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PAPER

Integration of GeoGebra Calculator 3D with Augmented Reality in Mathematics Education for an Immersive Learning Experience

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ABSTRACT

The use of augmented reality (AR) with GeoGebra allows for the contextualization of mathematical operations in real-world situations. In this approach, the teacher presents questions or problems that students solve using visualization and experimentation software. The objective of this work is to evaluate the impact of integrating the GeoGebra 3D calculator with AR. For the development of this study, the quasi-experimental method was employed, involving the comparison of results between two groups: the experimental group (EG) and the control group (CG). We worked with a population of 78 students. The study conducted confirms the use of the GeoGebra calculator in 3D with AR. AR effectively enhances mathematical learning. Seventy percent of the students in the EG achieved an outstanding level of performance, while 30% reached an expected level. In addition, a positive attitude towards mathematics was observed in 100% of the students. These results demonstrate that using the GeoGebra calculator in 3D with AR has a positive impact on mathematics learning. While in CG, 10% achieved the expected level of performance, 85% were in progress, and 5% were at the initial stage. Finally, it was concluded that the GeoGebra calculator in 3D with AR is very useful. It helps enhance the teaching and learning (TL) of mathematics and motivates students, making the development of class sessions more dynamic.

KEYWORDS

software, augmented reality (AR), GeoGebra, 3D calculator, teaching and learning (TL)

1 INTRODUCTION

The teaching and learning (TL) process of mathematics is an area of great importance in the academic training of students and constitutes a challenge for teachers due to its complexity [1]. UNESCO maintains that learning mathematics helps develop logical thinking and allows the study of objective reality through abstraction [2].

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Despite advances in educational technology, students still experience difficulties in understanding mathematical concepts and problem-solving [3]. Learning is defined as a relatively permanent change in knowledge, ability, or understanding due to experience [4]. The quality of education is a crucial issue worldwide, as it can either build or hinder a nation's future. While some countries boast highly effective education systems, others struggle due to various factors such as insufficient investment, a lack of resources, inadequate teacher training, and poor infrastructure [5]. Mathematics is an essential tool in everyday life and is applied in various professional areas and fields of study [6], [10]. Low academic achievement in mathematics is evident in international test statistics, indicating that students struggle with understanding mathematical concepts such as geometry. This includes comprehending and applying geometric concepts and properties, solving problems related to geometric shapes and patterns, and visualizing, and processing objects in space. Numbers: arithmetic operations, ratios, and proportions. Algebra involves solving equations, identifying patterns and regularities, representing, and analyzing functions. Statistics and probability involve interpreting and analyzing statistical data, making inferences and predictions based on the data, understanding probability concepts, and applying them to relevant situations [7], [8], [9].

GeoGebra calculator 3D with augmented reality (AR) is a technology that is increasingly being utilized in various fields [10]. This technology allows for the superimposition of digital information in the real world through devices such as smartphones, tablets, or computers [11]. In the educational field, mathematics is usually one of the subjects that students like the least, and teachers, due to a lack of updating, are not able to incorporate new technological tools to make the TL process interesting [12], [13]. In this context, GeoGebra with AR becomes a valuable tool that will allow students to interact with three-dimensional models, graphics, and other visualizations so that they can understand mathematical concepts more effectively. This can contribute to the development of problem-solving skills [14–16]. GeoGebra with Calculator 3 is a free software tool that combines geometry, algebra, and calculus functionalities. The tool is often used in mathematics education to help students visualize and understand complex mathematical concepts [17]. In recent years, GeoGebra and 3D calculators have incorporated AR technology into their platforms, enabling users to visualize mathematical models and graphs in a 3-D environment [18]. AR technology can assist students in comprehending abstract mathematical concepts by enabling them to interact with models more intuitively and visually [19].

