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PAPER

Integrating LLMs into Mathematics Laboratories Alongside CAS

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ABSTRACT

This paper proposes a structured approach for integrating large language models (LLMs) and computer algebra systems (CAS) in mathematics laboratories to enhance learning in higher education. By combining LLMs with CAS, this method provides a balanced environment where students engage actively in solving complex mathematical problems. LLMs assist in conceptual understanding through natural language interaction, while CAS ensures precise computational accuracy. Aligned with inquiry-based learning (IBL) principles, the approach guides students through stages that encourage exploration, collaboration, and reflection. This integration maximizes both pedagogical and functional opportunities, providing a balanced approach that enhances problem-solving skills and prepares students for the evolving use of LLMs in mathematics education.

KEYWORDS

large language models (LLMs), computer algebra systems (CAS), inquiry-based learning (IBL), mathematics education, higher education

1 INTRODUCTION

Advancements in technology have continuously reshaped the landscape of mathematics education, offering new tools and approaches that promote active learning and support complex problem-solving. Central to this evolution are mathematics laboratories (math labs), which serve as both dedicated spaces and pedagogical frameworks for students to explore mathematical concepts hands-on, testing hypotheses and developing critical thinking skills [1], [2]. In the digital era, math labs have adapted to incorporate powerful software platforms such as computer algebra systems (CAS), dynamic geometry software, and statistical packages [3]. These tools have enabled students to tackle more sophisticated mathematical problems, transforming traditional instructional methods and enhancing opportunities for inquiry-based learning (IBL).

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The deployment of CAS in particular has transformed mathematics education [3]–[9], enabling the performance of sophisticated calculations and visualizations that were previously impractical. These tools extend the range of solvable problems within the curriculum, supporting everything from basic arithmetic to advanced symbolic manipulations. Although the integration of CAS into educational frameworks has broadened both pedagogical and functional opportunities, it is not without its challenges—especially in maintaining a balance where these tools support, rather than replace, foundational mathematical skills.

As CAS continues to influence mathematics education, recent advancements in generative artificial intelligence (GenAI), particularly large language models (LLMs), are opening further possibilities for transforming learning environments [10]–[25]. LLMs, such as OpenAI's ChatGPT and Google's Gemini, process human-like text and can assist students and educators across various academic tasks, from concept clarification to interactive problem-solving. Nonetheless, challenges surrounding LLM accuracy and the need for verification mechanisms are critical considerations for their use in educational contexts.

While prior research has examined the role of LLMs in education and the integration of CAS in mathematics instruction, these technologies have largely been studied independently. There remains a critical gap in studies for combining the computational rigor of CAS with the language-based adaptability of LLMs, aligning both with established pedagogical principles. To contribute to addressing this gap, this study introduces ILAC (Integrate LLMs alongside CAS), a novel structured approach designed to incorporate both CAS and LLMs within a mathematics lab setting. Based on the principles of IBL [26], [27], the ILAC approach aims to foster exploration, collaboration, and reflective learning, enabling students to tackle complex mathematical challenges in a supportive, inquiry-driven environment. IBL is closely related to other active learning strategies, such as discovery learning, project-based learning, and problem-based learning, all of which have been successfully applied in engineering and mathematics education (e.g., [28]–[30]). However, unlike project-based learning, which often revolves around long-term projects with tangible outcomes, or problem-based learning, which is typically centered around real-world problem scenarios, IBL fosters a flexible, inquiry-driven structure where students refine their understanding through progressive exploration. This makes it particularly well-suited for structured engagement with AI-driven tools in a mathematics laboratory setting.

The remainder of this paper is organized as follows: Section 2 reviews the roles of CAS, math labs, and LLMs in mathematics education. Section 3 introduces the ILAC approach, while Section 4 presents an example problem to illustrate its application. Section 5 discusses ILAC's contributions, potential impact, and limitations. Finally, Section 6 restates the study's objectives, summarizes key contributions, and outlines future research directions.

