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PAPER

Didactic Interventions in a Large Group Mathematics Classroom with a Focus on Tutorial Sessions

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ABSTRACT

This paper examines how first semester tutorial sessions in engineering mathematics can be effectively designed to support student learning. The study focuses on managing the challenges of large-group teaching while maintaining high learning outcomes. Central to the approach are three practices developed by Peter Liljedahl: the use of vertical surfaces, structured problem-solving, and visibly random groupings of three students. The implementation of these strategies in tutorial settings is analysed through qualitative interviews with two associate professors and several teaching assistants (TAs), alongside quantitative data from student surveys, previous academic records, and final course results. Students in the middle group, according to their high school mathematics grades, are found to benefit significantly from attending tutorial sessions. The paper concludes with five recommendations for educators aiming to improve learning in large-scale tutorial environments based on our findings.

KEYWORDS

engineering mathematics education, tutorial sessions, thinking classroom, teaching assistants (TAs)

1 INTRODUCTION

As more students pursue degrees in engineering, the demand for scalable and effective higher education in the field continues to grow [1] and a focus on cost-efficiency measures has led to a significant expansion in class sizes, shifting from smaller cohorts of students to larger student cohorts [2]. This scaling introduces challenges in maintaining educational quality, as traditional teaching methods often struggle to engage students effectively in such environments [3]. Engineering students, who typically thrive in active learning environments, may find passive lecture formats less supportive of their learning needs [4].

Mathematics, which is a cornerstone of engineering education, poses significant challenges for students globally, with high dropout rates in mathematics-related courses being a recurring issue [5]. For instance, at the Latvia University, over 25%

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of engineering students withdrew after the first year mainly because of difficulties in mathematics [5].

To address both the growing number of students and the increasing demands for cost-efficiency measures, it is essential to implement strategic initiatives that optimise resource allocation while maintaining educational quality. To address these challenges, universities are increasingly relying on teaching assistants (TAs), who are often senior undergraduate or PhD students, to facilitate tutorial sessions, which now play a crucial role in supporting student learning in large classes [6]. However, this shift raises questions about how these tutorial sessions can be effectively designed and executed to maximise learning outcomes.

Already in 2023, the authors explored different didactic approaches in two separate mathematics courses within the engineering program. The electro-mathematics course had approximately 30 students enrolled, while the robot-mathematics course had around 80 students. These earlier studies received highly positive feedback from both students and teaching staff about the didactical approaches in the lectures and TA-sessions [7]. However, in 2024, these smaller course formats were replaced by a significantly larger combined course with 360 students, marking a substantial shift in scale and how to organise the newly developed didactic approaches.

This study seeks to explore how different didactic approaches can be integrated into mathematics teaching within engineering programs, with a specific emphasis on tutorial sessions led by TAs under the guidance of associate professors. This is achieved by examining the experiences of students, TAs, and associate professors to assess the effectiveness and outcomes of teaching practices, focusing on the enabling conditions and limiting constraints in this larger course. By examining these aspects, this paper seeks to address the following research question:

RQ: How can tutorial sessions in first semester mathematics be structured and conducted in a way that enhances learning outcomes while accommodating the complexities associated with larger student groups?

2 BACKGROUND

The theoretical foundation for this study draws on theories of educational scalability, TAs and learning environments in higher engineering education.

2.1 Scalability in higher engineering education

The expansion of student enrolment in universities poses challenges for maintaining effective learning environments. As class sizes increase, personalised interaction diminishes, and new instructional approaches are required. The theory of instructional scalability emphasises the need for adapting pedagogical techniques to ensure quality and student engagement in large cohorts [8]. The concept of scalability in higher education has become increasingly relevant as universities worldwide face a significant rise in student enrolment, a phenomenon often referred to as the massification of higher education [9]. This massification represents the transition from elite education, where only a small, select group of students attended universities, to mass and universal access, which has made higher education accessible to a broader segment of the population. While this expansion is a positive step towards educational equality and economic development, it simultaneously introduces significant challenges for maintaining effective and engaging learning environments.

One of the key challenges of scalability is the diminished opportunity for personalised interaction. In small class settings, associate professors and TAs can provide individualised support, respond to student queries in real time, and facilitate dynamic discussions. However, as class sizes increase significantly—e.g., well over 100 students—these interactions become logistically unfeasible. Large classes tend to prioritise lecturing above active learning, as the latter requires more time, organisation, and resources [10]. This shift can result in reduced student engagement, lower participation, and increased feelings of anonymity among learners, which are all factors that negatively impact the quality of education [11].

To address these challenges, [10] emphasises the importance of adapting pedagogical techniques and learning environments to suit larger cohorts without compromising quality. This involves identifying strategies that maintain the essential components of effective teaching—student engagement, interaction, and feedback—while accommodating the limitations of large-scale education. For instance, blended learning approaches, which integrate online platforms with face-to-face sessions, have been proposed as a scalable solution to improve interaction and support active learning in large classes [12]. Moreover, effective scalability often relies on the strategic involvement of TAs. TAs play a crucial role in large classes by facilitating smaller group tutorials, problem-solving sessions, and practical exercises. This enables universities to provide structured, interactive learning opportunities while balancing resource constraints. However, the role of TAs introduces its own challenges, such as ensuring they receive sufficient training, resources, and support to deliver quality instruction [13]. Poorly managed TA involvement can exacerbate inconsistencies in teaching quality and student experiences across different sections of a course [13].

Traditionally, TAs were seen primarily as administrative support, assisting associate professors with grading, preparing materials, and occasionally leading small discussion sections. However, in the context of large, lecture-based courses, their role has evolved to encompass a more interactive and student-centered function [13]. Research has shown that students in large classes are more likely to participate and seek clarification, when they can interact with TAs in smaller, more manageable groups. This interactive role allows TAs to bridge the gap between lecture content and practical application, thereby enhancing the learning experience [10].

The increasing involvement of TAs in the instructional process also requires proper training and professional development to ensure their effectiveness. Research has highlighted the importance of providing TAs with the necessary pedagogical training [14]. Such training ensures that TAs are equipped to handle the complexities of large class teaching and maintain the quality of the student experience.

However, the evolving role of TAs also presents challenges. TAs may face difficulties in balancing their responsibilities as TAs with their own academic commitments, particularly when they lack sufficient support or guidance from faculty members [13]. Furthermore, the effectiveness of TAs in promoting student engagement and learning depends on how well their role is integrated into the overall course design and teaching strategies. If their contributions are not well-aligned with the course's pedagogical goals, their impact may be limited. In fact, it might even be detrimental, potentially disrupting the learning process or creating confusion among students.

