Designing Math Trails for Enhanced by Mobile Learning Realistic Mathematics Education in Primary Education

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Georgios Fessakis (✉), Paschalina Karta, Konstantinos Kozas
University of the Aegean, Rhodes, Greece
gfesakis@rhodes.aegean.gr

Abstract—Seeking a systematic combination of the pedagogical model of m-learning with the Realistic Mathematics Education (RME) approach, this study concerns the use of math trail as a learning activity model that can take the advantages of mobile computing devices for the design of effective learning experiences in an authentic context. The paper presents the design and the case study of the first pilot implementation of a math trail, using mobile devices for primary school students. In this math trail, the students are guided, through a digital map, to a sequence of preselected sites of a park where they solve specially designed math problems using data from the environmental context. The students measure real objects' dimensions either with conventional instruments or by measurement applications of their tablet. According to the findings of the study, students solved the puzzles by applying mathematical knowledge, discussion and collaboration. The students applied and reinforced their knowledge through an effective and engaging learning activity. Moreover, the students were puzzled about the differences of the measurements by conventional and digital instruments and this confusion triggered social negotiation. Further research is needed for a grounded theory development about m-learning design for RME.

Keywords—Learning design, m-learning, Realistic Mathematics Education, Math Trails

1 Introduction

The widespread use of various kinds of mobile devices (e.g. Tablet PC, Smartphones, iPad) and their integration into the daily life of children have created new facts on the potential of the pedagogical model of mobile learning (m-learning). More and more students have their own mobile device, which they use frequently and bring it to school. Due to the extensive use of technology inside and outside the school settings, today's children are classified as "i-generation students" [1]. The broad use of mobile devices creates new challenges for the learning design. Learning technology researchers and teachers are trying to integrate mobile devices into schools in a meaningful way. In this paper, we focus on the learning design of meaningful m-learning activities for elementary school mathematics. The authors are interested in the idea that m-learning is consistent with the principles of the Realistic Mathematics Education (RME) approach and it can improve its applications. In particular, according to the principles of Realistic Mathematics Education, mathematics is a natural human activity, thus, during its teaching process, it is recommended to focus on authentic activities, in which children do math and they are gradually led to mathematization [2][3]. Although mobile learning seems to be consistent with realistic mathematics, the bibliographical review renders only few published papers, so far, which combine mobile learning and realistic mathematics. Moreover, the research of mathematics education applications for mobile devices produces only a few applications that adopt the realistic approach, and/or deploys the advantages of mobile devices, such as freedom of movement and increased interaction with the context.

This paper proposes the use of the math trail as a learning activity model for systematic mobile learning design for mathematical education [4] in elementary school students. The rest sections of the paper, present first a brief introduction of mobile learning and math trails, and then follows the analysis of the case study of the math trail that was designed for this research.

2 Mobile learning

Widely spread portable computing devices and wireless internet are now radically transforming the notions of discourse and knowledge [5] and make possible new learning models such as m-learning [6]. M-learning, which is defined as the use of wireless handheld devices in order to engage participants in some form of meaningful learning [7], as a component of formal or informal education [8]. During m-learning experiences, the learners have access anytime and anyplace to information in order to perform authentic activities [9]. In other words, m-learning constitutes a relatively new pedagogical model in which students learn as they move, interacting with each other, their environment as well, through the mediation of applications that run on varying types of mobile digital devices [10]. As it is supported by UNESCO, m-learning gives literal meaning to the principle that “the world is a classroom” [11]. In fact, the mobile devices are equipped with a number of sensors and applications that can be used in order to improve the user's interaction with the environment. For example, given the user’s local position by the GPS sensor, an application can propose relevant information by taking into consideration context sensitive factors e.g. time, temperature, orientation, and movement of the user. Therefore, we do not consider that a simple transfer of applications from personal computers to mobile devices without deploying their full possibilities of mediation between the student and the environment is not consistent with the concept of mobile learning. Until recently, the context “sensitive location depended learning” has been applied mainly to the informal learning because of the cost of the devices and the requirement to move outside the school environment [12]. With the broad use of the mobile devices and the wireless internet access, mobile learning functions bridges the gap between formal and informal learning [10].