This paper presents a solution to improve the mathematics TL process by integrating the GeoGebra 3D calculator with AR. The use of AR enables real-time visualization of mathematical objects in both virtual environments and the real world, thereby enhancing the comprehension and acquisition of mathematical concepts. The objective of this study is to assess the impact of integrating the GeoGebra 3D calculator with AR in the mathematics TL process for students aged 11 to 13 in the 2nd year of secondary school at Educational Institution 0037 Santa Rosa-Lima in the city of Lima. The study includes a sample of 78 students. This work is focused on the mathematical TL process. This work aims to contribute to the development of new strategies to optimize mathematics TL that can be applied in different educational environments.

Nowadays, with technological advances, it is imperative to innovate in TL strategies, especially considering that mathematics subjects are inherently complex, and students often struggle to solve activities. Therefore, the need for motivation and interest on the part of both teachers and students is vital to creating an environment conducive to meaningful learning. By breaking away from traditional teaching paradigms and utilizing a GeoGebra 3D calculator with AR, we can create a more engaging and interactive learning experience that fosters students' genuine interest in mathematics.

2 RELATED WORK

There is research suggesting that AR technology is a relevant and appropriate tool to achieve specific cognitive objectives in TL [20]. For example, in [21], GeoGebra and STACK were implemented with the purpose of enhancing the academic performance of students. The study involved 50 students, who were divided into two groups: control and experimental. The results revealed a significant improvement in the students in the experimental group who utilized two innovative technological tools. Finally, they concluded that the two technological tools have unique and valuable features that can enhance students' learning experiences. Likewise, in a study by Smith et al. [10], the aim was to assist students in developing their geometric reasoning skills through visualization, perception, and intuition. As a result, they found that the use of GeoGebra is highly effective in stimulating students' geometric thinking, thereby enhancing their geometric perception and visualization skills. Finally, they concluded that GeoGebra facilitated the development of the students' reasoning abilities. The students engaged in thinking, simulating, strategizing, verbalizing ideas and conjectures, and learning from their perceptions.

Additionally, in [22], mathematics learning tools were developed using AR for high school students. An agile research and development methodology was adopted in the study. SketchUp software was used for creating geometric objects, while an AR application was used for their three-dimensional visualization. The participants in this experiment were 15 ninth-grade students. Data were collected through expert evaluation and student responses. The results indicate that the tool is a valid and highly versatile instrument that can be easily integrated into the learning process. Moreover, it is capable of sparking students' interest and assisting them in comprehending geometry content. Similarly, in [17], a study was conducted to evaluate the impact of GeoGebra dynamic mathematics software on the comprehension of coordinates. A quasi-experimental design was used with two groups, each consisting of 42 participants. They randomly selected 84 participants from the population and assigned them to different groups using a purposive sampling technique. They used the t-test for pre-test and post-test activities. As a result, students who used GeoGebra achieved higher scores compared to those who followed the conventional methodology. This study concluded that the use of GeoGebra is effective for learning polar coordinates.

Also, the work [23] involved implementing four different approaches using various XR technologies, including projector-based AR, mobile-based AR, HMD AR, and HMD VR. With the aim of assisting students in tackling the challenges of the business world, such as planning, organizing, managing, problem-solving, critical thinking, creativity, and teamwork, the effectiveness of four approaches in developing entrepreneurial skills was evaluated among a group of 20 students. The results showed that the stories, scenarios, and educational intentions used were highly valued by the students. Finally, they concluded that the selection of technologies applied is crucial to achieving cognitive objectives. Likewise, in the study [24], researchers analyzed the perceptions of middle- to high-level students at a private educational institution in Mexico regarding the implementation of AR-based mathematics teaching strategies through the Metaverso mobile application. The work carried out was a descriptive, survey-based study. A total of 192 first-semester students taking the subject Fundamentals of Mathematics participated. According to the results, there has been an improvement in the pass rate compared to the previous school year. The participants have expressed their interest in the use of AR and have noticed improvements in learning by implementing strategies mediated by this tool. In conclusion, the implementation of RA in the teaching of mathematics has a positive impact on the learning process.