2 RELATED WORK

2.1 Math labs and CAS in higher education mathematics

A mathematics laboratory, according to [1], is both a dedicated space for mathematical experiments and practical activities and a flexible approach to teaching mathematics. As a physical space, it serves as a room equipped for hands-on exploration. Beyond the room itself, however, it represents a teaching method that encourages students to work informally, engage in discussion, select their materials and methods, and actively discover mathematical concepts for themselves. More recently, Maschietto and Trouche [2] emphasize that math labs are equipped with a variety of tools—from physical manipulatives to advanced digital technologies—creating an environment where students can test hypotheses, investigate mathematical properties, and develop problem-solving skills in an interactive setting.

The integration of digital technologies into math labs—and into mathematics education more broadly—has evolved significantly over recent decades. What started with basic tools such as calculators has expanded to include powerful software platforms such as CAS, dynamic geometry software, and statistical packages, allowing students to explore complex mathematical concepts in ways that were once unattainable [3]. CAS, in particular, are systems specifically designed to handle primarily symbolic as well as numerical computations, and they have been the subject of research since the 1960s [4]. The advancements in these tools have introduced new functional and pedagogical opportunities, triggering transformative shifts in teaching methodologies, problem-solving strategies, and mathematics education overall [3], [5], [6].

Further research has taken a more detailed view of how CAS are used to enhance mathematical learning, delving into both the benefits and challenges of their integration into mathematics education. Marshall et al., in their review study [7], identified that CAS are used to help students visualize and experiment with mathematical concepts, as well as explore real-world and complex problems, potentially benefiting student comprehension, preparing students for future careers, and making more advanced mathematics accessible earlier in their education. However, they also noted several challenges to CAS integration, such as the cost of CAS, course time restrictions, and the complexity of syntax.

Establishing a strong pedagogical foundation is essential for the effective integration of technology, such as CAS, into mathematics education. Leinbach et al. [8] emphasize that for CAS to have a meaningful impact, its use must be anchored in strong pedagogical practices with well-defined educational objectives. They advocate for CAS as a tool that transforms students into active learners, enabling them to independently plan and execute problem-solving strategies by using CAS as a collaborative partner in the learning process. Their approach is aligned with a classification scheme for mathematical tasks, demonstrating how educational goals can be effectively met through CAS integration in teaching. In a recent review, Ní Shé et al. [9] synthesize insights from over 300 research articles, underscoring the paucity of studies specifically focused on student engagement with technology in undergraduate mathematics education. They also highlight the lack of a framework describing both pedagogical aspects and educational contexts of technology integration in mathematics and call for more structured research to deepen the understanding of technology's role in education.

As the integration of CAS has transformed mathematics education by offering students powerful tools to enhance problem-solving and conceptual understanding, a new wave of technological advancements is reshaping the landscape. LLMs, a form of GenAI, are increasingly being adopted in educational environments. These models, much like CAS, present opportunities to further revolutionize the way mathematics—and other disciplines—are taught and learned. The following section explores the role of LLMs in mathematics education, examining their potential to support both students and educators in a variety of academic settings.

2.2 LLMs in mathematics education

Large language models, a form of GenAI, are advanced systems designed to process and generate human-like text based on vast language datasets. Technologies like OpenAI's ChatGPT and Google's Gemini exemplify this innovation, which has attracted considerable attention for its potential to impact nearly all areas of human activity, education included [10]. In fact, the literature reveals a growing number of studies on the utilization of LLMs in teaching and learning. A closer examination of the possibilities presented by an LLM like ChatGPT reveals a comprehensive range of potential applications for both students and teachers. For students [11], ChatGPT emerges as a valuable tool that supports personalized learning, encourages creative thinking, enhances assessment, and strengthens reading and writing comprehension. For teachers [11], [12], ChatGPT proves useful in organizing and enriching instructional methods, facilitating student learning and assessment, and fostering improved communication with both parents and students.

Recent studies have explored the diverse applications of LLMs in higher education, highlighting their potential to transform learning experiences across various STEM disciplines. Tsai et al. [13] examine the integration of LLMs, particularly ChatGPT, into chemical engineering education, demonstrating how these tools support the construction of virtual models for problem-solving, promote critical thinking, and enhance students' problem-solving abilities. Similarly, Liu and Yang [14] investigate the application of LLMs in system modeling and simulation courses in engineering education, showing how LLMs assist students with programming tasks and mathematical logic analysis, facilitating cross-disciplinary learning and improving their understanding of complex concepts. Additionally, recent studies on the integration of LLMs into programming education have highlighted both the benefits and limitations of these technologies in aiding students with tasks such as code comprehension and software development. Vadaparty et al., [15] introduced an LLM-focused instructional design in a foundational computer science course, reshaping curriculum and assessments to utilize LLMs for code understanding, debugging, and testing. This shift enabled students to engage with open-ended projects, where LLMs facilitated problem decomposition and iterative learning. Nam et al. [16] investigated the use of LLMs specifically within an IDE for code comprehension tasks, finding that students benefited from prompt-less, contextually relevant interactions, which enhanced task completion rates by reducing the need for context-switching and minimizing search time for coding information.

