Discovery Learning and the Computational Experiment in Higher Mathematics and Science Education: A Combined Approach

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Abstract—**In this article we present our research for Discovery learning in relation to the computational experiment for the instruction of Mathematics and Science university courses, using the approach of the computational experiment through electronic worksheets. The approach is based on the principles of Discovery learning expanded with the principles of constructivist, socio–cultural and adult learning theories, the concept of computer based cognitive tools and the aspects on which the computational experiment is founded. Applications are presented using the software Mathematica and electronic worksheets for selected domains of Physics. We also present a case study, concerning the application of the computational experiment through electronic worksheets in the School of Pedagogical and Technological Education (ASPETE) during the spring semester of the academic year 2008–2009. Research results concerning the impact of the above mentioned issues on students' beliefs and learning performance are presented.**

Index Terms—**Discovery learning, Computational science, Cognitive tools, Electronic worksheets**

I. INTRODUCTION

Contemporary research in Mathematics and Science Education points out the significance of the students' engagement in active discovery learning activities, learning in environments that have characteristics of active participation, self–action, exploration and experimentation. Also the significance of social learning is highlighted via the cooperation of the students with one another and with the teacher. The characteristics mentioned above come in contrast to the traditional teaching approach in higher education, the narration–based teaching.

Moreover Information Age has changed the needs of today's citizens, who are systematically trained and further educated. Businesses and organizations are developed and reorganized, organizing their personnel in autonomous groups with increased responsibilities. Employees who take initiative and introduce diverse points of view are in great demand. Communications are conducted mainly via networks and emphasis is laid on customization [11], [16]. Also technology is rapidly and constantly developed and initial education and training proves shortly to be inadequate.

Computers in Education originated in the early 1970's as Learning from computers (Computer Assisted Instruction–CAI), where computers were programmed to teach the students, keeping the principles of traditional instruction. In 1980's the emphasis was placed on Learning about computers, when numerous courses on mechanical parts of computers and programming emerged in almost every Secondary and Tertiary Education's Syllabus. Today the model of Computers as Cognitive tools is widely supported, where computers are used as intellectual partners of the students as they are involved in Discovery learning – constructivist activities [18].

Computational science is a quickly emerging field at the intersection of Science, Computer science and Mathematics because much scientific investigation now involves computing as well as theory and experiment [33], [28].

One of the crucial components of that research field is the correct abstraction of a physical phenomenon to a conceptual model and the translation into a computational model that can be validated. This leads us to the notion of a computational experiment where the model and the computer take the place of the "classical" experimental set–up and where simulation replaces the experiment as such.

This paper presents Discovery learning through the computational experiment, in teaching Mathematics and Science university courses, in particular the computational experiment through electronic worksheets. Moreover it studies the approach's impact on the students' learning performance and the students' beliefs concerning Physics and Mathematics.

In section 2 the theoretical framework is presented including the principles of Discovery learning, the concept of computer based cognitive tools and the views on which the computational experiment is founded. In section 3 Discovery learning through the computational experiment using electronic worksheets designed and implemented in Mathematica is presented and an application of teaching oscillation synthesis is presented. In section 4 the methodology of a case study is presented, concerning the application of the approach in the School of Pedagogical and Technological Education (ASPETE) during the spring semester of the academic year 2008–2009. In section 5 the results of the case study are presented. In section 6 some concluding remarks are discussed.

II. THEORETICAL FRAMEWORK

A. Discovery learning

Discovery learning emerged initially in Bruner's theory [1], [2], according to which the teacher's main role is to help and encourage his/her students to discover the various concepts and ideas and to develop an aspect of exploration and experimentation towards knowledge. The constructivist and socio–cultural theories of learning added to the theoretical foundations of Discovery learning focusing on and expanding different aspects of the theory. In particular, according to Constructivism, developed by Piaget, Von Glasersfeld and other contemporary theorists and researchers, students construct knowledge actively, using their pre–existent knowledge [35], [38]. Learning is activated through the learner's actions in problematic situations [39]. According to Von Glasersfeld, knowledge is a process of adaptation with the world of experiences and not the discovery of a pre–existent world, independent to the learner.

Vygotsky's Social Development Theory, added to the social and cultural aspect of Discovery learning, claiming that every function in the learner's cultural development appears twice: initially in a social level and then in a personal level [42]. Humans in order to communicate with their social environment use "tools" (such as speech, written speech, cognitive tools); the internalization of these tools leads to higher order thinking skills [6].

Discovery learning in Higher education should include conditions of adult learning theories, since students who attend university lessons have both the characteristics of adolescent students (studied by Pedagogy) and adult learners, as stated by Cross [7] and Zemke and Zemke [44]. According to Knowles' Andragogy, adults are self–directed and want to participate in the programming and evaluation of their teaching. Adult learners have a wealth of experience that becomes a resource for learning, so teaching should take into consideration the wide range of different backgrounds of the learners. Adult learners learn better when the object of learning is directly related to their work or personal life. Adult learners' motives to learn are basically internal, so the teacher's role is to facilitate learning, rather than to lecture or to grade the learners [21], [22].