Ultimately, scalability in higher education is not just about increasing the capacity to teach more students but ensuring that the learning experience remains effective and engaging. Successful scalability depends on innovative approaches to teaching and the strategic distribution of instructional and TA roles. It requires a shift from traditional teaching models to ones that prioritise active, student-centred learning.

2.2 Learning environments in tutorial sessions

The book Building Thinking Classrooms [15] introduced 14 practices for increasing thinking in classrooms. These practices are developed to improve the teaching and learning environment in mathematics classrooms. The practices are ordered in four toolkits according to when to introduce them in the classroom. The practices in the first toolkit include the use of vertical surfaces, the use of random visible groups of three, and to use problem-oriented tasks. Each of these practices plays a crucial role in fostering active participation, collaboration, and critical thinking among students [15]. This approach has been used and tested in many primary and secondary schools, but less so at the university level [7].

LeSage [16] examined the effects of the Building Thinking Classroom on students' attitudes and academic performance in a first-year business mathematics course, focusing on collaborative problem-solving using vertical non-permanent surfaces. Their findings showed that students rated the collaborative classroom experience positively and achieved significantly higher average grades than those in the control group. The study particularly highlighted benefits for lower-performing students: while 46% of students in the control group received low grades (responding to D or F), only 14% did so in the intervention group, which also had a higher proportion of students earning top grades (responding to A and B).

Despite these promising results from [16], there remains a lack of research on how this approach can enhance university mathematics in engineering education. Further studies are needed to explore both instructor and student experiences with this method at the tertiary level.

Vertical surfaces. Research by Liljedahl [15, 17] highlights how vertical non-permanent surfaces, such as whiteboards, create a dynamic learning environment that fosters active thinking and productive discussions. Writing on vertical surfaces encourages students to engage physically with the material, increasing focus, persistence, and participation. Furthermore, using these surfaces supports knowledge mobility by making students' thought processes visible to their peers, thereby facilitating collective problem-solving [17].

Liljedahl's [17] research demonstrates the significant impact of vertical non-permanent surfaces on students' learning behaviours. He measured key aspects such as the time elapsed between task assignment and the initiation of mathematical discussions, the duration of group discussions, and the extent of engagement with mathematical content. His findings indicate that vertical non-permanent surfaces are more effective than traditional group work methods, such as discussions around horizontal paper or computers, in fostering deeper mathematical thinking. Students working in groups at vertical non-permanent surfaces should not only encourage experimentation and risk-taking but also allow for rapid revisions of their solutions. This adaptability strengthens problem-solving strategies and ensures that all students contribute to class discussions, promoting an inclusive and collaborative learning environment [17].

Additionally, vertical non-permanent surfaces can play a crucial role in developing mathematical communication skills. Working on whiteboards requires students to articulate their mathematical reasoning clearly, aligning with Sfard's

[18] concept of "commognition," which merges communication and cognition. Since learning mathematics involves mastering its communicative practices, engaging in discussions around vertical non-permanent surfaces should enhance students' conceptual understanding and strengthen their sense of classroom community.

Random groups of three. Group work has long been a common pedagogical practice [19, 20], but research has also highlighted its challenges, particularly in how groups are formed [21]. Traditionally, students at the secondary and university levels often form their own groups, usually based on existing friendships. However, [15] points out that socially formed groups tend to reinforce pre-existing roles, and friendship-based groups do not necessarily lead to effective collaboration. An alternative approach is for the instructor to determine group composition, either to ensure diversity in terms of gender or competencies, to encourage social integration, or to structure groups based on academic performance.

[15] argues that randomly assigning students to groups of three in a visible process fosters a more open, diverse, and equitable learning environment. In our course, students work in randomly assigned groups of three on various tasks and new groups were formed every hour.

[17] argues that the visibility creates a sense of collective responsibility, encourages active participation, and enhances peer learning. Furthermore, continuously reshuffling group compositions ensures that students interact with a wide range of peers, broadening their perspectives and exposing them to different problem-solving approaches. This method also helps minimise social hierarchies and power dynamics, ensuring that all students have equal opportunities to contribute [17].

Another key advantage of random grouping is its impact on knowledge mobility [17]. As students regularly engage with new peers, they gain access to different ways of thinking and alternative strategies, enriching their learning experience [15]. Pruner and Liljedahl [22] argue that this approach not only fosters collaboration but also contributes to a stronger classroom community by reducing social barriers and mitigating conflicts.

Problem solving. The integration of problem-oriented tasks into tutorial sessions aims to align with Liljedahl's [15] Thinking classroom framework, fostering deeper engagement and independent problem-solving. These tasks should be designed to encourage higher-order thinking, collaboration, and application of theoretical knowledge. Instead of reinforcing routine exercises, the approach is expected to cultivate cognitive flexibility and problem-solving competencies [23]. A critical objective by using problem solving is to promote relational understanding over instrumental learning [24]. Rather than focusing solely on procedural fluency, students should be guided toward recognising connections between mathematical concepts and their broader applications [25]. Encouraging active participation in problem-solving is anticipated to develop reasoning skills and conceptual depth [26].

To maximise effectiveness, tasks should be introduced at the beginning of sessions following Liljedahl's [15] "low-floor, high-ceiling" principle. This structure is expected to ensure accessibility for all students while also maintaining sufficient complexity to challenge advanced learners. Research suggests that such an approach fosters meaningful engagement and enhances mathematical resilience [27]. Additionally, shifting from traditional instructor-led explanations to student-driven inquiry is hoped to mitigate dependency on external guidance [28], allowing students to develop autonomous problem-solving skills.

3 COURSE STRUCTURE AND IMPLEMENTATION IN THE ENGINEERING PROGRAM

In a Danish engineering program, a decision was made to consolidate ten study programs into one single large mathematics cohort in the first semester of the program named MAT1. The programs included: Chemical Engineering and Biotechnology, Civil and Architectural Engineering, Electronics, Energy Systems, Game Development and Learning Technology, Health Informatics and Technology, Mechanical Engineering, Product Development and Innovation, Robot Systems, Software Engineering, resulting in a combined class of approximately 360 students.

For the class it was chosen to implement the use of vertical non-permanent surfaces, visibly random groups of three and the use of problem-oriented tasks as suggested by [15]. By incorporating vertical non-permanent surfaces into the tutorial sessions, we aim to foster a more interactive and engaging learning experience, supporting students in developing both their mathematical reasoning and their ability to communicate complex ideas effectively [17, 15, 18]. With the use of visible random groups of three in the tutorial sessions, we aim to create an inclusive and dynamic learning environment that supports student engagement, encourages diverse perspectives, and enhances collaborative problem-solving [15, 22]. By embedding problem-oriented tasks into tutorial sessions, our goal was to create an environment where students actively explore multiple solution pathways and build confidence in their mathematical abilities. This instructional shift was expected to enhance both cognitive and affective learning outcomes, reinforcing the value of independent thinking and deep mathematical understanding [15, 23, 24, 25].