In mathematics education, researchers have highlighted the potential of LLMs to enhance student engagement and comprehension. Based on teachers' perspectives, Taani and Alabidi [17] found that ChatGPT's ability to generate examples, explain complex concepts, and support problem-solving fostered an interactive learning environment that increased student engagement and understanding. Torres-Peña et al. [18], on the other hand, focused on students and conducted a classroom study integrating tools such as ChatGPT, MathGPT, and Wolfram Alpha into calculus instruction. They found that AI-assisted feedback on foundational topics such as derivatives and rates of change led to deeper student engagement. Additionally, Gouia-Zarrad and Gunn [19] demonstrated the benefits of project-based activities in undergraduate mathematics, where students used ChatGPT to tackle complex numerical tasks and code solutions for differential equations, further enhancing their engagement, coding confidence, and active involvement in learning.

Several studies [20]–[23] have also underscored the role of LLMs in developing critical thinking and problem-solving skills, with a particular focus on the importance of verification mechanisms. Barana et al. [20] found that while ChatGPT assists in outlining steps for combinatorial problems, students must critically engage with and verify AI-generated content due to the possibility of incomplete solutions. Similarly, Yoon et al. [21] introduced the Students' Interactive Proving Experience with AI (SIPE-AI) framework to help students critically assess AI-driven proofs, promoting independent reasoning over passive acceptance. Kumar et al. [22] further caution against over-reliance on LLMs, emphasizing that while detailed AI explanations can support learning, verification mechanisms are crucial to ensuring accurate and reliable understanding. Focusing specifically on numerical computations, the authors of [23] advocate for integrating CAS as a verification mechanism alongside LLMs, leveraging CAS's computational accuracy to reinforce AI-generated insights and enhance student problem-solving.

The potential drawbacks of LLMs in educational settings have also been noted in recent studies. Sánchez-Ruiz et al. [24] present a blended learning case study, demonstrating that while ChatGPT can enhance skills such as critical thinking and teamwork in engineering mathematics, concerns remain about potential gaps in core engineering competencies. Taking a broader perspective, Schei et al. [25] reviewed student perceptions of AI chatbots, including ChatGPT, and found that while students appreciated AI's role in increasing motivation and efficiency, they were also wary of its impact on originality and critical thinking. These findings underscore the importance of responsible LLM integration, ensuring that educational benefits are carefully balanced against risks to academic integrity and independent skill development.

Collectively, the studies reported in this section highlight both the transformative potential and the challenges of integrating LLMs into mathematics education. They indicate that while AI can significantly enhance learning experiences and skill development, its integration must be thoughtfully managed to foster not only knowledge acquisition but also essential critical-thinking skills.

The following section proposes a structured approach for integrating LLMs into a math lab alongside CAS, creating a collaborative environment where students can actively explore, solve problems, and critically engage with these advanced technologies.

3 A STRUCTURED APPROACH FOR INTEGRATING LLMS INTO MATH LABS

The studies reviewed above recognize both the educational potential and challenges of integrating LLMs into mathematics education, underscoring the need for a structured approach that supports teachers in incorporating these tools effectively and guides students toward responsible, meaningful use. In response, this study presents the ILAC approach—a structured method for incorporating LLMs and CAS into math labs. Designed to foster exploration, problem-solving, and reflective engagement, ILAC aligns with the constructivist principles of IBL, which encourage students to actively construct knowledge through exploration and self-directed inquiry [26], [27].

3.1 Workflow of the ILAC approach

Under the ILAC approach, math lab sessions offer students hands-on, computer-based problem-solving experiences that combine the intuitive, language-driven exploration of LLMs with the computational precision of CAS. The core objectives are to enhance students' problem-solving abilities, foster technological and AI proficiency, and deepen their understanding of how AI-driven tools complement traditional mathematical methods. By integrating CAS tools such as Maxima, Mathematica, and Maple with LLMs like ChatGPT and Gemini, students gain the ability to approach complex mathematical problems from multiple technological perspectives.