B. Cognitive tools

Traditional views concerning the use of computers in teaching and learning (learning from computers) resulted in the definition of educational software as the means of the educational process that aims at the facilitation of learning using computers as the basic tool. Kemmis, Atkin and Wright [19] initially proposed a framework for Computer Aided Learning (CAL), including all the activities via which computers contribute to the learning process in various ways, defining four paradigms: a) Instructional model or Computer Assisted Instruction – CAI, b) Relevatory model or Simulation, c) Conjectural model or Modeling and d) Emancipatory model.

According to Scardamalia & Bereiter, computer learning environments, if appropriately designed, can support constructivist and exploratory learning, giving learners more agency in the learning process [32], [12]. Especially simulation–based learning involves learning performed in a computerized environment, in which the learner interacts with the entities of the environment and gradually infers the features of the concept model whilst he/she proceeds through the simulation, which may lead to changes in his/her original concept [41].

The discussion on whether computerized environments improve learning performance is currently continuing. There are studies' outcomes according to which simulation–based learning does not significantly improve the test results of learners [29], [31], other outcomes that do not prove significant differences between simulation–based and narration–based teaching [3] and others that prove significant advantages of simulation–based learning [4], [5], [25].

Today's most appealing model of Computer Aided Learning is "Computers as cognitive tools". According to Jonassen [18], cognitive tools or mindtools are learning environments and computer based tools that have been developed or adjusted, in order to function as "intellectual partners" of the students, in order to activate and accommodate critical thinking and higher order learning.

Cognitive tools have the following characteristics:

- They are generalizable computational tools [18].
- They reorganize (radically reconstruct) the way learners think [26].
- They aim in activating and facilitating the cognitive process [23].
- They support, guide and extend the thinking processes of their users [9].
- They are not just accommodating tools [18], neither "fingertip" tools [27].
- They are critical thinking devices [18].
- The tools create an "Intellectual Scaffolding" towards "meaningful thinking" [18].
- They support knowledge construction and transferable learning [18].
- They have simple, powerful formalism and they are easily learnable [18].

C. The computational experiment

According to Sloot [36], Computational Science (CSE) aims to create reliable computational experiments. The functional stages in the development of a computer experiment are: Physical phenomenon, Mathematical model, Discrete algebraic approximation, Numerical algorithm, Simulation, Computer experiment.

Sloot [36] identifies three major phases in the process of the development of a computer experiment, each with its own challenges:

- The modeling phase. The first step to simulation is the development of an abstract model of the physical system under study.
- The simulation phase. Here we refer to (mathematical) methods that make the underlying physical models discrete and a rough distinction can be made between solvers for Discrete Event systems and solvers for Continues systems. The more conventional solvers are Finite Difference, Finite Element/Volume and a large class of linear algebra solvers.
- The computational phase. In this phase we concentrate on the mapping of the different solvers to the machine architecture. Since the types of problems we

are interested in are computationally very demanding, lots of research effort in the efficient use of modern architectures for simulation is going on in this field.

Landau et al. [24] suggest an approach similar to Sloot's approach. They organize the steps of the Computational science in the framework of a scientific problem–solving as follows:

Problem↔Theory↔Model↔Method↔Implementation↔Assessment.

They state that Computational Physics, Computational Mathematics, Engineering etc. are subfields of Computational Science (CSE) and as multidisciplinary subjects, they combine aspects of Physics (Engineering, Biology, etc.), Applied mathematics and Computer Science (CS) with the aim of solving realistic problems.

We have to distinguish between computational science and computer science since, although related, computational science is not computer science. Computer science studies for its own intrinsic interest and develops the hardware and software tools that computational scientists use. Likewise, Applied Mathematics develops and studies the algorithms that computational scientists use. As much as Mathematics and Computer Science are interesting for their own sakes, our focus is on solving physical problems; we need to understand the CS and math tools well enough to be able to solve our problems (for education) correctly.

Landau et al. [24] emphasize their computational science focus in the form of a problem to solve, with the components that constitute the solution separated according to the scientific problem–solving paradigm. In this framework, being able to transform a theory into an algorithm requires significant theoretical insight, detailed physical and mathematical understanding and a mastery of the art of programming and the actual debugging, testing, and organization of scientific programs is analogous to experimentation, with the numerical simulations of nature being essentially virtual experiments. The scientific paradigm should include modeling and simulation as an additional dimension in order to create computational experiments.