3 METHODOLOGY

In this section, we will describe the methodology employed in this work. We utilized the exploration-based approach, a methodology that encourages active and profound learning. In this approach, students independently explore and discover mathematical concepts using visualization and experimentation tools like the GeoGebra calculator 3D with AR. The teacher provides questions or problems for students to solve without giving a solution beforehand, offering them the opportunity to investigate and come to their conclusions through active exploration. This approach allows students to work at their own pace and skill level, fostering the development of critical thinking and problem-solving skills. To implement this approach, the teacher guides students through the process of exploration and experimentation, helping them identify mathematical patterns, relationships, and concepts. This teacher encourages students to share their observations and conclusions with the rest of the group. The quasi-experimental research methodology involves comparing the results of a group of students who will use the GeoGebra calculator 3D with AR with the results of a group of students who will not use it. The research variables in this study include TL as the dependent variable and "GeoGebra calculator 3D with AR" as the independent variable. To answer this question, an EG of 39 students and a control group of 39 students were selected. The EG used the GeoGebra calculator 3D with AR during TL sessions, while both groups participated in the same sessions designed by the teachers. Activities related to geometry and algebra problems. The experiment will last for three months. At the end of this period, the survey technique will be applied using a questionnaire as the instrument. The questions will be related to the topics covered in the class sessions. Didactic materials were developed for creating geometric figures using tools such as points, lines, segments, arcs, circles, ellipses, and polygons, as well as translation, rotation, and reflection operations. In addition to solving mathematical problems related to geometry, instruments were also designed to evaluate the participants' level of satisfaction. These instruments addressed questions on various aspects, such as the ease of use of the tool, perceived usefulness, impact on learning, and overall satisfaction with the tool.

3.1 GeoGebra calculator 3D with AR

It is an interactive mathematics software that allows the visualization and manipulation of objects and figures in a three-dimensional environment [25], as illustrated in Figure 1. The GeoGebra 3D calculator with AR is a dynamic program specifically designed to facilitate the learning and teaching of mathematics by integrating elements from various areas such as arithmetic, geometry, algebra, analysis, calculus, probability, and statistics [26]. This program is easy to learn and can be downloaded for free from its official website. It is compatible with different operating systems, such as Windows, macOS, Linux, and Solaris, due to its development in Java. One of the most outstanding features of GeoGebra is its collaborative approach, which provides access to resources, forums, wikis, and help maintained by users around the world. In addition to being free and easy to use, GeoGebra's most notable feature is its ability to represent objects in two distinct views: a graphical view that focuses on geometry and an algebraic view that focuses on algebra. This dual representation makes it possible to establish a direct connection between algebraic symbols, numerical values, and geometric graphs. For example, GeoGebra can simultaneously display a point on a Cartesian plane along with its numerical coordinates, a circle with its corresponding equation, or the graph of a function with its symbolic expression [27].

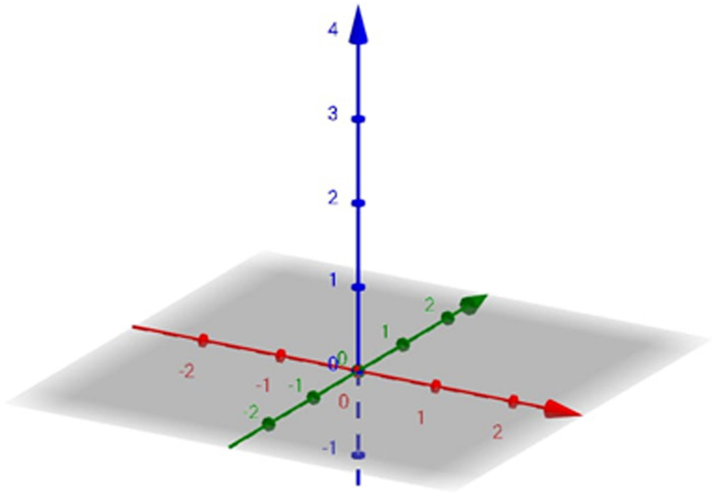

Fig. 1. Representation of GeoGebra calculator 3D with AR

3.2 Teaching and learning mathematics

It is a field of study that focuses on how students acquire knowledge and skills in mathematics and how educators can teach these concepts effectively [28]. Mathematics is a fundamental discipline that is utilized in a wide variety of fields [29]. Therefore, it is essential for students to comprehend and master basic mathematical concepts [30]. Teaching mathematics involves the selection and use of effective pedagogical strategies and techniques to help students learn and apply complex mathematical concepts [1].