3 Mobile learning and realistic mathematics

Freudenthal [13] the main representative of RME movement, has argued that mathematics had to be taught in such a way to become useful for solving everyday-life problems. He was a strong supporter of "Mathematics for All", trying to make mathematics accessible to everybody and he also advocated problem-solving as a teaching method. Pupils should have a genuine interest about the problem which would be preferable to be a part of children's experiences [14]. According to RME, learners should find the starting point of their learning in rich, complex structures of the real world and afterward they must continue to the abstract structures of the world of symbols. Other RME researchers consider important to view concepts as problem-solving tools and discover them through their application in authentic contexts [15]. With the use of digital technology, learners can develop deeper understanding of mathematics, because technology as a mindtool can support learners' inquiry, decision making, reflection, reasoning, and problem-solving capacity [16]. The principles of RME constitute a context in which digital technology could provide significant assistance in teaching and learning [17], because digital technology itself influences the kind of mathematics that are taught and enhances students' learning [16]. Furthermore, networked mobile devices contribute to mathematics education by fostering the collaboration between participants [18].

4 Research review for mobile learning and realistic mathematics

A systematic research review regarding mobile learning applications in mathematical education produced, relatively, a few studies with mixed results concerning the effectiveness and the impact of mobile learning in mathematics education. The small number of mobile learning research for Mathematics was confirmed also by other researchers [19]. As their review showed, only 1.9% of research published between 2009 and 2014 had a subject on investigations mobile learning for Mathematics. In the case of m-learning, it is even more limited [20] [21] [22] [23]. The results of our research review regarding m-learning applications in mathematics education have shown positive results a) in the performance in basic mathematical concepts and arithmetic in Kindergarten [17][24] and the first grade of elementary school [25], b) in engagement and confidence of secondary education students [22] and finally c) in team collaboration and understanding of graphs [26]. Some of the studies [27][28][25] examine in particular, whether educational mobile applications, compatible with the Realistic Mathematics Education, help Kindergarten and first grade Elementary School students to improve their performance in basic mathematical concepts and arithmetic with positive results. Additionally, another research [29] investigated the effect of the application of mobile devices concerning the improvement of math skills as well as the intrinsic motivation of Elementary School students had also positive results. In this study [30], the researchers dealt with solving mathematical problems with the use of mobile devices by Elementary School students and they found out that mobile devices provided students with opportunities to find, test and explore the problem solutions. As far as secondary school education is concerned, we mention as an indicative current research the papers [22] and [26]. The study [22] explores the effect of handheld devices in the engagement and confidence of Irish students aged 15-17 years students in math activities. Finally, in the study [26], the researchers examine the combination of mobile devices with Realistic Mathematics Education for the teaching of graphs for the chapter "Time, Distance and Velocity" in the textbook of mathematics. The research [26] was conducted in High School students in Indonesia.

Summarizing the research review, the studies mentioned above do not take in advance key distinctive features of the pedagogical model of m-learning, such as students' movement and their interaction with the environment. On the contrary, they concern the use of mobile devices as a more flexible type of electronic devices for the delivery of conventional learning applications. Moreover, the search of educational applications, for elementary school mathematics, in mobile devices (e.g. in app store and iTunes services) showed that most of available applications do not utilize the advantages of mobile devices and m-learning, while these applications were slightly based on the principles of Realistic Mathematics Education. Most applications concern arithmetic drill and practice instructional games based mainly on behaviorism theory for learning. In other words, the available educational applications could be well used on a desktop PC and mobile devices as long as they do not require moving in space and do not utilize the sensors of portable devices. This is totally logical as the production of location based applications does not consist a priority due to its commercialization difficulties. Hence, a research gap is ascertained regarding the combination of mobile learning with RME.

