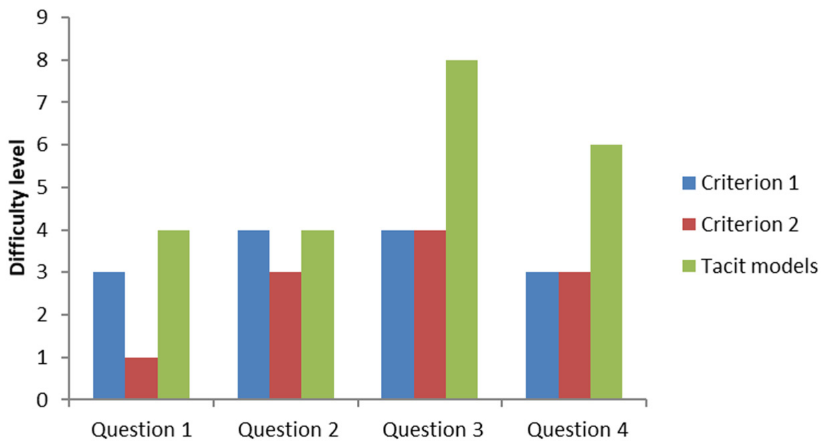


Fig. 6. Difficulty levels according to the two criteria, in comparison to the difficulty level indicated by the number of *tacit models* (*Sierpinski triangle*) (N = 77)

The correlation between the percentage of correct answers and the number of these models observed for each question could be considered relatively strong (r = –0.76), although it is not significant. Also, a positive correlation (r = 0.63) (neither significant) was observed between the mean difficulty levels given by the subjective evaluation of the students and the number of these *tacit models* observed.

5 DISCUSSION

The analysis carried out showed strong correlations between difficulty levels of the questions (measured through a subjective criterion by the students and a behavioral one, given by the percentage of correct answers) and compared them with the presence of *tacit models* for the three questionnaires.

The correlation between the mean difficulty levels given by the students and the percentage of correct answers was strong and negative, although it was only significant for the *Achilles and the tortoise* questionnaires. The values of the Pearson correlation coefficient were r = –0.90 ($p < 0.05$) for the *Achilles and the tortoise* questionnaire, r = –0.67 for the *Cantor set* questionnaire, and r = –0.69 in the case of the *Sierpinski triangle* questionnaire. Thus, as was expected, it is found that the higher the difficulty level given by students, the lower the percentage of correct answers obtained for a question.

However, the analysis identified that students perceived the difficulty levels of the questions above the difficulty levels indicated by the objective measurement of the percentage of correct answers for some questions in the *Cantor set* and the *Sierpinski triangle* questionnaires, which indicates that a higher complexity level was consciously perceived by them in those cases. Furthermore, according to their evaluation, students found the *Cantor set* and the *Sierpinski triangle* questionnaires more difficult than the *Achilles and the tortoise* questionnaires (*Achilles and the tortoise*, mean = 4.35, *Cantor set*, mean = 5.77, *Sierpinski triangle*, mean = 5.43).

The subjective assessment of the difficulty levels of a question is related to the perception of the students and could be affected by many factors. Nevertheless, the greater number of *tacit models* observed in the *Cantor set* and the *Sierpinski triangle* questionnaires than in the *Achilles and the tortoise* questionnaires could be the reason for this difference in perception. The fact that this subjective evaluation consciously recognizes a higher level of difficulty present in a task makes the emergence of these models natural, corroborating that these constitute ways of thinking in a simplified form that are used to facilitate de-resolution tasks.

Indeed, there was a strong positive (and significant, except in the case of the *Sierpinski triangle*) correlation between the difficulty level given by the students and the number of *tacit models* observed in their reasoning, showing that students actually perceived the complexity level of the question in each case, which is precisely what induces them to resort to these simplified models. Specifically, the values of the Pearson correlation coefficient were r = 0.70 ($p < 0.05$) for the *Achilles and the tortoise* questionnaire, r = 0.8 ($p < 0.05$) for the *Cantor set* questionnaire, and r = 0.63 in the case of the *Sierpinski triangle* questionnaire. Thus, it can be said that these models could be taken as an index of the difficulty level present in the task being solved.