By implementing these three practices, the tutorial session aimed to create a more interactive and engaging learning environment. A further rationale for this approach is the recognition, that the task of the TAs becomes significantly more manageable when their role is well-defined and expectations are clearly communicated. Ensuring a clear structure and guidance for the TAs is therefore a key responsibility of the associate professors responsible for the course, as it directly supports the effective execution of the tutorial sessions. The following sections will explore how these practices were applied in the tutorial sessions and discuss their impact on student engagement, collaboration, and learning outcomes.

To manage the large cohort, the students were further divided into eight tutorial groups. Each week the course had this structure:

1. Preparation phase: Students were provided with online exercises, instructional videos and reading materials to engage independently before attending classes.
2. TA-Led tutorial sessions: Students participated in exercise sessions led by TAs. During these sessions, they worked on problems and exercises closely connected to the preparatory content. Each session took two hours, and they were designed following Peter Liljedahl's three practices:
   2.1 Use of Vertical Non-Permanent Surfaces (whiteboards),
   2.2 Visible Random Grouping groups of three students
   2.3 Problem-oriented tasks that encourage active problem-solving.
3. A two-hour lecture: Following the tutorial sessions, students attended formal lectures that expanded on the material. This also included different interactive approaches such as visualizations in GeoGebra, use of student response systems based on drawings and think-pair-share sessions.

4. Weekly assignments: At the end of each week, students were encouraged to submit a video based on a specific assignment. The videos were subsequently peer-reviewed by fellow students. The focus of the assignments was to let the students explain their calculations.
5. Finally, there was a homework café. The homework café was managed by students on their 3rd and 5th semester. The purpose of the homework cafe was to provide a place to get help if the regular teaching was not enough.

Each tutorial session was planned for 45 students and manned by two TAs. While the TAs were primarily responsible for conducting the tutorial sessions, they could get help from an associate processor when challenges or issues arose.

The TAs had somewhat different backgrounds, but they were all enrolled in an engineering education on either bachelor or master level. TA1 and TA2 had been TAs in smaller courses based on Liljedahls ideas in 2023, four TAs (TA3, TA4, TA5 and TA6) had been students in one of these courses. The remaining four TAs had no previous experience with the ideas of Liljedahls approach. TA7 and TA8 had earlier been TAs on more traditional courses. An overview of the TA's, their previous experience with being a TA and the ideas from Building Thinking Classrooms are given in Table 1.

To familiarise the TAs with the instructional approach by Liljedahl [15], all ten were invited to participate in a workshop. The session began with a brief introduction to the method, after which the TAs experienced the approach themselves by engaging in activities as if they were students. This was followed by an opportunity to discuss the method and ask questions. Having tried the approach first-hand, all the TAs responded very positively to the approach.

4 METHODOLOGY

The choice of a mixed-methods approach allows for a more nuanced understanding of the research problem by integrating the complementary strengths of qualitative and quantitative data. The qualitative interviews provide in-depth insights into TA's experiences, reflections, and challenges, which may not be fully captured through numerical measurements alone. Conversely, the quantitative survey data enable the identification of patterns, trends, and correlations across the entire student cohort. This methodological combination enhances the validity of the study and offers a broader and more robust basis for evaluating both the effectiveness of tutorial sessions and the factors influencing learning outcomes [29].

4.1 Interview process and participant selection – the qualitative data

Before the course began, we conducted interviews with five TAs, selected from the ten invited, to understand their expectations and perceived challenges in teaching large groups. These five participants were those who responded positively when all ten were asked if they were willing to participate. These interviews focused on their readiness, anticipated difficulties, and their views on the course structure.

Midway through the course, a second round of interviews was held with two out of ten TAs (both of whom had also participated in the pre-course interviews). Again, these two were those who responded positively when all ten were asked if they were willing to participate. These interviews aimed to gather feedback on how the sessions were progressing, the challenges they felt that the students faced, and the effectiveness of teaching strategies, including random grouping and vertical surfaces. After the course there was a final interview with two out of the ten TAs (one of them had also participated in the pre-course and midway interview). Again, these two were those who responded positively when all ten were asked if they were willing to participate in an interview. To this last interview they were asked if there was anything they would do differently and about their reflection after finishing the teaching. At the end of the course, there was also a follow-up interview with two faculty members – the two associate professors (AP1 and AP2) associated with the course which also are authors on this paper. They were interviewed about the effectiveness of the course structure and their instructional methods.

Table 1. An overview of the TAs participating in the interviews

Id	Experience with BTC/TA	Pre-Course Interview	Midway Interview	After Course
TA1	BTC TA			
TA2	BTC TA			x
TA3	BTC student	x		
TA4	BTC student	x	x	x
TA5	BTC student	x		
TA6	BTC student	x		
TA7	TA Mathematics	x	x	
TA8	TA Statistics			
TA9	None			
TA10	None			

Note: BTC TA: have been TA in a course based on BTC. BTC student: have experienced BTC as a student.

All interviews were conducted online and recorded via Microsoft Teams. Informed consent was obtained from all participants, ensuring confidentiality.

The interview data were analysed using thematic analysis [30], which identifies recurring themes within the data to provide a rich understanding of participants' experiences. The analytical approach will be elaborated more upon in the analysis section.

4.2 Analysis of qualitative data

Based on all qualitative data – including open-ended responses from the survey and interviews with TAs and associate professors – we conducted a thematic analysis following Braun and Clarke's six-phase framework [30], in order to identify key themes that reflect both the strengths and challenges associated with the tutorial sessions structured around Liljedahl's three practices. The quantitative results were already known at the outset of the qualitative coding process, which informed but did not predetermine the development of codes and themes.

First, we engaged in familiarisation with the data through repeated readings and initial notetaking to ensure a comprehensive understanding of the content.

During the second phase, when generating initial codes, we paid particular attention to the three practices but were open to what we found important and interesting in connection to the complexities associated with larger student groups. Third, these codes were collated into potential themes by grouping related codes that reflected patterns in the TA's experiences and perceptions. One particularly prominent theme, which emerged consistently across both survey responses and interviews, was the issue of alignment – that is, the perceived coherence between teaching activities, expectations, and assessment. Given the centrality of this concern, a theme addressing alignment has been included alongside the three practices from Liljedahl [15].

Fourth, the four themes were then reviewed in relation to both the coded extracts and the full dataset to ensure internal coherence and consistency with the overall narrative. Fifth, the four themes were clearly defined and named, with careful attention to the conceptual boundaries between themes. Finally, the four themes were organised into a coherent account, illustrated with representative quotations, to provide a rich and nuanced interpretation of the qualitative findings. This systematic process ensured methodological rigour and transparency, thereby strengthening the validity of the qualitative component of the study.