The ILAC workflow, illustrated in Figure 1, adapts stages from Matzakos et al. [23] while aligning with the IBL phases for collaboration with GenAI, as outlined by Moundridou et al. [12]. This adaptation tailors the IBL framework for math labs, fostering structured and collaborative engagement with LLMs and CAS tools.

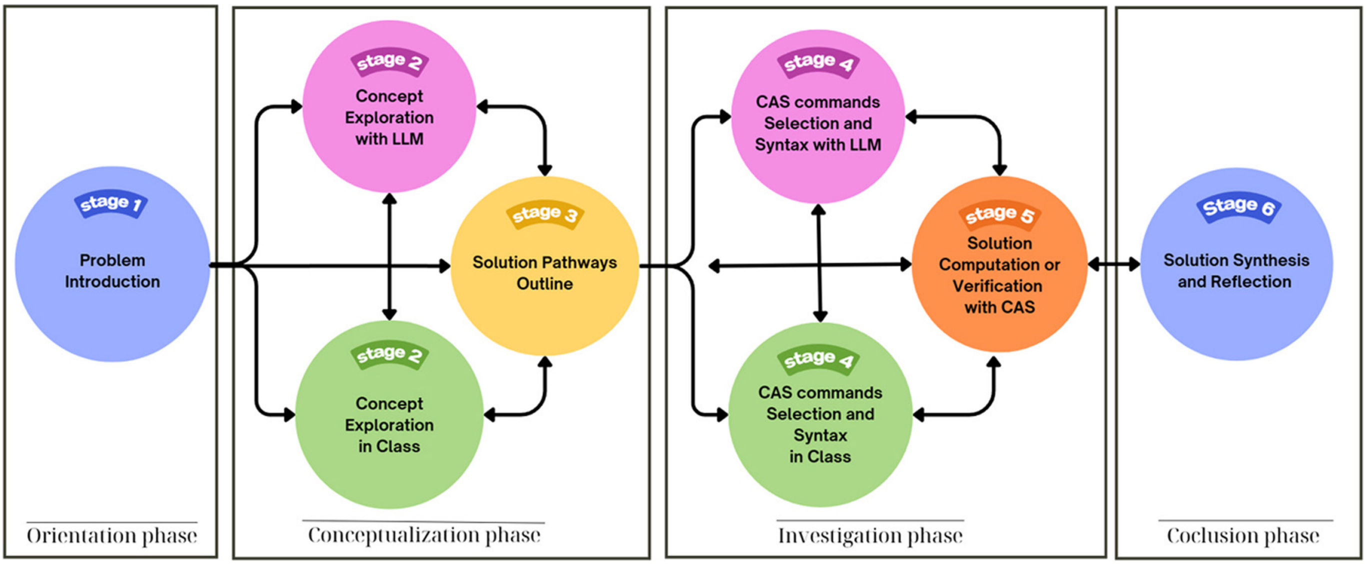

Fig. 1. Workflow of the ILAC approach for integrating LLMs and CAS in math lab settings

The following is a breakdown of the ILAC approach, demonstrating how LLMs and CAS can be jointly utilized to support inquiry-based problem-solving in educational contexts:

***Stage 1*: Problem introduction:** Aligned with the *orientation* phase in IBL, this stage begins with the instructor introducing the problem to spark students' interest, inspire curiosity, and establish a foundational understanding. The instructor presents relevant background material, offers an overview of the topic, and engages the class in discussion to encourage initial engagement with the problem.

***Stage 2*: Concept exploration:** Corresponding to the *conceptualization* phase in IBL, this stage encourages students to identify and articulate their questions or knowledge gaps related to the problem. Students may participate in class discussions or consult an LLM to clarify key concepts, deepen their understanding, and prepare for further exploration. This stage sets the groundwork for building a strong conceptual foundation before proceeding.

***Stage 3*: Solution pathways outline:** This stage mirrors the *investigation* phase in IBL, as students outline a structured approach for solving the problem. Here, they plan their methods and identify key steps that will guide them through the subsequent stages of the process. By defining their solution pathways, students develop a strategic approach to problem-solving, fostering a sense of ownership and confidence in their methods.