5 Mobile learning, Realistic Mathematics Education and math trails

The development of mobile learning applications for young ages and for use in typical learning environment consists a challenge for learning design. In an attempt to explore models for the use of space and context sensitive applications of mobile devices in learning designing in general and for mathematics learning more specifically, the authors consider various models of activities such as treasure hunt games [31] and math trails. As stated in Ref. [32], math trail is a walk (a tour) towards the discovery of Mathematics. More typically, the math trail includes a pre-planned route, which is defined by a sequence of stops in which students examine the environment in a mathematical way [4]. A remarkable collection of mathematical trails is available from the MathCityMap project of the University of Frankfurt (https://mathcitymap.eu). The math trail constitutes a learning activity model that combines: *movement, communication, collaboration, problem-solving, links of school knowledge with the real world and other school subjects, practical application of knowledge and skills in the conceptual and natural environment* [33]. Concerning the advantages of the mathematical trail, as stated in *Ref. [33]: as they take place outside the classroom they create an* *atmosphere of adventure and exploration. The common sense of expectation and discovery that is developed, it leads naturally to the communication of mathematical ideas in which the trail is focused. Students observe measure, collect and record data to process and interpret in the classroom. As they complete activities in the trail, the students use mathematical concepts that were taught in the classroom and discover the various uses of these concepts in everyday life.*

6 Research rational

The compatibility of the mathematical trails features with the principles of Realistic Mathematics Education is evident. Moreover, mobile devices provide possibilities for advanced interaction between the user and the environment. The above statements highlight the need to research the possibility of the application of mobile learning in order to improve mathematics trails and realistic mathematics. In this direction, the present research studies the design and the first implementation of a mathematical trail enhanced with mobile learning for the realistic approach of mathematics by Elementary School students. The purpose of this study is to examine whether the mathematical trail is a learning activity model that facilitates the learning design of mobile learning applications. These mobile applications should take advantage of the advantages of mobile learning that are related to free movement and the interaction between the user and the environment. To this end, a mathematical trail for Elementary School students that focuses on the concepts of length, perimeter, area and measurement was designed and implemented experimentally. The following sections present the design of the application and the experimental application.

7 The mathematical trail "The Fairy of the Waterfalls"

For research purposes, a math trail was designed, which was named "The Fairy of the Waterfalls". The site chosen is the waterfalls' park of Edessa city in Greece. The estimated duration of the trail was approximately 1-2 hours. To make it more attractive, a story was integrated into the math trail. According to the legend, there is the famous Fairy of Waterfalls in the park, who appears only at midnight. The students were requested to help the famous Fairy of the Waterfalls to solve a series of mathematical problems so that she could keep on her nocturnal wanderings at the waterfall's park. As soon as the students solve a problem, they earn some pieces of the fairy's puzzle. In the end, after they provide correct answers to all problems, they can assemble the puzzle and see the image of the famous fairy. There are six problems in the trail located in corresponding stops of the park area. In order to facilitate the children to find these six places, a Google Map was made for the specific mathematical trail. The math trail map that was given to the students is available on: https://goo.gl/hKamxR.

As shown in Figure 1, each stop of the math trail is marked by a corresponding pinpoint on the map. Arriving at each stop of the trail, students can see on the digital map, a photo of the stop and a text with instructions about the problem. The use of

Google maps facilitates the implementation of mathematical trails thanks to the use of GPS and the marking of the current position of the tablet, they can easily find the stops. At each stop, the students are requested to solve the problems in Table 1.

For the required measurements, students were provided with a conventional tape measure and the application "Object Height" (see Figure 2). The measurement applications usually require the user's height and then they permit the measurement of the user's distance from a specific target point on the camera of the device. They also measure the height of objects on the camera, by applying the Pythagorean theorem for a given angle of inclination which they measure with the help of the tilt sensor of the device. The "Object Height" is a specific mobile application for measuring both height and length. In order to use the app, the user has to follow three steps: Step1. Insert his/hers height, Step2. Take a photo so that the red line that appears on the camera (see Figure 2) is at the base of the object to be measured, and Step3. Take a photo so that the red line of the camera is on the top of the object. This will show the height of the object and the user's distance from it. The application works by using the same principles as a simple theodolite, or as a more modern geodetic measurement station.