4.3 Quantitative surveys

Quantitative data on student participation in tutorial sessions (the number of times they attended), completion of assignments, student average grade in high school mathematics, and their final exam performance (n = 282) was collected. This allowed us to examine the correlations between engagement in learning activities and exam performance. Each student could participate in 12 tutorial sessions and complete 12 assignments. The maximum number of points in the written part of the exam was 100. The average high school mathematics grade in Denmark is a number in the range from –3 to 12 (higher is better).

At the exam, the students first took a one-hour multiple-choice question-based test followed by a two-hour written exam. In the second (written) part of the exam, the students were given ten questions to answer. The answer for each question was during grading assigned a score between zero and ten points. Each of the ten questions focused on a specific mathematical topic, and for each topic, students had one lecture, one tutorial session, and one assignment dedicated to exploring the content.

Quantitative data were also gathered through two surveys administered to the students: a mid-term evaluation (n = 221) and a final evaluation (n = 156). These surveys were designed to capture students' experiences with and utilisation of various elements of the teaching process, including their perceptions of the course structure, teaching methods, and learning outcomes. The students assessed the effect of learnings elements on a scale from 0 to 4, where 0 means no effect and 4 means very big effect. Additionally, students were asked to self-assess their perceived level of proficiency in mathematics, categorising themselves as belonging to the highest, lowest, or middle tier, both during high school and at the university level. In all surveys, multiple open-ended response options were included. These open-ended responses are also incorporated into the qualitative analysis.

Attendance in tutorial sessions, assignment completion and exam performance. In the following, the relation between how students performed in the written part of the exam and their course activity during the semester is investigated. The following linear model was fitted to the data, where the number of completed assignments (n_a), the number of attended tutorial sessions (n_{ts}) and the average high school mathematics grade (*HSMath*) all were used as predictors to estimate the students score on the written part of the exam (*wp*).

$$wp = \alpha \cdot n_{ts} + \beta \cdot n_a + \gamma \cdot HSMath + \delta$$

The statistics software R was used to perform the statistical analysis, in which the values of the parameters α, β, γ, and δ delta were determined.

Contributions from related tutorial sessions and assignments on exam questions. A fixed effect model is used to investigate the relative importance of tutorial sessions and assignments. For each student the performance on the ten questions of the written exam was related to the student attendance of the tutorial session related to the exam question and the completion of the assignment related to the exam question. The model is as follows:

$$Y_{ij} = \alpha \cdot ass_{ij} + \beta \cdot tut_{ij} + \gamma_i + \delta_j + \epsilon_{ij}$$

Where Y_{ij} is the number of points that student i earned on question j in the exam. ass_{ij} is one if student i completed the assignment related to question j otherwise the value is set to zero. tut_{ij} is one if student i attended the tutorial session related to question j, otherwise the value is set to zero. γ_i is a term reflecting the proficiency of student i and δ_j reflects the difficulty of question j on the exam. ϵ_{ij} is an error term that should be minimised.

Effect on student groups. To see whether certain groups of students benefit more from attending the tutorial sessions, the analysis is repeated by dividing the students into six ability groups according to their average high school mathematics grade. The effect of attending the tutorial sessions is then estimated for each of these groups.

5 RESULTS

In this section the obtained results from the quantitative data are presented.

5.1 Results from quantitative data

Attendance in tutorial sessions, assignment completion and exam performance. The estimated parameters and their standard errors are as follows: $\alpha = 1.00 \pm 0.32$, $\beta = 2.10 \pm 0.37$, $\gamma = 4.69 \pm 0.44$ and $\delta = -5.5 \pm 4.1$. There is a positive impact of attending the tutorial sessions ($p < 0.01$) and completing the assignments ($p < 0.001$). There is no significant difference between the impact of tutorial sessions and assignments.

Contributions from related tutorial sessions and assignments on exam questions. The values and standard errors of the parameters α and β were estimated to be $\alpha = 0.19 \pm 0.24$ and $\beta = 0.67 \pm 0.24$. We see that attending the tutorial session related to an exam question raises the evaluation of the answer for that question by 0.67 ($p = 0.006$). The effect of assignments is not significant.

Effect on student groups. The estimated effects are shown in Figure 1. Students with an average high school mathematics grade in the range 7 to 11 are seen to benefit the most from the tutorial sessions. Students outside this range did not benefit significantly from the tutorial sessions.

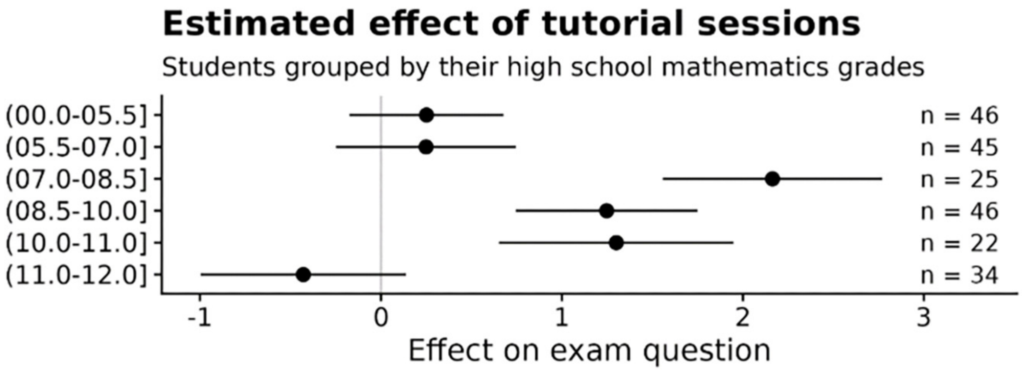

Fig. 1. Estimated effect of tutorial sessions on different larger student cohorts according to a fixed effects model

Students with an individual average high school mathematics grade below 7, do not benefit significantly from the tutorial sessions, whereas students with an average grade between 7 and 11 have a huge benefit. The number of students in each group is shown on the right.

Summary of results. The analysis of the quantitative data uncovered several important insights. First, the data showed that completing the assignments and attending the tutorial sessions during the course had a significant positive impact on student performance to the final exam.

Additionally, attending the tutorial session relevant to a specific exam task significantly improved students' performance on that task. While completing the assignment relevant to a specific exam task had a positive effect, it was not statistically significant. These findings emphasise the value of active engagement through assignments and exercise sessions in supporting student learning and academic success. Finally, it is seen in Figure 1, that students in the middle range (with average high school mathematics grades between 7 and 10) benefit the most from attending tutorial sessions, while the students with a grade higher than 10 seem to benefit the least.