3.3 Augmented reality

It is a tool that enables students and educators to visualize and manipulate mathematical objects and figures in a real-world environment [31], as depicted in Figure 2. Users can create and experiment with three-dimensional figures like cubes, spheres, cones, and pyramids. Another advantage is its user-friendly interface, which makes it easy to use and intuitive. This makes the tool accessible to a wide variety of students and educators [32–34].

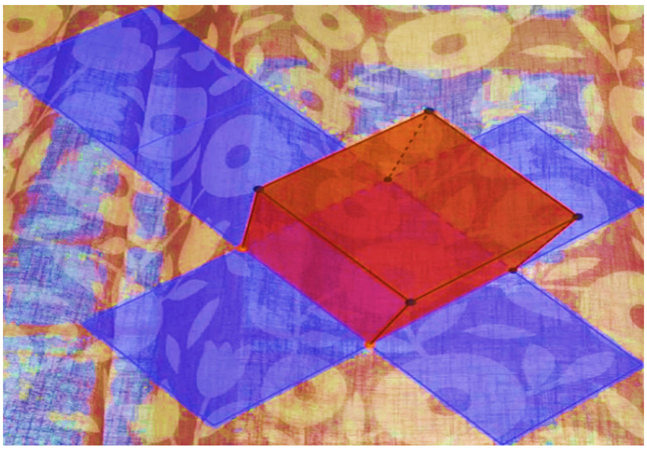

Fig. 2. Exploration of the sections of a cube

3.4 Understanding case study

UNESCO stresses the fundamental role of mathematics in modern science and technology, from AI to the optimization of transportation networks and health research [35]. It also recognizes the importance of mathematics as a tool for international cooperation and intercultural dialogue. However, there are still significant barriers to group inclusion in mathematics education and research. In addition, the shortage of competent mathematics teachers poses a threat to the future of the discipline, and its application is still limited to certain fields. Therefore, efforts are needed to foster inclusion and education in mathematics and expand its application in addressing the challenges of today's world [36]. Low academic achievement in mathematics is a complex and urgent issue that demands effective solutions. The aim of this study is to evaluate the impact of integrating a 3D calculator with AR in teaching mathematics. With this software, users can create and manipulate three-dimensional figures such as cubes, spheres, cones, and pyramids, and can also work with functions and equations in three dimensions, as shown in Figures 3 and 4. For this purpose, an experiment is conducted to compare the performance of a group of students using the integrated tool with the performance of another group using traditional teaching methods. It is developed as applied research at an explanatory level, utilizing a quantitative approach and a quasi-experimental design. It works with a population of 78 students aged between 11 and 13 years old. Instruments for data collection, such as written tests and surveys, were utilized. Also, tools such as Minitab and Excel are used for data analysis, specifically for the student's t-test hypothesis.