The Scientific Paradigm of Landau et al. is summarized as follows:

Problem (from science), Modeling (discrete, continuous), Simulation Method (numeric, symbolic), Implementation (Mathematica, Java etc.) and finally Assessment and Visualization/exploration.

The format of Computational Science proposed by Sloot [36] and Landau et al. [24] places the subject matter in its broader context and indicates how the steps are applicable to a wider class of problems. Most importantly, educational assessments and surveys have indicated that some students learn science, mathematics and technology better when they are presented together in context rather than as separate subjects. Likewise, some students who may not profess interest in Mathematics or CS are motivated to learn these subjects by experiencing their practical value in science problem solving.

This view is also supported by Guzdial [15]. He states that today, education researchers are more interested in programming as a medium, as a way of thinking about and

exploring disciplines other than computer science [10], [37].

In his paper Guzdial [15] says that we are still interested in having students learn about programming, because we view programming as an important skill and as a medium of communication. But now we are even more interested in having students learn through programming because we recognize that programming is a good lever for understanding many domains.

D. Students' beliefs on Physics and Mathematics

Redish and Hammer [30] classify students' beliefs along three dimensions: Independence/authority, Coherence/pieces and Concepts/equations. Redish and Hammer's definitions on the students' beliefs on Physics and Mathematics are presented in Table I.

TABLE I.
DEFINITION OF STUDENTS' BELIEFS

	Beliefs on Physics and Mathematics	
	Favourable	*Unfavourable*
Independence	Learns independently, believes in their own need to evaluate and understand	Takes what is given by authorities (teacher, text) without evaluation
Coherence	Believes physics needs to be considered as a connected, consistent framework	Believes physics can be treated as separated facts or "pieces"
Concepts	Stresses understanding of the underlying ideas and concepts	Focuses on memorizing and using formulas
Reality link	Believes ideas learned in physics are useful in a wide variety of real-world contexts	Believes ideas learned in physics are unrelated to experiences outside the classroom
Math link	Considers mathematics as a convenient way of representing physical phenomena	Views Physics and Mathematics as independent with no strong relationship between them

III. THE COMPUTATIONAL EXPERIMENT USING ELECTRONIC WORKSHEETS IN MATHEMATICA

A. Discovery learning through the computational experiment

According to de Jong [8], inquiry (discovery) learning is defined as "an approach to learning that involves a process of exploring the natural or material world and that leads to asking questions, making discoveries and rigorously testing those discoveries in the search for new understanding".

Our challenge is to combine scientific inquiry and the computational experiment as the path to create/organize a student–centered instructional design. Scientific practice for students should contain the modeling, the algorithmic approach and the simulation of the phenomena which is the realm of the computational science [36].

Scientific practice involves also the construction, validation and application of scientific models, so science instruction (using the scientific inquiry learning) should be designed to engage students in making and using scientific models which are considered as coherent units of structured knowledge [17]. Therefore, the structure of scientific

knowledge can be made more explicit for students by organizing course content around a small number of basic models. The ability of students to make and use models depends on the representational tools at their command. Students learn transferable modeling skills by applying given models to a variety of situations to describe, explain or predict physical events or to design experiments.

Scientific practice for students (at all levels of education) should provide students with basic conceptual tools for modeling physical objects and processes, especially mathematical, graphical and diagrammatic representations. In addition to that, scientific inquiry for students should help them to develop insight into the structure of scientific knowledge by examining how models fit into theories and to show how scientific knowledge is validated by engaging students in evaluating scientific models through comparison with empirical data.

Models are simultaneously important outcomes of science and relevant components of scientific methodology and through modeling, students should: (i) be introduced to the most significant scientific models; (ii) become able to appreciate the scope and limitations of such models; and (iii) be engaged in both modeling activities and discussions concerning the use of the models that they produce [13].

In this framework, Instructional design should be organized into the sequences of the computational experiment, which should engage students in all phases of model development, evaluation and application in concrete situations – thus promoting an integrated understanding of modeling processes and acquisition of coordinated modeling skills.

Students would also be encouraged to present and justify their conclusions in oral and/or written form, including a formulation of models for the phenomena in question and evaluation of the models by comparison with data.

B. The computational experiment via electronic worksheets in Mathematica

We propose the creation and use of electronic worksheets containing the phases of the computational experiment, in the environment of a cognitive tool.

A very promising cognitive tool in teaching Mathematics and Science courses is Mathematica [43], [40], [14], mainly because:

a) Its mathematical operations' notation and objects are similar to the standard mathematical notation,

b) It has a function–based structure, which allows us to define and study objects and quantities as real functions of real variables and

c) It offers possibilities in plotting graphs easily, quickly and precisely and in making complex calculations quickly and accurately.

Electronic worksheets created in the environment of Mathematica function both as word processors and as dynamic, interactive notebooks of Mathematica, that means the students can run the commands and the programs that are handed out by the worksheet, they can alter the parameters, the plot margins and the expressions of the commands and the programs and run them again and they can write their answers to the questions posed typing in the free spaces of the worksheet.