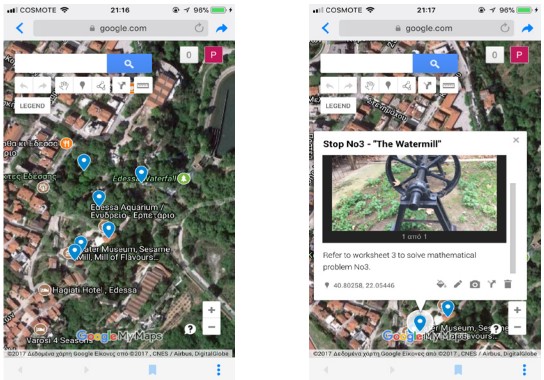

Fig. 1. Map with points of the path in Google Maps

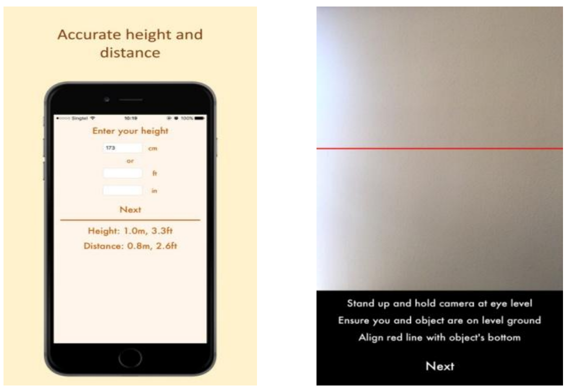

Fig. 2. Object Height app's user interface screenshots

Table 1. The math trail's stops and problems

Stop's name	Photo	Problems
No. 1 "The circle"		Calculate the perimeter of the circular square: 1. With the tablet apps and 2. With the measuring tape (Work in 2 groups)
No. 2 "The church"		1. Calculate the height of the church. 2. Calculate the height of the door of the church 3. Calculate the surface of the door (Work in one group)
No. 3 "The watermill"		1. Calculate the radius of the circular pulley, 2. Calculation of the diameter of the pulley, 3. Calculation of the angle in degrees of each section of the pulley (Work in one group)
No.4 "The stair"		1. Calculate the height of the stairs. 2. Calculate the height of each step, 3. Calculate the steps going up 2-2 stairs (Work in one group)
No. 5 "The shelter"		1. Calculate the area of the covered rectangular square with tape, 2. Calculate the same area as the Object Height embodiment (Work in 2 groups)
No. 6 "The waterfall"		Open-ended problem. Recommended ways to calculate the height of the Karan's waterfall.

The students can learn initially to use the application in order to make measurements without fully understanding how exactly calculation of the device functions. More specifically, the students use the application as a measuring instrument. However, the details of the operation of which is a black box to them.

The process of measuring while theoretically substantiated, it is practically sensitive to the accuracy of the tilt sensor of the device, to the point in which the tablet is held in relation to the one stated in the embodiment, to the alignment-targeting red guide line showing the embodiment of the camera with the ends the objects to be measured and to the slope of the counter stop. In order to produce more accurate measurements, the tablet can be placed on a photographic tripod equipped with spirit levels. In any case, the measurements will vary from group to group and from those made with the tape measure. The variation of measurements is the basis for the learning value of the enhancement of the trail with the portable application. Besides the use of the map, the aim of the paper is to consider the measurements and the errors in the calculation of lengths, circumference, areas and heights of various objects. It is expected that students in higher grades will be able to reflect collaboratively on algorithms and theorems that the Object Height type applications relate or, even try to build their own similar applications, thereby developing computational thinking skills combined with mathematical knowledge.

8 Methodology, Research questions, and research conditions

The following research questions were posed before the study:

RQ1: Can math trails, enhanced by mobile technologies, help primary school students in the development of mathematical concepts, such as the length, the circumference, the area and their measurement?

RQ2: Can math trails, enhanced by mobile technologies, be effective, attractive and feasible for primary school students?

RQ3: Can math trails, enhanced by mobile technologies, foster collaboration among the learners?