***Stage 4*: CAS commands selection and syntax:** Also aligned with the *investigation* phase, this stage involves students determining the specific CAS commands and syntax needed to implement their solution pathways. Students may discuss their choices with peers, consult an LLM, or seek guidance from the instructor to ensure command accuracy and appropriateness. This stage bridges the gap between conceptual planning and computational execution, helping students transition from strategy to action.

***Stage 5*: Solution computation or verification with a computer algebra system:** Continuing within the *investigation* phase, this stage requires students to use a CAS to compute a solution to the problem or to verify results generated by an LLM in earlier stages. Applying the CAS commands selected in Stage 4, students engage with the technical aspects of problem-solving, ensuring precision and accuracy in their computations. At this stage, students actively validate their work, reinforcing their solution pathways and preparing for final reflection.

***Stage 6*: Solution synthesis and reflection:** Aligned with the *conclusion* phase in IBL, this final stage prompts students to consolidate their findings and reflect on the entire problem-solving process. After verifying their solution in Stage 5, students assess the validity and completeness of their final answer, considering its alignment with the original problem requirements. They are encouraged to reflect on the strategies they employed, explore possible alternative approaches, and assess the effectiveness of both LLMs and CAS tools in achieving their solution. Instructors facilitate this reflection by guiding discussions on challenges encountered, insights gained, and connections to broader mathematical concepts. This stage not only reinforces the learning objectives but also cultivates metacognitive skills, as students critically analyze their process and results.

The ILAC workflow is designed as a flexible, cyclical process that supports iterative exploration, allowing students to revisit and refine earlier stages as needed. Although the workflow consists of sequential stages aligned with the phases of IBL, it incorporates multiple pathways for students to move back and forth between stages based on the insights and challenges they encounter. For example, after *Stage 5* (*Solution Computation or Verification with CAS*), students may discover gaps in their understanding or potential errors, prompting them to return to earlier stages such as *Concept Exploration* or *Solution Pathways Outline*. This cyclical structure reinforces the iterative nature of problem-solving in mathematics, where insights gained in one stage often necessitate adjustments in previous stages.

Additionally, certain stages within the *conceptualization* and *investigation* phases (i.e., *concept exploration* and *CAS commands selection and syntax*) provide two parallel pathways—consulting an LLM or engaging in class discussions. This dual-pathway approach not only allows students to access diverse resources but also fosters collaborative learning and encourages critical evaluation of AI-assisted solutions. Finally, *Stage 6* (*solution synthesis and reflection*) consolidates the cyclical process by prompting students to assess their solution comprehensively and reflect on their approach, thus completing the inquiry cycle while promoting metacognitive skills. Overall, the cyclical design of the ILAC workflow encourages deep, reflective engagement with both mathematical concepts and problem-solving strategies. By enabling students to iteratively refine their understanding, the ILAC approach supports a robust IBL environment that mirrors the dynamic and recursive nature of real-world problem-solving.

3.2 Functional and pedagogical opportunities of the ILAC approach

The ILAC approach integrates both CAS and LLMs, leveraging their complementary strengths to enhance mathematics education. By combining these tools in a structured, inquiry-based workflow, ILAC creates opportunities that go beyond the individual functionalities of CAS and LLMs, offering a holistic framework that fosters both technical proficiency and conceptual understanding. Building on the exploration of CAS in mathematics education [3], which highlighted the functional advantages of computational accuracy and the pedagogical benefits of supporting conceptual understanding, the ILAC approach further extends these opportunities through the integration of LLMs. As demonstrated in [23], LLMs add value by enabling natural language interaction and supporting adaptive learning pathways, which enhance student engagement and exploration.

Functional opportunities. The functional opportunities of the ILAC approach are most evident in the later stages of the workflow, where students engage directly with CAS and LLM tools to solve complex mathematical problems. In *Stage 4: CAS Commands Selection and Syntax*, students consult an LLM for guidance on the appropriate commands to use within a CAS. This interaction harnesses the functional strength of LLMs in natural language processing, enabling students to communicate their needs in everyday language rather than requiring detailed command syntax knowledge. This accessibility facilitates a user-friendly approach to problem-solving, making it possible for students to focus on the problem rather than on the technicalities of CAS commands [23].