While these findings may not be entirely surprising, as active engagement in the different learning activities is often assumed to benefit learning, they are nonetheless valuable in providing clear, empirical evidence to support this assumption.

5.2 Student reported effects and their use of teaching elements

In the last lecture, as part of the end of course evaluation, the students were asked to evaluate the learning effectiveness of several elements in the course. These results are shown in Figure 2, where the student responses are divided according to their perceived math level. In the figure the teaching elements are arranged in order of the student evaluations. The elements videos, assignments, online exercises and lectures are all evaluated similarly by the students across their perceived math levels. This is not the case for the tutorial sessions, which are rated very highly by the students in the top math skill group. The students in the middle and low math skill groups rate the tutorial sessions somewhat lower. About one third of the students in these two groups rate the tutorial sessions to have a "very big effect", which is a much higher fraction than the lectures to have a "very big effect". This indicates that there is significant potential in the tutorial sessions. However, about a fourth of the students in the middle and low math skill groups rate the tutorial sessions to have "no effect" or "small effect". This indicates that there is a group of students for which the tutorial sessions did not work that well.

The students also answered questions related to their use of the different teaching elements in the course. These results are shown in Figure 3. The teaching elements in the figure are ordered by the students' usage of the elements, such that the most used elements are at the top. Very high utilisation of the lectures and assignments are seen. The numbers related to the utilisation of the lectures are probably biased, as the end of course evaluation took place during the last lecture. The students seemed to prioritise lectures, assignments and videos above participation in the tutorial sessions. Finally, very few students are seen to use the homework cafe.

In Figure 4, the student assessed effect of the tutorial sessions from the course in 2024 are compared with the assessed effect in the two smaller courses from 2023. The students evaluated effect in 2024 is somewhat lower than the assessed effects from 2023, but still quite high. From 2023 to 2024 the class size increased significantly, which made it more difficult to conduct the tutorial sessions.

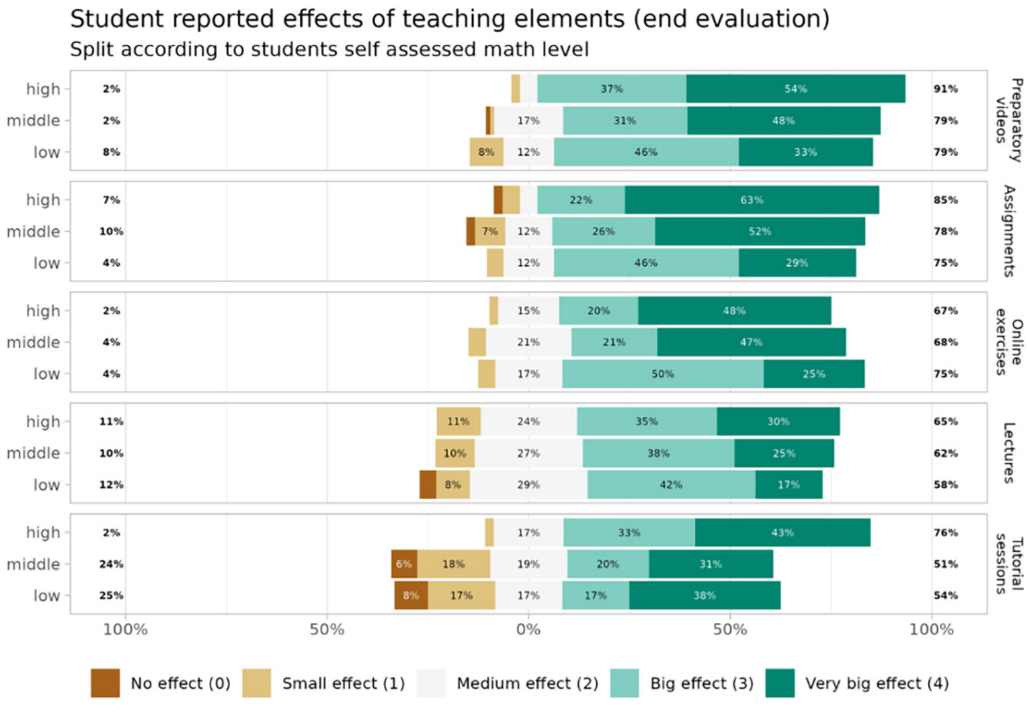

Fig. 2. Students self-reported effects of various teaching elements used in the course

The students have been split into groups according to their self-evaluated math levels. The tutorial sessions are rated lowest, but it is also the first year where this setup is used. The other teaching elements have been refined over several years. The most math-proficient students assign the highest estimated effect to the tutorial sessions.

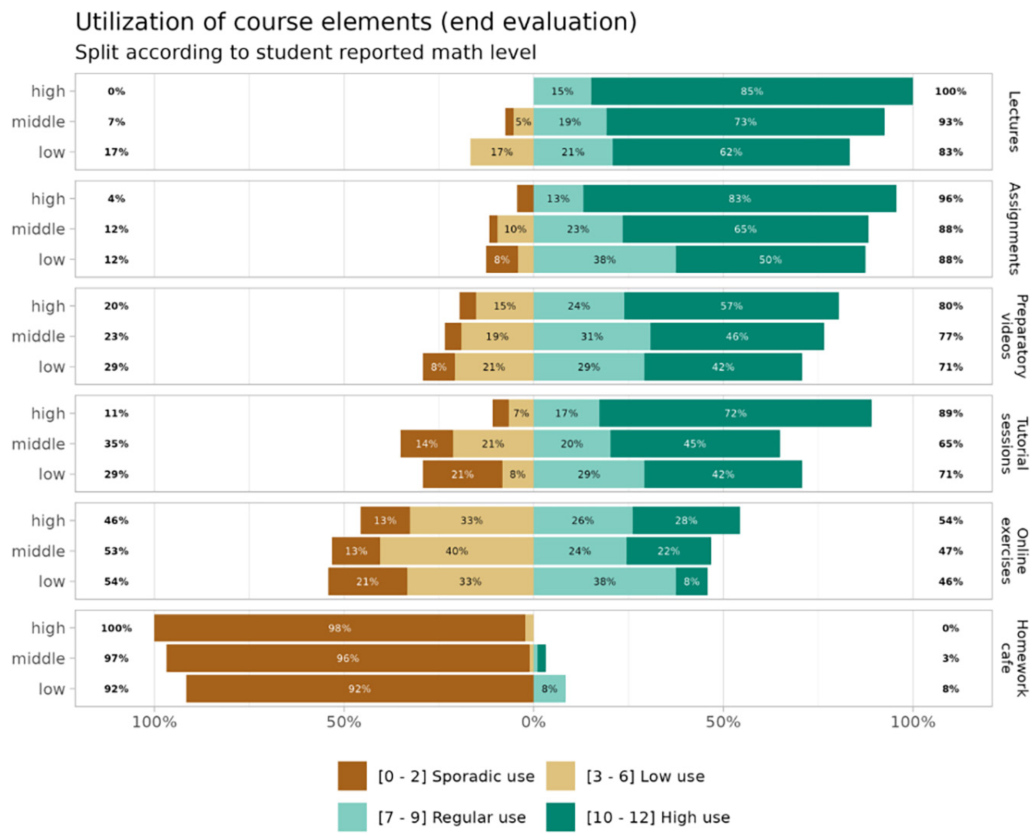

Fig. 3. Student self-reported utilization of different course elements

Note: Be aware that the source of data for the chart was the end of course evaluation, which was conducted during the last lecture, which might skew the collected data.