The methodology chosen is a combination of the design experiment [34] and exploratory case study [35]. For the collection of research data questionnaires, the students' worksheets, the interviews and the researcher's observation log have been used. The second author participated in the implementation of the trail as the teacher. The participants of the research were four children of the sixth grade of a public Elementary school in Edessa. The details of the participants are shown in Table 2. The rules of research ethics were followed the children's and their guardians' consent was given after they were informed about the purpose and the research process. All children knew how to use a tablet while children who had their own tablet mentioned that they use it 1-3 hours per day for gaming and web browsing.

Table 2. The participants of the research

A/A	Alias	Gender	Age	Owns tablet
1	P.A.	M	12	Yes
2	E.K.	F	12	Yes
3	X.A.	M	12	Yes
4	L.K.	F	12	No

Children implemented the math trail on Sunday, 18/12/2016 and the total duration of the process was 75 minutes. In order to prepare the children, a three-hour meeting took place, one day before the trail implementation. In this meeting, the students became familiar with the use of the map on tablet and the measuring tape. In addition, students' previous knowledge of the concepts of the trail was examined by written exercises using paper and pencil. The examination showed that the children knew the formulas for rectangles dimensions but they faced problems with the formulas for the cycle. More particularly, none of the students could remember the formula for the calculation of the circumference of the circle. It was observed that most of the students confused the diameter with the radius of the circle. During the preparation of the students, previous knowledge was recalled, questions were answered and misconceptions were worked out. The students already knew how to use a tablet pc. They learned quickly to use the Google Maps, the Object Height application and they were eager to implement the trail.

9 Findings

The students completed the trail and solved the problems that corresponded to all six stops. They also kept their interest and enthusiasm during the entire activity. The students collaborated, discussed and made common decisions and initiatives. All the students used the tablet and the applications, Google Maps and Object Height. The difficulties that appeared in the solution of the problems were expected for this level of education. More specifically, children mixed up the concept of the radius with the diameter during the calculations related to the circle (stops 1 and 3). However, they came up with a solution, by discussing the problem and referring to the preparation material. Students were familiar almost with the whole park area and they found some of the stops without using the map. Nonetheless, in some cases (e.g. stops No 3 & No 4) they used the map and their GPS position trace to find the point that they were looking for. The automatic navigation of Google Maps was not used by the students even they were allowed to. Students noticed discrepancies in the representation of the GPS position on the map, when the internet connection was not strong enough and they critically interpreted these discrepancies based on the position of the known landmarks. The dialogues that took place in stop 4 (the staircase) are characteristic:

L.K.: *"Miss, the map shows that we are moving, it doesn't show where we are"*.

L.K: *"Miss, we are here [at stop No 4] but the sign on the map for stop No 4 is farther away"*.

The ability to integrate multimedia information and instructions for each stop into the map is a clear improvement compared to the conventional math trails where printed map is used. The digital map significantly improves the mathematical trail. Additionally, it provides easier navigation and information on the communication and cooperation between different groups which work synchronously or asynchronously. The groups can leave traces and results on the map which makes the participation more fun. These features were not used in this case, however, they were emerged as ideas for a future use.

As it was expected, students came up with different results in measurements with the measuring tape and the Object Height App. Table 3 shows characteristic comments of the students on the differences in measurements in various stops. Generally, the students trusted more the measurements with the measuring tape instead of the tablet, since for them this kind of a measuring procedure is like a black box.