Shunn and Klahr [34] and Klahr and Dunbar [20], in order to describe discovery learning as a search process, introduced spaces in scientific discovery learning that include the hypothesis space and the experiment space. In Klahr and Dunbar's model the hypothesis space contains all rules and variables describing the specific domain while the experiment space consists of all experiments that can be implemented within this domain.

Van Joolingen and De Jong [41] extended Klahr and Dunbar's model introducing different sub–domains in hypothesis space and proposed a taxonomy to describe relevant search operations in every space. The knowledge representations introduced by Van Joolingen and De Jong's model are not static, but they involve a time dependent evolution when students gain knowledge about the underlying model leading to a shift from their learner domain space toward the target conceptual model.

Van Joolingen and De Jong [41] consider that the universal hypothesis space (the set of hypotheses that can be stated in principle) and the target conceptual model (the set of hypotheses that describes the domain to be discovered) are fixed and that during discovery learning approach there could be a change only for the learner's hypothesis space and the learner's domain space. According to Van Joolingen and De Jong the change in learner's hypothesis space could happen, for example, when the learner discovers new relations that can be used in creating new hypotheses, while the learner's domain could change during experimentation.

Based on the principles mentioned above, the electronic worksheets in Mathematica, should include:

A) The hypotheses space, where the students in cooperation with the teacher decide, clarify and state the hypotheses of the problem/ problems or the subject domain to be studied.

B) The experiments space, where the computational experiment actually takes place, that includes simulation based discovery learning activities via which the students, through discussion and social interaction with their peers and the teacher, actively construct and formulate conclusions, generalizations of results and solutions on the problems or subjects under negotiation.

C) The predictions space, where the results, conclusions or solutions formulated in the experiments space are checked with the analytical (mathematical) solutions of the problem/ problems or the analytical negotiation or the subject domain to be studied, in order to check their credibility.

C. Electronic worksheet on oscillation synthesis

Below we present an electronic worksheet in Mathematica that contains the computational experiment on oscillation synthesis as presented above. The electronic worksheet basically contains two characteristic problems on oscillation synthesis. The first and second problems involve the study of oscillation synthesis for oscillations with different widths – same frequencies and same widths – different frequencies correspondingly.

The electronic worksheet contains the following parts (see Figures 1.a., 1.b., 1.c., 1.d., 1.e., 1.f., 1.g. and 1.h.):

1. Hypotheses space

a. Activity on oscillations

b. Setting the problem of oscillation synthesis

   c. Setting the hypotheses, the variables and the relations between variables of the 1st problem

   d. Setting the hypotheses, the variables and the relations between variables of the 2nd problem

2. Experiments space

   a. Experimental solution of the 1st problem

   b. Experimental solution of the 2nd problem

3. Predictions space

   a. Analytical solution of the 2nd problem

   b. Credibility of the results of the 2nd problem

IV. THE METHODOLOGY OF THE CASE STUDY

A. Description of the case–study

The case–study involves the application of the computational experiment via electronic worksheets in Mathematica in ASPETE (School of Pedagogical and Technological Education) and the study of the approach' s impact on students' scores and the students' beliefs on Physics and Mathematics. The application of the approach was implemented in a class of 20 students, attending the course "Pedagogical Applications of Computers".

Two research questions were posed and are answered by this paper:

1st research question: Does the computational experiment using electronic worksheets in Mathematica have an impact on students' scores in tests concerning the subjects taught?

2nd research question: Is there a shift on students' beliefs regarding Physics and Mathematics that can be attributed to the approach?

Two questionnaires were designed and implemented, being handed out to the students the first before and the second after the application of the approach. Both questionnaires included two parts: The first part aimed at evaluating students' understanding of the cognitive subject taught and the second part aimed at evaluating students' beliefs on Physics and Mathematics, as defined by Redish and Hammer [30].

The cognitive subject selected is oscillation synthesis, a subject that all students have been taught both in high school, being included in the content that the students were prepared to be examined at the formal state Greek Entry Examinations, but also in the students' studies in ASPETE. In this way differences in students' scores in the two tests before and after the application of the approach can be attributed to the approach. Also differences in students' beliefs on Physics and Mathematics before and after the application of the approach can also be attributed to the approach.