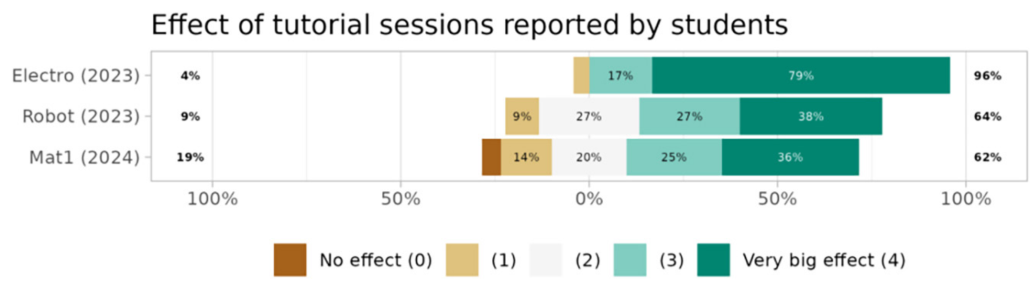

Fig. 4. The student self-reported effect of tutorial sessions in three different courses that all are based on the ideas of Peter Liljedahl

The electro (2023) course had about 30 students enrolled, and the Robot (2023) course had about 80 students enrolled. A very positive reported effect of the tutorial sessions is seen in all three courses.

6 QUALITATIVE DATA – THEMES IN DIALOGUE WITH THEORY AND PRACTICE

The following four sections present an overview of the themes identified in the analysis of the qualitative data. Each section integrates findings from the data and includes selected illustrative quotes (with English translations) to provide a representative and nuanced account of the participants' experiences. While not all quotes could be included, the sections reflect the full range of benefits and challenges identified in the empirical material. In addition to presenting empirical patterns by using quotes, the section of each theme also incorporates an analysis of how these findings relate to relevant theoretical perspectives, thereby situating the results within existing scholarly frameworks. This integration of empirical evidence and theoretical interpretation allows for a deeper understanding of the identified themes and their implications for the design and facilitation of tutorial sessions.

6.1 Student frustration with problem-solving

The students' engagement with problem-solving tasks generally proceeded well, yet the process entailed both strengths and challenges. One key advantage of the open-ended task structure is that it ensures continuous engagement. As AP1 explains, "In principle, there are infinitely many tasks. Our aim is not necessarily for students to complete them all—there should always be something extra to work on." This aligns with Liljedahl's [15] emphasis on fostering a dynamic learning environment that encourages students to remain engaged and actively think through problems.

However, several challenges also emerged. A recurring issue was the varying pace among student groups, with some progressing rapidly while others struggled. As TA4 notes, "Some groups move forward quickly, while others remain stuck on the initial problems for a long time." This discrepancy can lead to frustration, particularly when stronger students spend significant time explaining concepts to their peers rather than solving problems themselves. TA6 reflects, "If you work with groups that are not strong in mathematics, you spend more time explaining than actually solving the tasks." Some students feel they are not making enough progress. While students perceive this as a drawback, one of the associate professors (AP1), highlights in the interview that explaining concepts to peers is an integral part of the learning process: "The students may not realise it, but they learn a lot by explaining to others." However, AP1 acknowledges that this benefit may not be sufficiently clear to the students and should probably be better communicated. Additionally, weaker students may tend to withdraw, allowing their peers to dominate. AP1 notes, "In some groups, the weakest student withdraws slightly, enabling the others to move ahead at an overly fast pace."

These findings resonate with student feedback from the midterm evaluation, where perspectives on tutorial sessions were mixed. While many students valued the opportunity to apply theoretical knowledge practically in a problem-solving process—stating that it deepened their understanding—others expressed frustration with the emphasis on independent thinking. A key concern was the perceived lack of constructive guidance from the TAs. Some students reported that when seeking help, TAs often provided vague suggestions, such as, "Try to think more about it". As one student noted, "It gave me the feeling that I might as well have worked from home." This left some students feeling unsupported, particularly when tackling unfamiliar topics, highlighting the need for more direct and actionable assistance.

The observed challenges suggest that some students may not be fully accustomed to problem-solving environments that emphasise independent exploration over instructor-led explanations. This is consistent with prior research indicating that transitioning from procedural learning to conceptual understanding can be met with resistance [25]. While the current approach aims to shift students away from reliance on external guidance [28], the empirical data indicate that many students still expect more direct instruction.

Furthermore, TAs acknowledged that facilitating students' learning and providing the right hints at the right time was particularly challenging. TA2 expressed awareness of how proficient the two associate professors were at this and hoped to gradually improve in this regard. However, as [23] points out, mastering this skill takes time and experience, as effective scaffolding requires deep pedagogical insight and an ability to assess students' thinking in real-time. According to [17], a certain level of frustration is expected in problem-solving, as it stimulates deeper thinking and learning. However, guidance is essential, and students must understand the rationale behind this approach. The findings suggest that the students may not be accustomed to problem-solving environments, which could explain their discomfort.

6.2 Group collaboration: Strengths and challenges in learning

Group collaboration emerged as a central theme in both the interviews and the survey data, with students expressing a strong appreciation for its pedagogical benefits while also voicing concerns about its inherent challenges. The opportunity to work in randomly assigned groups during tutorial sessions was widely regarded as a significant strength. As one student enthusiastically shared in the survey: "I love solving problems together with other students during the exercise sessions." This collaborative approach not only fostered a sense of shared responsibility for problem-solving but also encouraged peer learning, allowing students to be exposed to diverse perspectives and strategies. TA6 emphasised this pedagogical value, stating, "Arranging people in groups and getting them to talk about mathematics instead of just working alone with numbers and letters – that is one of the most important things." The social and cognitive benefits of group work were also highlighted by TA4, who noted that random groupings prevent social isolation: "It's good because it stops students from always sticking to the same people," while also pushing students to adapt their communication styles: "When you meet many new people, you also have to find different ways of explaining things."