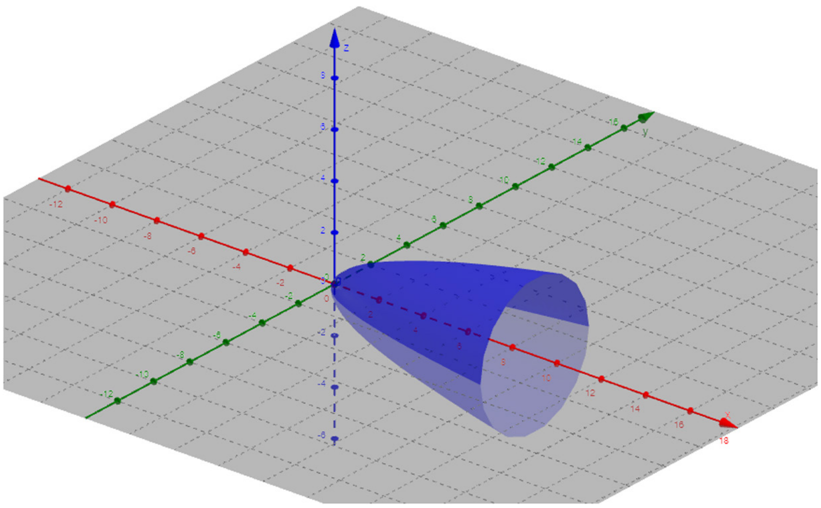

Fig. 3. Solid of revolution formed when the area is rotated around the x-axis

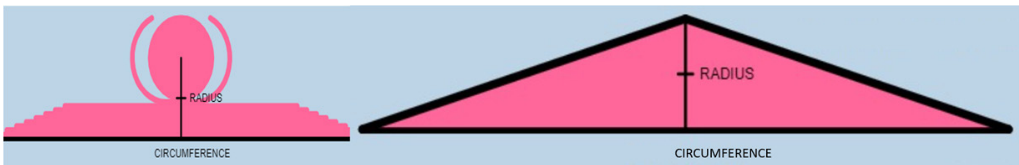

Fig. 4. Area and radius of a circle

In terms of the instruments, a questionnaire will be utilized to gather information after the completion of the sessions over a three-month period. The GE will use the 3D calculator with AR during the TL sessions, while the CG will develop the sessions using the traditional method. The test is administered to both the GE, consisting of 40 students, and the CG, consisting of 40 students.

4 RESULTS

The objective of this study is to evaluate the impact of integrating 3D calculators with AR in the TL of mathematics. A written test was conducted. The study considered the main challenges students face. It focused on classifying geometric figures and finding measurements like area, perimeter, volume, length, height, and trigonometric functions. Following the specified criteria:

Learning (KPI-1), attitude towards mathematics (KPI-2), and level of satisfaction with the use of a 3D calculator with AR (KPI-3). Table 1 presents the consolidated performance of the instruments for identifying the criteria KPI-1, KPI-2, and KPI-3.

Table 1. Consolidated data processing

#	Control Group						Experimental Group					
	KPI-1		KPI-2		KPI-3		KPI-1		KPI-2		KPI-3	
	Score	Rating Scale	Score	Rating Scale	Score	Rating Scale	Score	Rating Scale	Score	Rating Scale	Score	Rating Scale
1	5	In Beginning	5	Positive Attitude	3	Satisfied	18	Expected Achievement	5	Positive Attitude	4	Very Satisfied
2	14	Expected Achievement	5	Positive Attitude	3	Satisfied	18	Expected Achievement	5	Positive Attitude	4	Very Satisfied
3	13	In Process	5	Positive Attitude	6	Very Satisfied	23	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
4	7	In Process	5	Positive Attitude	5	Very Satisfied	17	Expected Achievement	5	Positive Attitude	5	Very Satisfied
5	9	In Process	5	Positive Attitude	3	Satisfied	18	Expected Achievement	5	Positive Attitude	4	Very Satisfied
6	12	In Process	5	Positive Attitude	3	Satisfied	17	Expected Achievement	5	Positive Attitude	4	Very Satisfied
7	13	In Process	5	Positive Attitude	3	Satisfied	23	Outstanding Achievement	5	Positive Attitude	4	Very Satisfied
8	7	In Process	5	Positive Attitude	3	Satisfied	20	Outstanding Achievement	5	Positive Attitude	4	Very Satisfied
9	9	In Process	5	Positive Attitude	3	Satisfied	17	Expected Achievement	5	Positive Attitude	4	Very Satisfied
10	12	In Process	5	Positive Attitude	6	Very Satisfied	21	Outstanding Achievement	5	Positive Attitude	4	Very Satisfied
11	5	In Process	5	Positive Attitude	6	Very Satisfied	21	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
12	12	In Process	5	Positive Attitude	6	Very Satisfied	19	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
13	15	Expected Achievement	5	Positive Attitude	3	Satisfied	18	Expected Achievement	5	Positive Attitude	5	Very Satisfied