B. The teaching approach

The experiment lasted for 6 hours. The lessons took place in the computer laboratory, with the students working in groups of 2 students per computer. The students cooperated with the members of their group, with members from other teams and with the teacher in dealing with the problematic situations they came across while working with the electronic worksheet. They were given the opportunity to set into discussion questions, conjectures and conclusions to the community of the class and to ask at any time the help of the teacher, regarding the understand-
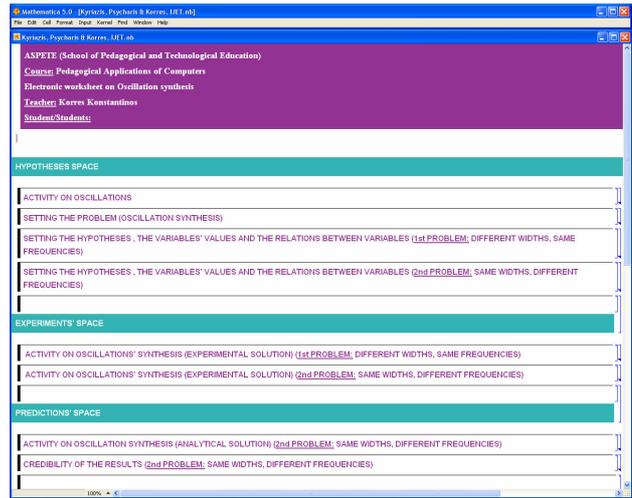

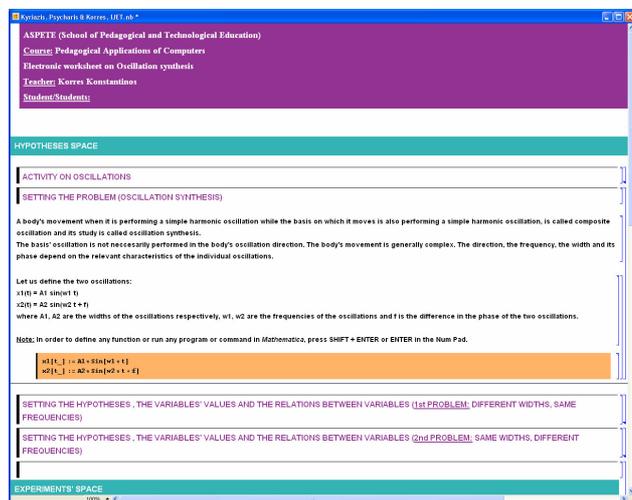Figure 1.a. Electronic worksheet on oscillation synthesis – Starting screen

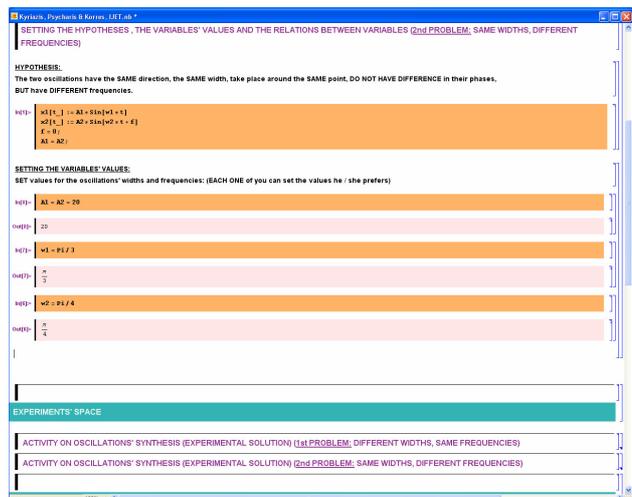Figure 1.b. Electronic worksheet on oscillation synthesis – Hypotheses space: Setting the problem

Figure 1.c. Electronic worksheet on oscillation synthesis – Hypotheses space: Setting the hypotheses, the variables and the relations between variables of the 2nd problem (same widths, different frequencies)
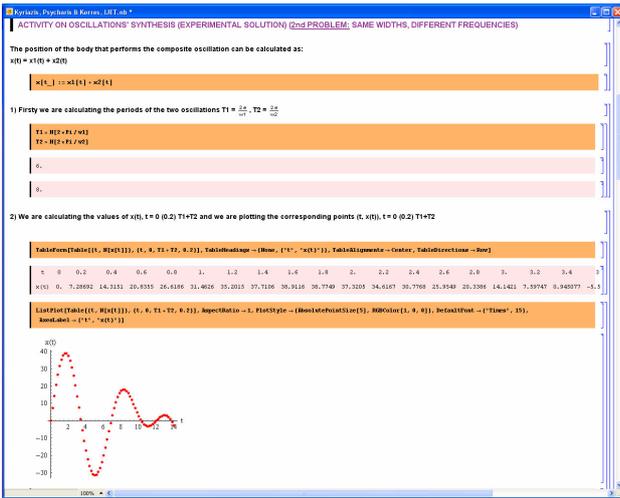
Figure 1.d. Electronic worksheet on oscillation synthesis – Experiments space: Experimental solution of the 2nd problem (1st screen)
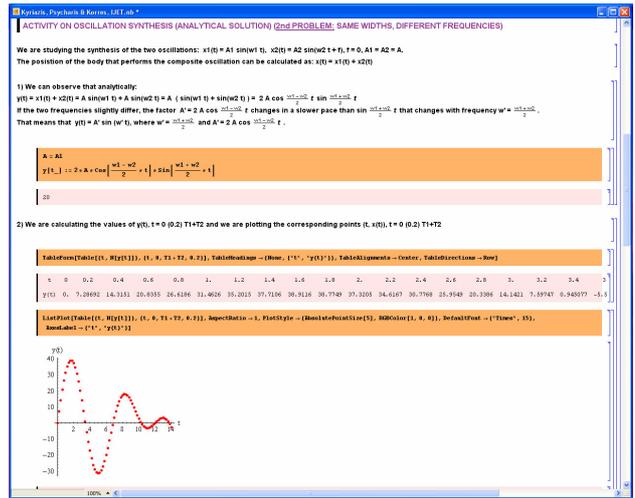