In *Stage 5: Solution Computation or Verification with CAS*, students leverage the computational precision of CAS to perform calculations or verify solutions generated in previous stages. This use of CAS aligns with the functional opportunities identified by Pierce and Stacey [3], who emphasized CAS's capacity for accuracy and efficiency in mathematical computations. By allowing students to validate their work with CAS, ILAC ensures that they not only reach accurate solutions but also gain confidence in their computational abilities. Together, these stages illustrate how ILAC combines the functional strengths of both tools, creating a seamless and interactive learning experience that fosters efficiency, accuracy, and deeper engagement with mathematical concepts.

Pedagogical opportunities. The pedagogical opportunities of the ILAC approach are deeply rooted in its alignment with IBL, which emphasizes active exploration, collaboration, and reflection. As highlighted in [3], one of the strongest pedagogical benefits of CAS is their ability to support conceptual understanding by allowing students to explore mathematical relationships and visualize complex operations. ILAC builds on this by adding LLMs to the learning environment, expanding the pedagogical scope to include adaptive, language-based interactions that enhance student understanding. As noted in [23], LLMs facilitate exploratory learning by allowing students to pose questions and receive targeted guidance, fostering a personalized learning experience.

Throughout the ILAC workflow, students engage in activities that align with the pedagogical goals of IBL. For instance, in *Stage 2: concept exploration*, students have the option to consult an LLM or participate in class discussions, enabling a flexible, student-centered inquiry that caters to diverse learning preferences. This dual-pathway approach, available in both the conceptualization and investigation phases, fosters collaboration and critical evaluation by exposing students to multiple resources and perspectives. In *Stage 6: Solution Synthesis and Reflection*, students are encouraged to reflect on their strategies and evaluate the effectiveness of the tools they used, reinforcing metacognitive skills essential for mathematical thinking and lifelong learning. This stage embodies IBL's conclusion phase, where students consolidate their understanding, assess their approach, and connect their learning to broader mathematical concepts.

The cyclical design of ILAC further strengthens its pedagogical impact by allowing students to revisit and refine previous stages based on insights gained in later ones. As students' progress through the workflow, they may encounter new challenges or perspectives that prompt them to adjust their initial approach, promoting an iterative, reflective learning process. This design not only supports deeper conceptual engagement but also cultivates critical thinking and adaptability, as students learn to evaluate and improve their methods continuously. By fostering a learning environment where students actively engage with both CAS and LLMs, the ILAC approach promotes a balanced integration of procedural skills and conceptual understanding, equipping students to tackle the complex problem-solving demands of real-world mathematics.

To illustrate the ILAC approach in a real-world higher education setting, the next section presents an example problem that students might encounter in a mathematics lab. This example demonstrates how the outlined stages guide students through a structured, inquiry-based process, leveraging both LLMs and CAS tools to solve a complex mathematical problem.

4 APPLYING THE ILAC APPROACH: A SAMPLE PROBLEM FOR A MATH LAB

To demonstrate the ILAC approach in action, this section presents an example problem from a calculus course designed for undergraduate engineering students. This example illustrates how the structured stages of the ILAC framework can guide students through a calculus problem, integrating LLMs for conceptual exploration and CAS tools for precise computation.

Problem: Minimizing material cost in structural design. An engineering team is designing a truss structure for a bridge and needs to minimize the material costs. The cost function for the material distribution across two key components, x and y, where x and y represent the quantities of material allocated to each component, is given by: $C(x, y) = 8x^2 + 6y^2 - 2xy$. The goal is to determine the optimal allocation of materials that minimizes the overall cost $C(x, y)$. This requires finding the critical points of the cost function and classifying them to determine whether they represent a minimum, maximum, or saddle point.

Applying the ILAC workflow to the example problem

Stage 1: Introducing the optimization problem: The instructor introduces the problem to students, presenting the goal of minimizing material costs in a truss structure for a bridge. The instructor provides relevant background on the importance of optimization in engineering and the use of cost functions to model material expenses. By discussing the cost function $C(x, y) = 8x^2 + 6y^2 - 2xy$, students understand that x and y represent the quantities of material of the bridge structure and that their goal is to determine the optimal allocation that minimizes cost. This stage serves to engage students and ensure they understand the real-world context and mathematical objectives of the task.