Table 3. Vignettes from the math trail implementation

<table>
<tr><th>Stop</th><th>Students Comments</th><th>Answers</th></tr>
<tr><td>No.1
«The circle»</td><td>P.A. (Group B): “Miss, why did we measure 5,10m while the girls (Group A) found 4,77m? Did not we measure correctly?”
X.A. (Group B): “Miss, maybe we should measure it again to be certain”</td><td><table><tr><th>Group A</th><th>Group B</th></tr><tr><td>r = 4.77m.</td><td>r = 5.1m.</td></tr><tr><td>c = 29.96m.</td><td>c = 31.4m.</td></tr></table></td></tr>
<tr><td>No.3
«The watermill»</td><td>P.A.: “Miss, this cannot be 3m, it seems to be much less”</td><td>a) circle, b) diameter = 1,10m. c) radius = 0.55m.
d) 6 parts, e) $360^o/6 = 60^o$</td></tr>
<tr><td>No.5
«The shelter»</td><td>E.K.: “Miss, a minus sign appears in front of the result of the measurement. Is this because it is dark in here and the tablet did not measure it well? We should do it again”.
E.K.: “Aaah!, now the result is similar to the result of the other group so it must be ok”.</td><td><table><tr><th>Group A</th><th>Group B</th></tr><tr><td>a = 11.7m</td><td>a = 11m</td></tr><tr><td>b = 8.7m</td><td>b = 8.9m</td></tr><tr><td>A = 101.7 m^2</td><td>A = 97.9m^2</td></tr></table></td></tr>
<tr><td>No.6
«The waterfall»</td><td>P.A.: "Maybe we can only measure the point that the waterfall starts until the point we are standing now"
E.K: “We cannot do it with the measuring tape either”</td><td>They concluded that:
1. It was not possible to measure the height with the measuring tape.
2. Only one part of the height could be measured with the tablet application.
3. Maybe the measurement of the height can be achieved with the use of other instruments that they are not aware of.</td></tr>
</table>

In the case of discrepancies, they tried to re-measure so that tablet measurements approximate the values of the measuring tape. The cases of stops No 3 and 6 (table 3) are more interesting where the children could only use the tablet. In the case of stop 3, the students were asked to measure from a distance the diameter of a circular metal structure. The first measurement was rejected as false by the children because they empirically considered it to be very large. Also in the case of stop 6, students concluded that the height of the waterfall cannot be measured with the measuring tape so they just estimated a part of it by using the tablet. Children were amazed when the teacher told them that this was possible using analogy in photos, in the discussion after the trail walk. Finally, students gained important experience that facilitated them to understand concepts that are related to the approach, the error and the measurement of height and distance by applying the Pythagorean theorem or the use of proportions in photos. These activities, by examining the measurements with alternative methods could be an extension of the trail in the classroom. Moreover, students realized that the use of electronic devices, when it comes to measurements in the real world, does not always provide values with absolute precision. This realization is the foundation for building concepts of computational thinking.

10 Results

This section, based on the findings, gives answers to the research questions.

RQ1: Can math trails, enhanced by mobile technologies, help primary education students in the development of mathematical concepts, such as the length, the circumference, the area and their measurement?

The students practiced in measuring length, the distinction of the radius from the diameter and the circumference from the area. They also applied all the knowledge acquired in school to realistic situations of everyday life in the natural environment in the area calculation of a rectangle and a circle transferring. In addition, the students became familiar with the concept of accuracy and measurement error. With the use of the digital map, the students practiced in reading the map and its use to navigate in space. Math trail and the use of the tablet can contribute to the strengthening of the knowledge of mathematical concepts. Furthermore, the students realized the limits of their ability to make accurate measurements and calculations. This experience improved significantly their mathematical and computational thinking.

RQ2: Can math trails enhanced by mobile technologies be effective, attractive, and feasible for primary school students?

As far as efficiency is concerned, the experiment results are positive since the learning objectives of the math trail, such as the application of mathematical concepts in authentic environment, the use of a tablet as a learning tool, the radius distinction from the diameter, the comparison of the measuring tape with the application Object Height, the use of a digital map, as well as the collaboration were achieved satisfactorily by all students.

The application of the mathematics trail seems to be feasible and worth of the extra effort, time and preparation that is required by the teacher. This is not only due to its effectiveness but also due to its impact on the quality of the students' interaction. The application of mathematical trails requires preparation and training for the proper management of teaching. During its implementation, it also requires additional care and dedication on behalf of the teacher in order to avoid unexpected situations. A careful choice of the site, the stops and the number of participants can prevent several undesirable situations and reduce risks. The development and the documentation of a shared collection of mathematical trails for a specific region can improve their applicability in formal or informal settings (parents, associations, teachers). The smooth, non-mandatory, enrichment of math trails with technology could help to their gradual transition to augmented reality versions which are more attractive and effective for students. Finally, the technical problems that may occur are the only parts that hinder the applicability of the math trail.