Figure 1.g. Electronic worksheet on oscillation synthesis – Predictions space: Analytical solution of the 2nd problem
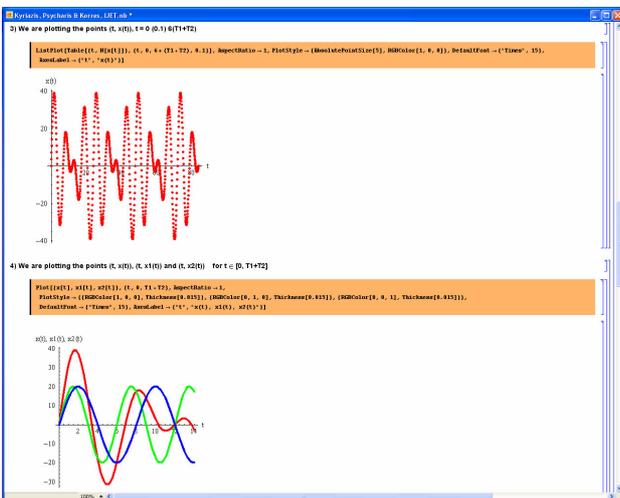

Figure 1.e. Electronic worksheet on oscillation synthesis – Experiments space: Experimental solution of the 2nd problem (2nd screen)
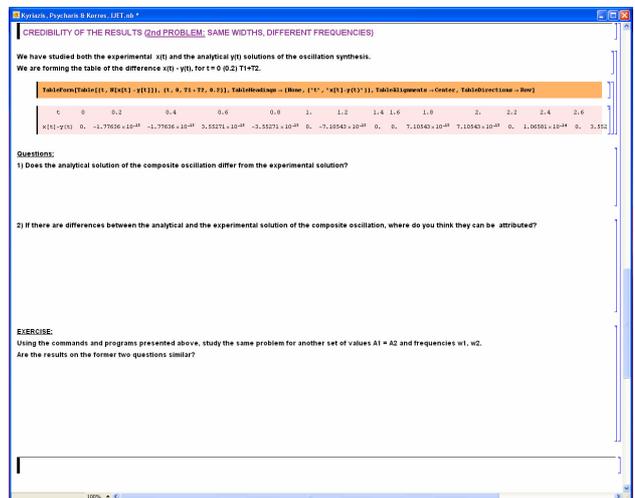

Figure 1.h. Electronic worksheet on oscillation synthesis – Predictions space: Credibility of the results of the 2nd problem

Figure 1.f. Electronic worksheet on oscillation synthesis – Experiments space: Experimental solution of the 2nd problem (3rd screen)

ing of the concepts referred to at the lesson and the use, the syntax and the function of the software's commands. The teacher wrote on the board the elements of theory he regarded as necessary, the commands and programs of Mathematica, the conjectures, the hypotheses and the conclusions formulated by the students and with the use of a video – projector, he presented to students static and dynamic representations (animations), relevant to the subjects under negotiation. Also notes were handed out to the students at the start of every lesson containing the lesson's main elements

The computational experiment via electronic worksheets in Mathematica on oscillation synthesis had the following stages:

At the first stage, the students were guided to work in groups of two students with a basic problem on oscillations and afterwards to discuss the setting of the problem of oscillation synthesis at the Hypotheses space, at the start of the lesson.

The second stage involved the students working with the 1st problem in Hypotheses space (Setting the hypothe-

ses) and Experiments space (Experimental solution), with the teacher basically teaching those activities and the students working at the same pace all together and discussion taking place during that process.

At the third stage, the students were guided to work in groups of two students with the 2nd problem individually in Hypotheses space (Setting the hypotheses), in Experiments space (Experimental solution) and in Predictions space (Analytical solution) and then they discussed with the teacher and the whole class the Credibility of the results' part in Predictions space.