Stage 2: Exploring key concepts: In this stage, students identify and address any questions or gaps in their understanding of key concepts, such as optimization, critical points, and classification of points as minima, maxima, or saddle points. They may participate in class discussions or consult an LLM to clarify terms and methods. Example queries to an LLM at this stage might include: "How do I use partial derivatives to identify the critical points of a two-variable function?", "What is the second derivative test, and how can it help me determine if a point is a minimum or maximum?" This exploration helps students prepare for the technical problem-solving steps that follow and ensures they have a solid conceptual foundation for working with the cost function.

Stage 3: Outlining solution pathways: Students outline a structured approach to solving the problem, planning the steps they will take to find and classify the critical points of the cost function. At this stage, students decide to calculate partial derivatives of $C(x, y)$ with respect to x and y, set these derivatives to zero to find critical points, and use the second derivative test to classify these points. This planning process fosters strategic thinking and allows students to map out their approach before delving into calculations.

Stage 4: Selecting CAS commands and syntax: Students now focus on selecting the specific commands and syntax needed to implement their solution pathway in a CAS tool. Consulting an LLM, if needed, students identify commands for calculating partial derivatives, solving equations to find critical points, and performing the second derivative test. Example queries to an LLM at this stage might include: "What CAS command should I use to find the partial derivative of $C(x, y) = 8x^2 + 6y^2 - 2xy$ with respect to x ?", "What command is used to perform a second derivative test in a CAS?". They may also discuss these choices with peers or the instructor to confirm the accuracy and suitability of their selected commands. This stage bridges the conceptual planning from Stage 3 with the computational execution they'll perform in the next stage.

Stage 5: Computing and verifying solutions with CAS: With their commands ready, students use a CAS to carry out the necessary computations. They calculate the partial derivatives of $C(x, y)$ with respect to x and y, solve for the critical points and apply the second derivative test to classify these points. As they verify each step, students actively engage with the technical aspects of the problem and ensure the accuracy of their results. This stage provides them with computational precision and confidence in their solution, preparing them to interpret their findings in the final stage.

Stage 6: Synthesizing and reflecting on the solution: In the final stage, students consolidate their findings by evaluating the cost function's critical points to determine which allocation minimizes the material cost. Students assess whether the solution makes sense in the context of material distribution for bridge design. They are encouraged to reflect on the problem-solving process by discussing any challenges encountered—such as understanding the cost function's structure or applying the second derivative test—and insights gained from using both the LLM and CAS. Instructors guide this reflection by prompting students to consider alternative approaches, such as different ways of setting up the cost function, and to think about how this optimization process could apply to similar engineering tasks where cost-efficiency is essential.

5 DISCUSSION

The ILAC approach represents a novel framework for integrating LLMs alongside CAS in mathematics education. By structuring the learning process around inquiry-based stages, ILAC facilitates a hands-on, exploration environment that combines the computational precision of CAS with the adaptive, language-based interactions of LLMs. This dual integration allows students not only to solve complex mathematical problems but also to engage critically with both computational and conceptual aspects of mathematics. Through this approach, students are encouraged to adopt an investigative mindset, applying inquiry-based methods that support deep learning and reflective engagement.

The ILAC workflow, with its cyclical design, aligns with the question, explore, discover (QED) paradigm introduced by Damaševičius and Zailskaitė-Jakštė in [31], which emphasizes iterative exploration and discovery as core to AI-integrated education. Both QED and ILAC highlight the importance of questioning and investigation in building knowledge, allowing students to move between stages as new insights emerge. In ILAC, stages such as Concept Exploration and Solution Synthesis and Reflection mirror the QED framework's focus on encouraging students to pose questions, use AI to explore those questions, and synthesize findings to reinforce understanding. This cyclical, adaptive process not only enhances students' problem-solving abilities but also promotes critical thinking and metacognitive skills, fostering a self-directed learning environment that is increasingly relevant in modern education.