On the other hand, the objective percentage of correct answers for each question verified this subjective perception of the students and the presence of these models in students' reasoning, and according to the analysis, it was the most objective measure of the actual level of difficulty.

There was a strong negative correlation (and significant, except in the case of the *Sierpinski triangle*) between the percentage of correct answers and the number of these *tacit models* observed in students' reasoning, thus confirming that these models constitute obstacles in the proper understanding of students and can be taken as an index of the difficulty level of the task. In particular, the values of the Pearson correlation coefficient were r = –0.80 ($p < 0.05$) for the *Achilles and the tortoise* questionnaire, r = –0.93 ($p < 0.05$) for the *Cantor set* questionnaire, and r = –0.76 in the case of the *Sierpinski triangle* questionnaire.

In general, the percentage of correct answers in the *Sierpinski triangle* questionnaire was relatively high (mean = 58.45%), compared to the *Cantor set* questionnaire (mean = 43.21%) and to the *Achilles and the tortoise* questionnaire (mean = 35.74%), even though ten of the twelve *tacit models* were found in this case. It may indicate that when students were answering this last questionnaire, some of them were already aware, at least, to some extent, of their use of these erroneous models, and they were already learning to deal with them. In other words, they were able to replace them with more appropriate schemes that allowed them to arrive at the correct answers. These divergences are similar to results obtained from other studies on which subjective and behavioral criteria have been used (for example, see [22, 23]).

These results are also relevant when selecting proper instructional strategies for Calculus courses and specific learning outcomes in teaching engineering. Nevertheless, our study does not consider other important factors such as a weak background in mathematics and the lack of motivation influencing the cognitive processes of engineering students, which have been considered in some research conducted before [24, 25], therefore it has some limitations.

6 CONCLUSION

Based on the results previously discussed, it is concluded that there are strong and significant correlations between the numbers of *tacit models* observed in students' cognitive processes and the task difficulty level categories established by both types of criteria (a subjective one, through an evaluation made by the subjects, and a behavioral one, through the obtention of the correct solution).

Thus, the analysis showed the recognition of these unconscious patterns in students' cognitive mechanisms is relevant when solving mathematical tasks that require information processing at a higher level and could play a role in assessing the difficulty levels of a task related to the study of mathematical concepts in Calculus courses, which constitutes the main novelty of this study. Based on the results obtained, it can also be affirmed that the use of these models by students could be considered an indicator of mental effort according to the cognitive load theory and could be instrumental in the design of didactic strategies for the classroom.

More specifically, these results could allow us to improve our teaching practice by using them to develop didactic activities to identify these models and guide students to overcome these difficulties. From a metacognitive point of view, it could help to stimulate students to become aware and to reflect on their thoughts and intuitions regarding certain mathematical concepts and the incoherences associated with them. At the same time, it could show them the validity of these incoherences by revealing constraints and limitations imposed by our intuitions and previous experiences on our learning processes. All of this could also be meaningful in the enhancement of mathematical conceptual understanding in the context of a conventional university classroom or more unconventional settings, as suggested in [26].

Furthermore, establishing difficulty levels considering these models can help in the construction of scales and grading criteria for certain tasks, which is one of the issues usually examined in educational studies [27]. This is especially important when solving problems that involve processing information at a higher level, requiring the use of different cognitive mechanisms at once, and in this case, the unconscious ones.

Nevertheless, our work has some limitations: the role of these unconscious models and their automatic processes of formation must be further investigated concerning the difficulty levels of mathematical tasks and proper instructional strategies for Calculus courses from the point of view of the cognitive load theory [10], especially in the context of undergraduate education for engineering and science students. Also, further research is recommended; a more detailed analysis is needed to evaluate each of these *tacit models* specifically, considering their levels of difficulty and their influence and persistence in students' cognitive mechanisms.

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