(Continued)

Table 1. Consolidated data processing *(Continued)*

#	Control Group						Experimental Group					
	KPI-1		KPI-2		KPI-3		KPI-1		KPI-2		KPI-3	
	Score	Rating Scale	Score	Rating Scale	Score	Rating Scale	Score	Rating Scale	Score	Rating Scale	Score	Rating Scale
14	7	In Process	5	Positive Attitude	3	Satisfied	18	Expected Achievement	5	Positive Attitude	5	Very Satisfied
15	9	In Process	5	Positive Attitude	6	Very Satisfied	19	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
16	12	In Process	5	Positive Attitude	6	Very Satisfied	20	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
17	13	In Process	5	Positive Attitude	6	Very Satisfied	19	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
18	7	In Process	5	Positive Attitude	3	Satisfied	17	Expected Achievement	5	Positive Attitude	4	Very Satisfied
19	9	In Process	5	Positive Attitude	3	Satisfied	18	Expected Achievement	5	Positive Attitude	4	Very Satisfied
20	12	In Process	5	Positive Attitude	6	Very Satisfied	21	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
21	9	In Process	5	Positive Attitude	6	Very Satisfied	20	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
22	12	In Process	5	Positive Attitude	6	Very Satisfied	19	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
23	9	In Process	5	Positive Attitude	6	Very Satisfied	19	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
24	12	In Process	5	Positive Attitude	6	Very Satisfied	22	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
24	9	In Process	5	Positive Attitude	6	Very Satisfied	22	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
26	9	In Process	5	Positive Attitude	6	Very Satisfied	19	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
27	12	In Process	5	Positive Attitude	6	Very Satisfied	19	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
28	9	In Process	5	Positive Attitude	6	Very Satisfied	21	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
29	9	In Process	5	Positive Attitude	6	Very Satisfied	23	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
30	14	Expected Achievement	5	Positive Attitude	6	Very Satisfied	20	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
31	11	In Process	5	Positive Attitude	6	Very Satisfied	21	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
33	9	In Process	5	Positive Attitude	6	Very Satisfied	20	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
33	12	In Process	5	Positive Attitude	6	Very Satisfied	21	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied

(Continued)

Table 1. Consolidated data processing *(Continued)*

#	Control Group						Experimental Group					
	KPI-1		KPI-2		KPI-3		KPI-1		KPI-2		KPI-3	
	Score	Rating Scale	Score	Rating Scale	Score	Rating Scale	Score	Rating Scale	Score	Rating Scale	Score	Rating Scale
34	13	In Process	5	Positive Attitude	6	Very Satisfied	21	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
35	13	Expected Achievement	5	Positive Attitude	4	Very Satisfied	18	Expected Achievement	5	Positive Attitude	5	Very Satisfied
36	9	In Process	5	Positive Attitude	4	Very Satisfied	17	Expected Achievement	5	Positive Attitude	4	Very Satisfied
37	11	In Process	5	Positive Attitude	6	Very Satisfied	20	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
38	10	In Process	5	Positive Attitude	6	Very Satisfied	19	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied
39	9	In Process	5	Positive Attitude	6	Very Satisfied	23	Outstanding Achievement	5	Positive Attitude	5	Very Satisfied

The first indicator, learning, is assessed on a scale of beginning to outstanding achievement; the second attitude towards students' mathematics is measured on a scale of yes/no, and the third use of a 3D calculator with AR is measured on a scale of satisfied to very satisfied.

4.1 KPI-1

Figure 5 displays the results obtained from the experimental and control groups, indicating that the EG achieved a higher level of learning compared to the control group. It is evident that KPI-1 demonstrates the outcomes following the integration of the 3D calculator with AR during mathematics class sessions in the Experimental Group. Initially, 0% was achieved during the process; 0% was achieved, expected achievement reached 30%, and outstanding achievement was 70%. This shows a notable contrast with the results of the control group, which obtained 5% at the beginning, 85% during the process, 10% for expected achievement, and 0% for outstanding achievement.