From a functional perspective, the ILAC approach leverages the unique strengths of CAS and LLMs in complementary ways. CAS, as discussed by Pierce and Stacey [3], provides computational accuracy and efficiency essential for verifying solutions and handling complex calculations. In parallel, LLMs enable accessible, natural language interactions that help students query mathematical concepts more intuitively, as highlighted by Matzakos et al. [23]. This combination addresses key challenges noted in prior research, including students' difficulties with CAS syntax [7] and the need for reliable verification of LLM-generated solutions [20], [22]. By guiding students to engage iteratively with both tools, ILAC helps reduce cognitive load, enabling them to focus on understanding and applying mathematical concepts rather than struggling with technical barriers.

The ILAC approach also provides substantial pedagogical benefits, supporting the constructivist principles that underpin IBL. Pierce and Stacey [3] emphasized the pedagogical value of CAS in promoting conceptual understanding, while in [23] LLMs were identified as effective tools for personalized learning through intuitive natural language interactions. In ILAC, students are encouraged to explore and clarify their understanding (e.g., through LLM-guided concept exploration in Stage 2) and reflect on their solutions in Stage 6, promoting deeper conceptual engagement and long-term retention of mathematical principles.

While the ILAC approach presents substantial potential, several challenges and limitations must be acknowledged regarding both its implementation and impact. First, effective use of ILAC requires students to navigate between LLMs and CAS efficiently. While this flexibility is one of ILAC's strengths, it may also pose a challenge for students unfamiliar with either tool. Moving between conceptual exploration (via LLMs) and computational verification (via CAS) requires a certain level of digital literacy and mathematical maturity, which may not be evenly distributed among learners. As a result, students with limited experience in AI-assisted problem-solving may struggle to fully engage with ILAC, leading to variations in its effectiveness.

Second, helping students balance the use of CAS and LLMs—understanding each tool's strengths and limitations without becoming overly dependent on AI—requires thoughtful instructional design, along with continuous educator support and feedback. Consequently, ILAC's success largely depends on educator preparedness, which can be enhanced through comprehensive teacher training and professional development programs covering both the technical and pedagogical aspects of AI-enhanced learning [32], [33].

Third, several ethical considerations must be addressed. Students may be tempted to rely on LLMs for direct solutions rather than engaging critically with the problem-solving process, as ILAC intends. This raises concerns about academic integrity and highlights the need for strategies that encourage active engagement rather than passive AI reliance. Following established frameworks, such as UNESCO's 'Guidance for Generative AI in Education and Research' [34], can help institutions and educators implement LLMs responsibly, fostering an environment that respects both academic rigor and ethical standards.

Finally, ILAC has not yet been empirically validated, and its impact on learning outcomes remains an open question. Additionally, while ILAC is well-suited for structured problem-solving tasks, its applicability to other mathematical domains, such as proof-based courses or abstract mathematical reasoning, requires further investigation.

6 CONCLUSIONS

This study introduced ILAC as a structured approach that integrates LLMs and CAS within an IBL framework. ILAC capitalizes on both functional and pedagogical opportunities by combining the computational power of CAS with the language-based adaptability of LLMs in a structured workflow. This integration provides students with a dynamic learning environment that promotes conceptual exploration, problem-solving efficiency, and critical thinking. By leveraging the strengths of both tools, ILAC not only enhances mathematical reasoning but also supports the development of AI literacy, better preparing students for the evolving landscape of mathematics education.

While ILAC presents a pedagogically sound approach, its practical implementation and impact require further investigation. Future research should focus on empirical validation through case studies and controlled experiments in educational settings. Specifically, studies could assess student engagement, problem-solving efficiency, and conceptual understanding when using ILAC compared to traditional CAS-only or LLM-only approaches. Quantitative metrics such as problem-solving accuracy, time efficiency, and verification success rates could provide insights into its effectiveness. Additionally, qualitative analyses of student attitudes and learning experiences can help refine instructional strategies for optimal integration of AI-driven tools in mathematics education.

Beyond validation, future work should also explore ILAC's adaptability to different mathematical domains, such as proof-based courses and abstract reasoning, where AI tools may function differently. Investigating best practices for teacher training and professional development will be essential to ensure effective implementation. Furthermore, as educational policies evolve around AI usage, ongoing research should examine ethical considerations and academic integrity safeguards in ILAC-driven learning environments.

Exploring these research directions will be essential for refining ILAC and ensuring its effective and responsible integration into mathematics education, ultimately helping students develop the skills needed to engage with AI-driven problem-solving.

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