Despite these advantages, the practice of group work, particularly when organised randomly, is not without its drawbacks. A recurring issue was the unequal distribution of effort among group members. As one student noted in the survey, "Group work was challenging, as not everyone put equal effort into preparation." Such imbalances often led to frustration and hindered the effectiveness of collaboration. Moreover, the social dynamics of group work posed challenges for some students, especially when working with unfamiliar peers or when feeling academically vulnerable. One student shared in the survey, "I don't like standing with people I barely know when I'm struggling with mathematics", a concern also raised by AP2, who questioned whether this setup might unintentionally intimidate students: "Are we intimidating some students by exposing their weaknesses when they're standing up and working in front of others?" Additionally, as AP1 observed, group effectiveness varied widely: "Some academically weaker groups withdraw, while the stronger students move too fast."

In summary, the experiences with group collaboration reflect the dual nature of Liljedahl's [15] model of random visible groupings. On the one hand, their accounts illustrate how this approach can successfully promote peer learning, broaden perspectives, and foster a more inclusive and socially integrated learning environment—core aims highlighted in both Liljedahl's theory and supporting literature. On the other hand, the challenges regarding social discomfort, uneven participation, and group dynamics reveal important tensions that must be acknowledged when implementing such strategies in practice.

6.3 Vertical surfaces

Despite some logistical constraints, the integration of non-permanent vertical surfaces into university-level mathematics tutorials has been perceived as beneficial for student engagement, collaboration, and conceptual understanding. Many students emphasised how the act of standing and using a vertical space transforms the learning experience into something more dynamic and participatory. One student explained in the survey, “I learn more by standing up, solving problems, and finding the answer together with others,” while another reflected: “you use mathematics creatively and understand it better this way.” The physical format of vertical group work naturally promotes visibility, shared responsibility, and real-time feedback. TA4 noted, “It’s really great because everyone can see what’s going on … students should take turns writing, or if someone is struggling with a topic, they can write down what the others are saying. That way, everyone is engaged.” These practices support an inclusive and active learning environment, where responsibility for learning is distributed among group members.

Teaching assistants also highlighted the spatial and ergonomic benefits of vertical surfaces. TA2 commented on the clarity and orientation offered by whiteboards: “It’s super relevant to have a board where you can write large enough for everyone to see, and it faces the same way for everyone. It works really well for collaboration.” The design of the space facilitates not only group cohesion but also inter-group communication. As TA2 further added, “It’s easier for them to say, ‘We’re stuck on this task,’ and then just turn their heads and see what other groups have done. They can go over and ask others who have moved on.” This ease of informal peer support creates a learning culture based on openness and mutual help, which may be difficult to achieve in traditional seated or digital formats.

From the associate professors’ perspective, vertical formats are also seen as pedagogically powerful. AP1 observed, “It just does something to the way people interact. You can see what’s happening,” emphasising how visibility of process and group thinking facilitates both instruction and peer learning. AP2 likewise remarked on the activity the format encourages: “They’re standing up, so you’re naturally more active, and you can better see what each other is doing.” This visibility extends beyond content to foster interpersonal awareness, helping students remain aware of both their own progress and that of others. TA6 added a complementary perspective: “It’s a good idea also that they are physically active, so you don’t fall asleep,” highlighting the value of bodily movement in sustaining attention and cognitive engagement. Additionally, vertical surfaces offer a broader visual perspective on mathematical problems. As TA6 put it, “If you sit at a desk with paper, you see the math from one angle. But with a board, you can take a step back and evaluate if you’re solving it correctly or if there’s another way.”

However, despite these clear pedagogical advantages, several practical and structural issues were identified that may limit the effectiveness of vertical surface use if left unaddressed. A recurring concern was the physical configuration of the teaching spaces. TA2 noted, “Some students sat down while working at the boards because the boards were too low. It wasn’t optimal,” and AP1 added, “The whiteboards are not high enough, so it’s not comfortable to stand at them. There are also too many chairs, and they're too close." These physical constraints inadvertently reduce the level of bodily engagement that vertical setups are intended to foster. However, AP1 "We got some slightly higher whiteboards during the semester, and that helped." This illustrates that while spatial limitations may present obstacles, they are not inherent to the method but rather a matter of appropriate infrastructure.

In addition to physical constraints, equitable participation within group work remains an area of attention. Although the ideal is for students to alternate turns at the board or collaboratively narrate and write, some students may dominate the interaction while others become passive observers. TA4 says: "If someone is struggling with a topic, they can write down what the others are saying," which allows all members to engage in ways aligned with their level of confidence and understanding. Yet this requires ongoing facilitation and clear expectations from instructors to ensure that all students benefit equally from the format.

By incorporating vertical non-permanent surfaces into the tutorial sessions, our aim was to foster a more interactive and engaging learning experience, supporting students in developing both their mathematical reasoning and their ability to communicate complex ideas effectively [15, 17, 18]. The empirical insights from the tutorial sessions align closely with Liljedahl's research, which highlights how vertical surfaces support active thinking, foster persistence, and promote collaborative engagement. The practices observed during group work mirror [18] concept of commognition, in which learning is seen as a process of participating in specific discursive practices. Working at vertical surfaces requires students to verbalise their reasoning, respond to others' ideas, and construct shared representations of mathematical problems. This not only supports deeper conceptual understanding but also strengthens students' ability to engage in mathematical dialogue—an essential component of disciplinary learning. However, the effectiveness of vertical surfaces is not automatic. As highlighted in the observations, physical limitations such as poorly placed boards or insufficient space can hinder student engagement and undermine the intended benefits.

6.4 Alignment challenges: Tutorial sessions, lectures, and exams

A recurring concern among students was the perceived disconnect between the active, verbal nature of the tutorial sessions and the written format of the final exam. Many felt that the approach of solving problems verbally and collaboratively did not align with the skills required for the exam. As one student expressed, "I don't think it works to solve problems standing up and discussing them when the exam is written." This sentiment reflects a broader frustration with the perceived lack of direct preparation for the exam format. Some students suggested that focusing more on written tasks during the sessions would better equip them for the demands of the exam.

AP1 acknowledges that the exam ultimately requires individual understanding and the ability to construct written mathematical arguments. However, he further emphasises that the course focuses on "using mathematics as a language"—particularly on how to argue for different conclusions—which is best practiced through active engagement: "The best way to train your ability to argue, which is what we test in the exam, is to do it in practice—to stand at the board and discuss with others." Nevertheless, it is important to note that while these sessions emphasise verbal reasoning and collaborative problem-solving—skills that support conceptual understanding—the exam primarily assesses written argumentation and individual problem-solving. This distinction underlines the need for clearer communication regarding the purpose of the sessions and how they complement the exam format.