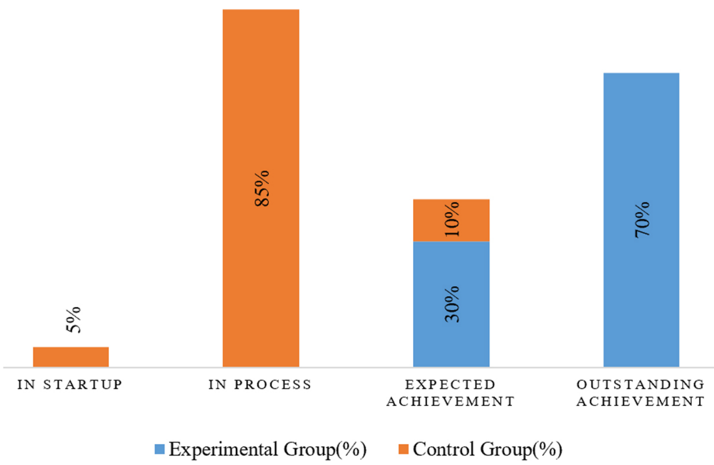

Fig. 5. Learning outcomes achieved

4.2 KPI-2

Attitude towards Mathematics: Figure 6 displays the results of KPI-2, indicating that students in the EG who used the 3D calculator with AR in GeoGebra have a 100% positive attitude. In contrast, in the control group, where traditional TL methods were followed, 68% of students had a negative attitude and 32% had a positive attitude towards mathematics.

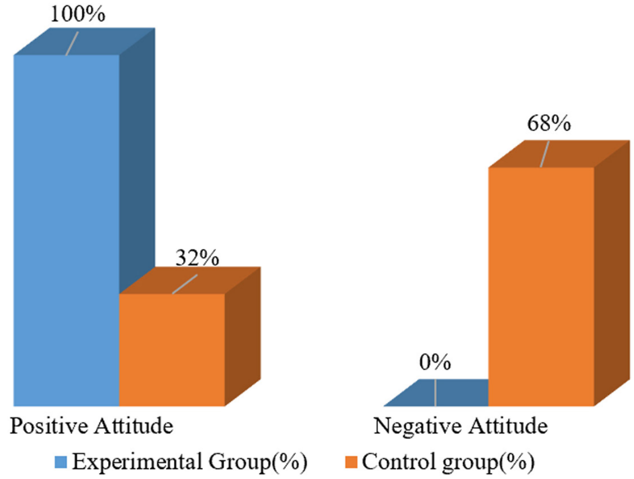

Fig. 6. Students' attitude towards mathematics

4.3 KPI-3

Level of satisfaction with the use of the 3D calculator with AR: Figure 7 displays the results of KPI-3, reflecting the students' satisfaction levels after using the 3D calculator with AR in GeoGebra. The experimental group, comprising students who used it during class sessions, reported that 72% were "Very satisfied" and 28% were "Satisfied."

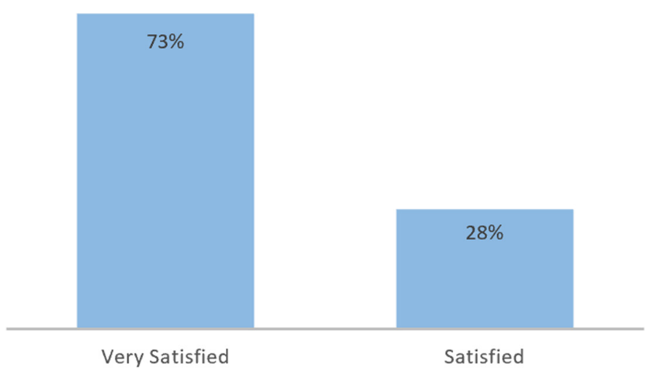

Fig. 7. Satisfaction on use of 3D calculator with AR

Hypothesis test: To test the hypothesis, a student's t-test was used with the data obtained from the independent samples. Also, the following statistical equations (1), (2), and (3) are applied.