The Mathematical trail that was applied seemed to be particularly attractive to the students as they had the ability to browse, wander and experiment in the space. Additionally, the students were anxious to locate the point where the next mathematical problem was set. On the question *"how they characterize the mathematical trail"* their answers are: *"good, fun, fantastic, and different"*. The playful structure of the math trails is a pleasant adventure for young students. The students also liked the fact that they wander in pleasant places, outside the school environment, such as the park or the waterfalls in this case. Thus the learning process is transformed into a more enjoyable and playful activity. Finally, the students stated that they would wish to participate again in such an activity in the future. the following statement of P.A is characteristic.: *"Miss, if you repeat this experiment, please choose us again to do it together!"*.

RQ3. Can math trails, enhanced by mobile technologies, foster collaboration among the learners?

The math trail with the specific script of action has fostered collaboration among the students for map reading and navigation as well as for the solving of math problems. The students had intensive on task discussions about navigation, measurements and problem-solving. Students overcame the difficulties that appeared in the solution of the problems through their collaboration. The distinction between the radius and the perimeter which are usually mixed-up, they constitute a striking example. However, a common dialogue and looking up the preparation material, the students were finally finding the right answer. Moreover, the mathematical structure itself was in such a way that it was demanding that the exercise was being conducted whether in 1 or 2 groups so that the mathematical riddles of the mathematical trail could be solved. The table 3 contains a characteristic episode of the quality of the interaction between students as well as between students and the teacher.

11 Summary-Discussion

M-learning is a modern pedagogical model which can make learning more enjoyable and effective as it utilizes mobile devices which are very popular. M-learning can support the construction of knowledge in the context of its application. These characteristics make mobile learning compatible with RME. However, the applications of mobile technologies in RME are fairly a few and they do not take full advantage of the pedagogical model. In order to facilitate the design of m-learning application for RME, the enhancement of a math trail with mobile technologies was explored. The math trail had a positive impact on the students learning and contributed to the transfer of school knowledge to authentic situations. The results of this research are in alignment to those of other researchers [21] [22] [31]. M-learning can foster collaboration, facilitates the math trail implementation, and makes it more efficient and attractive. The students applied and improved their map skills with the support of GPS and Google Maps application. Moreover, by the comparison of the measurements with a tape measure to the ones of the Object Height application, the students understood better the concepts of measurement accuracy, error and approximation. The students reached the limits of the potential practical applications of measurement with measuring tape and a tablet. Thus, they laid the foundations for acquiring new knowledge such as the applications of the Pythagorean theorem and measurements through ratios in photos. Generally, the researchers believe, according to the findings of this research, that mobile learning can contribute to the improvement and the enhancement of realistic mathematics education for primary school students. The research can be continued in the future with more trails, a larger sample of children and more detailed records of the interactions during implementation. It could also be the basis for developing an online community of practice that will share math trails with mobile technologies.

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13 Authors

Georgios FESSAKIS, holds a B.Sc. and a M.Sc. from Informatics and Telecommunications department of the N. K. University of Athens, and a PhD in Informatics Didactics from the University of the Aegean where he serves as an associate professor. His research interests include Technology Enhanced Learning Design for Computer Science and Mathematics.

Paschalina KARTA (psemdt14016@rhodes.aegean.gr) has a B.Sc. in Primary Education and a M.Sc. in Didactics of Mathematics, Science and ICT in education. Her research interest are didactics of Science and ICT in primary education, Learning Design and Mobile Learning.

Kontantinos KOZAS (psemdt15007@rhodes.aegean.gr) holds a B.Sc. in Physical Education and a M. A. in School Leadership. He is a PhD candidate at the University of the Aegean. His research interests include learning design with the use of ICT and augmented reality.

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