V. RESULTS

A. The students that participated in the study

The students that participated in the study were 20 students of ASPETE, attending the course "Pedagogical Applications of Computers". The participation in the study was on a voluntary basis and the questionnaires were answered anonymously, with the registration number of the students to be the only personal data filled in at the questionnaires, in order for pre and post questionnaires to be correspondent for every student.

Regarding gender, 40 % of the students were males and 60 % females. Regarding age, the students had mean age 20.16 with standard deviation 2.5, having minimum age 19 and maximum age 30 years.

B. Students' scores

The students' scores before and after the application of the approach present significant differences since the students before the approach had mean score 3.35 with standard deviation 2.80 while after the approach they had mean score 6.15 with standard deviation 2.48 (see Table II).

TABLE II.
DESCRIPTIVE MEASURES FOR STUDENTS' SCORES

	Descriptive measures				
	N	**Minimum**	**Maximum**	**Mean**	**St. Deviation**
Score (pre)	20	0.0	8.0	3.35	2.80
Score (post)	20	2.0	10.0	6.15	2.48

The difference between the students' scores before and after the application of the approach can also be validated by the results of the Inferential Analysis and specifically via the Paired Samples t–test.

The paired samples t–test has as a precondition the differences of the values of the students' scores before and after the approach to come from a population with normal distribution. One Sample Kolmogorov–Smirnov test for the differences d_i = Score(post)$_i$ – Score(pre)$_i$, i=1(1)20, showed: Z = 0.753, p = 0.622, so the precondition of paired samples t–test is satisfied.

The results of the paired samples t–test showed that Pearson's correlation coefficient is r = 0.820, p < 0.001, so there is a strong linear correlation between the two variables.

The mean value of the differences between students' scores equals to –2.8, with the corresponding 95% confidence interval to be from –3.55 to –2.05. Since the confi-

dence interval does not contain the value 0, there is difference in the mean scores of the students, with 0.05 probability of error. The value of t–test is t = –7.782, df = 19, p < 0.01, with the negative sign stating that the mean value in students scores before the approach is lower than the mean value of the scores after the approach.

By observing the students' scores distributions (see Table III), we can conclude that before the application of the approach 35 % of the students scored 5 and above compared to 65 % of the students after the application. Also before the application only 5 % of the students scored 8 and above and 0 % 9 or 10, compared to 35 % and 15 % of the students after the application of the approach correspondingly.

TABLE III.
DISTRIBUTION OF STUDENTS' SCORES

Scores	**Students' scores**			
	Score (pre)		**Score (post)**	
	Frequency	**Percent**	**Frequency**	**Percent**
0	7	35.0	0	0.0
1	0	0.0	0	0.0
2	0	0.0	1	5.0
3	1	5.0	1	5.0
4	5	25.0	6	30.0
5	2	10.0	1	5.0
6	2	10.0	1	5.0
7	2	10.0	3	15.0
8	1	5.0	4	20.0
9	0	0.0	0	0,0
10	0	0.0	3	15.0
Total	20	100,0	20	100,0

C. Students' beliefs on Physics and Mathematics

Students' beliefs on Physics and Mathematics regarding Independence do not present significant change compared before and after the application of the computational experiment via electronic worksheets. 30 % stated that they learn independently before the application of the approach compared to 40 % after the application (see Table IV).

That result is also validated by the results of the Inferential Analysis, since X^2 – Testing for Independence gave the results: X^2 = 6.706, p = 0.01 (< 0.05), so the two variables can be regarded as dependent for significance level 5 %.

Also McNemar test for equality of percentages (p = 0.625) showed that there is not a statistically significant difference in the students' beliefs regarding independence before and after the application of the approach.

TABLE IV.
STUDENTS' BELIEFS ON PHYSICS AND MATHEMATICS - INDEPENDENCE

Beliefs on Physics and Mathematics	**Independence(post)**	
Independence(pre)	Learns independently	Takes what is given by authorities
Learns independently	5	1
Takes what is given by authorities	3	11

Students' beliefs on Physics and Mathematics regarding Coherence before and after the application of the approach seem to present significant change, since 35 % of the students stated that Physics needs to be considered as a connected, consistent framework before the application of the approach compared to 63.2 % afterwards (see Table V).

That result is validated by the results of X^2 – Testing for Independence: $X^2 = 0.652$, $p = 0.419$ (> 0.05), so the two variables can be regarded as independent for significance level 5 %.

McNemar test for equality of percentages however ($p = 0.146$) showed that there is not a statistically significant difference in the students' beliefs regarding coherence before and after the application of the approach.

TABLE V.
STUDENTS' BELIEFS ON PHYSICS AND MATHEMATICS – COHERENCE

Beliefs on Physics and Mathematics	Coherence (post)	
Coherence (pre)	Physics needs to be considered as a connected, consistent framework	Physics can be treated as separated facts or "pieces"
Physics needs to be considered as a connected, consistent framework	3	3
Physics can be treated as separated facts or "pieces"	9	4

Students' beliefs regarding Concepts before and after the application of the approach do not present significant change, since 70 % of the students stated that they stress understanding of the underlying ideas and concepts before the application of the approach compared to 75 % afterwards (see Table VI).