Another recurring challenge identified in the study was the perceived lack of alignment between exercise sessions and course content, particularly because the tutorial sessions were scheduled before the corresponding lectures. Some students expressed confusion and a sense of unpreparedness when engaging with new material in these tutorial sessions. As one student noted in the survey, "I prefer to learn the theory first and then solve problems." This sentiment was echoed by others, who suggested that reversing the order—placing exercise sessions after lectures—would enhance their understanding and overall learning outcomes. For example, a student stated, "It would work better to have lectures first and then the exercise sessions afterward." Additionally, some students felt that certain assignments did not feel directly related to the lectures, highlighting the need for a stronger connection between theory and practice. Despite these challenges, the majority of students described the tutorial sessions as effective preparation for the lectures. These sessions introduced them to key topics and allowed them to refresh previously learned material, which in turn helped them feel better prepared and derive greater benefit from the lectures.

7 DISCUSSION

While the three practices from Liljedahl [15] – open-ended problem-solving, group collaboration, and the use of non-permanent vertical surfaces – demonstrate significant potential for enhancing student engagement and conceptual understanding in engineering education, they also present notable challenges that warrant careful consideration. Open-ended tasks promote sustained engagement and foster independent thinking, aligning with constructivist ideals of active learning. Group collaboration encourages students to articulate their reasoning, adapt to diverse perspectives, and build communication skills, while vertical surfaces contribute to a more dynamic and participatory learning environment that enhances visibility and interaction.

The quantitative findings strongly support the pedagogical value of these practices. A statistical analysis showed that both attendance at tutorial sessions and completion of assignments significantly and positively affected students' performance on the written exam ($\alpha = 1.00$, $p < 0.01$; $\beta = 2.10$, $p < 0.001$). Moreover, a fixed-effects model revealed that participation in a tutorial session specifically related to an exam question increased the score on that question by 0.67 points on average ($p = 0.006$), indicating a clear link between active participation in tutorials and improved learning outcomes. Interestingly, these benefits were not distributed equally among all students. In Figure 1 students with a mid-range high school mathematics grade (between 7 and 11) benefited the most from tutorial participation, while students in the highest and lowest skill groups saw less or no significant improvement. This pattern may, however, be influenced by a ceiling effect—a statistical phenomenon in which the majority of data points cluster near the upper limit of the measurement scale, leaving little room to detect further improvement. Specifically, some of the highest-performing students had already achieved full scores on assignments and answered all test questions correctly. As such, their lack of measurable progress may not reflect an absence of learning or benefit from the tutorial sessions, but rather a limitation in the assessment's ability to capture further gains. Had the tasks or exam been more challenging, it is possible that we would have observed a positive effect in this group as well. This finding however aligns with qualitative observations that less confident students may struggle to fully engage in group-based, open-ended tasks, potentially due to cognitive overload or lack of prior knowledge. Similarly, more advanced students may experience frustration from having to explain basic concepts repeatedly, which can inhibit their own learning progression.

Furthermore, survey results indicate varied perceptions of the tutorial sessions' usefulness depending on students perceived mathematical proficiency. While students in the top skill group rated the tutorials very highly, those in the middle and low skill groups were more divided: about one third rated the sessions as having a "very big effect," whereas approximately one fourth rated them as having "no effect" or a similarly low impact. These disparities suggest that the design and facilitation of tutorial sessions may need to be adjusted to better accommodate different student needs and learning styles.

In terms of utilisation, students reported prioritising lectures, assignments, and videos over tutorial sessions. While this could partially be attributed to the timing of the final survey (conducted during a lecture), it also points to a possible misalignment between students' perceptions of what is most effective and what data suggest is beneficial for performance. Notably, tutorials—despite their proven impact—were not among the most frequently used elements of the course.

Another point of concerns was the physical and structural setup. Vertical surfaces, while intended to support collaborative learning, were occasionally hindered by inadequate placement or insufficient space. Moreover, students expressed confusion when problem sets were introduced before the relevant lecture, and some noted a mismatch between the collaborative nature of tutorials and the individual, written format of the final exam. These issues underscore the need for pedagogical coherence and more targeted instructional support. To optimise the effectiveness of these practices while mitigating their limitations, we recommend the following strategies:

7.1 Recommendations for practice:

- **Improve communication with students:** Clearly articulate the pedagogical rationale behind practices and their relevance to students' academic and professional development early in the course.
- **Bridge the gap between tutorial sessions and exams:** Clarify for the students how collaborative, verbal activities contribute and align to the development of skills assessed in written exams.
- **Guidance and scaffolding:** Educate TAs to strike a balance between fostering student autonomy and offering timely, concrete support when needed in their problem-solving process.
- **Establish clear expectations for group work:** Define roles and responsibilities within student groups to promote equitable participation and shared accountability.
- **Enhance physical learning environments:** Ensure that classroom infrastructure supports vertical group work, with appropriately placed whiteboards and sufficient space for movement and collaboration.

Implementing these recommendations may help to create a more supportive and coherent learning experience, thereby maximizing the pedagogical value of the observed teaching practices.

8 CONCLUSION

This study demonstrates that well-structured and deliberately facilitated tutorial sessions can significantly enhance learning outcomes in university engineering mathematics education, even within the context of large student groups. Quantitative analysis provides clear evidence that both attendance at tutorial sessions and completion of related assignments positively correlate with improved exam performance. Furthermore, task-specific engagement – such as participating in a tutorial directly tied to a specific exam topic – was shown to have a measurable impact on performance in that area. Both findings are what can be expected. However, the effectiveness of tutorials is not uniform across all students. Students with either very high or very low mathematical backgrounds benefited less from tutorials, highlighting the need for differentiated instructional strategies. Additionally, student perceptions revealed that while some highly value the collaborative nature of tutorials, others found them less helpful – especially when tutorial activities seemed misaligned with assessments or were hindered by logistical constraints, such as inadequate classroom infrastructure.

To increase student buy-in and maximise effectiveness of the tutorial sessions, transparent communication about the pedagogical purpose of the tutorial sessions as well as alignment between tutorial sessions and course assessments is essential. To accommodate the complexities of large groups, tutorial sessions must be designed with careful attention to scaffolding, clear role distribution within groups, and physical learning environments that support collaboration.

In conclusion, tutorial sessions can substantially support learning in large mathematics courses when they are intentionally structured to balance autonomy and support, address diverse student needs, and are coherently integrated into the overall course design.

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10 DECLARATION OF INTEREST STATEMENT

No potential conflict of interest was reported by the author(s).

11 DATA AVAILABILITY STATEMENT

The data that support the findings of this study are openly available in Zenodo at https://doi.org/10.5281/zenodo.15790515, reference number 15790515.

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