Null hypothesis H0: There is no significant difference in TL between the EG that used the 3D calculator with AR and the control group that did not use the software.

Alternative hypothesis Ha: The EG that used the 3D Calculator with AR significantly improved their learning compared to the control group that did not use the 3D Calculator with AR. The results are presented in Table 2.

$$\bar{x} = \frac{\sum_{i=1}^{n} \times 1}{n} \tag{1}$$

$$S^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} \tag{2}$$

$$Z = (x - y) / \sqrt{(\sigma x2/n1 + \sigma y2/n2)} \tag{3}$$

Table 2. Hypothesis test result

Student's T-test	Experimental Group	Control Group
Mean	19.65	10.4
Significance Level $\alpha = 0.05$	5%	5%
Standard deviation	1.9	3.14
Variance	5.51	11.86
Student's t	7.83	
Degrees of freedom (n1 + n2–2)	38	
Critical value of t	± 2.024	

According to the results of the t-test, the calculated t-value exceeds the critical t-value. Therefore, the null hypothesis is rejected, and the alternative hypothesis is accepted. The experimental group, which utilized the 3D calculator with AR, demonstrated a significant improvement in learning compared to the control group, which did not use the 3D calculator with augmented reality.

5 DISCUSSION

The analysis of the results of the tests conducted on students aged between 11 and 13 in the mathematics course of the second grade of secondary education at the educational institution 0037 Santa Rosa-Lima indicates that the utilization of GeoGebra 3D calculators, along with AR, notably enhances mathematical learning. In fact, 70% of the students who participated in the EG reached an outstanding performance level, while the remaining 30% reached the expected level. Moreover, their attitude toward learning mathematics was 100% positive. These results coincide with the findings of previous studies, such as the one conducted in [10], which concluded that GeoGebra has the potential to stimulate students' geometric thinking and improve their perception and visualization skills. Similarly, the results of the study in [17] showed that AR helps to increase students' interest by 100% and understanding by 50%. Satisfaction levels ranged between 40% and 60%, with students expressing high levels of satisfaction with the implementation of AR. Overall, the use of GeoGebra and AR can be a powerful tool to enhance students' learning outcomes and increase their engagement with mathematics. Teachers should consider incorporating these technologies into their teaching practices to enhance their students' learning experience and promote a more positive attitude towards mathematics. The results obtained in this study provide compelling evidence for the effectiveness of these tools in promoting mathematics learning. Further research in this area is recommended. In a recent study [27], it was found that the use of GeoGebra with AR elicited a positive response, with a 75% satisfaction rate among participants. The study concluded that GeoGebra is a highly engaging resource that captures the attention and interest of most learners. This finding suggests that GeoGebra can be an effective tool for enhancing the learning experience, particularly when integrated with AR technology. With its intuitive interface and interactive features, GeoGebra offers a unique and immersive way of learning that can benefit learners of all ages and levels. No wonder GeoGebra has become a favorite among educators and students.

6 CONCLUSIONS

After analyzing the results, it can be concluded that the GeoGebra 3D calculator with AR is extremely useful for enhancing the TL process of mathematics. Moreover, it makes the classroom sessions more dynamic and engaging, thus motivating the students. Thanks to the significant contribution of the 3D calculator with AR, the students in the EG exhibited a higher level of interest in the evaluated variables. For example, all students exhibited a positive attitude towards mathematics, with 70% achieving outstanding performance and 30% achieving expected performance. In the control group, the test results revealed that only 10% of the students achieved outstanding performance, 85% were at the proficient level, and 5% were at the initial proficiency level. After conducting a thorough analysis of the results, it can be concluded that the incorporation of this application in the TL process of mathematics has been very significant for the students.

A 3D calculator with AR technology would be a game-changer for future research. It is crucial to provide training and resources on this innovative tool to all participants in the study. This will help educators adapt mathematics learning sessions to accommodate the diverse abilities and learning styles of their students. In this way, we can create a more inclusive learning environment that supports the unique needs of each learner and promotes academic success.

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