That result is validated by the results of X^2 – Testing for Independence: $X^2 = 7.937$, $p = 0.005$ (< 0.05), so the two variables can be regarded as dependent and McNemar test for equality of percentages ($p = 1.000$).

TABLE VI.
STUDENTS' BELIEFS ON PHYSICS AND MATHEMATICS – CONCEPTS

Beliefs on Physics and Mathematics	Concepts (post)	
Concepts (pre)	Stresses understanding of the underlying ideas and concepts	Focuses on memorizing and using formulas
Stresses understanding of the underlying ideas and concepts	13	1
Focuses on memorizing and using formulas	2	4

Students' beliefs regarding Reality link before and after the application of the approach are exactly the same, since the total of the students stated that ideas learned in physics are useful in a wide variety of real world contexts (see Table VII).

Students' beliefs regarding Math link before and after the application of the approach do not present significant change, since 90 % of the students stated that Mathematics is a convenient way of representing physical phenomena before the application of the approach compared to 95 % afterwards (see Table VIII).

TABLE VII.
STUDENTS' BELIEFS ON PHYSICS AND MATHEMATICS – REALITY LINK

Beliefs on Physics and Mathematics	Reality link (post)	
Reality link (pre)	Ideas learned in physics are useful in a wide variety of real world contexts	Ideas learned in physics are unrelated to experiences outside the classroom
Ideas learned in physics are useful in a wide variety of real world contexts	20	0
Ideas learned in physics are unrelated to experiences outside the classroom	0	0

TABLE VIII.
STUDENTS' BELIEFS ON PHYSICS AND MATHEMATICS – MATH LINK

Beliefs on Physics and Mathematics	Math link (post)	
Math link (pre)	Mathematics is a convenient way of representing physical phenomena	Physics and math are independent with no strong relationship between them
Mathematics is a convenient way of representing physical phenomena	18	0
Physics and math are independent with no strong relationship between them	1	1

That result is validated by the results of X^2 – Testing for Independence: $X^2 = 9.477$, $p = 0.002$ (< 0.05), so the two variables can be regarded as dependent and McNemar test for equality of percentages ($p = 1.000$).

VI. CONCLUSIONS

This paper presents a combined approach that is the computational experiment via electronic worksheets in Mathematica. The approach is based on the principles of Discovery learning expanded with the principles of constructivist, socio–cultural and adult learning theories, the concept of cognitive tools and the views on which the computational experiment is founded.

Mathematica is a very promising cognitive tool for Mathematics and Science Higher Education, since its notation, its function–based structure and the possibilities it offers in plotting graphs and making complex calculations make it an ideal tool–intellectual partner.

We propose the creation and use of electronic worksheets in Mathematica, containing the phases of the computational experiment, in the environment of a cognitive tool, electronic worksheets that include:

A) The hypotheses space, where the students in cooperation with the teacher state the hypotheses of the problem/problems to be studied.

B) The experiments space, that includes simulation based discovery learning activities.

C) The predictions space, where the results formulated through experimentation in the experiments space are checked with the analytical solutions of the problem/problems, in order to check their credibility.

This paper also presents a case study concerning the application of the computational experiment via electronic worksheets in ASPETE, during the spring semester of the

academic year 2008–09. 20 students participated voluntarily in the study, attending the course "Pedagogical Applications of Computers", 40 % of who were males and 60 % females.

The students' scores before and after the application of the approach present significant differences, making a shift from mean score 3.35 before the approach to 6.15 afterwards.

The difference between the students' scores before and after the application of the approach was also validated by the results of the Inferential Analysis and specifically via the Paired Samples t–test, as the Pearson's correlation coefficient showed a strong linear correlation between the two variables and both the 95% confidence interval and the value of t–test showed that there is difference in the mean scores of the students, with the mean value in students' scores before the approach to be lower than the mean value in students' scores after the approach.

Students after the approach showed higher percentages both in passing grades (5 and above) and higher (8 and above) and excellent grades (9 or 10).

Students' beliefs on Physics and Mathematics do not present significant change compared before and after the application of the computational experiment via electronic worksheets regarding Independence, Concepts, Reality link and Math link as studied by the corresponding Crosstabulation Tables. These results were also validated by the results of the Inferential Analysis, both by X^2 – Testing for Independence and by McNemar test for equality of percentages.

Students' beliefs regarding Coherence seem to present significant change before and after the application of the approach, by the corresponding Crosstabulation Table, a result validated by X^2 – Testing for Independence. However McNemar test for equality of percentages did not show a statistically significant difference in the students' beliefs.

In answering the two research questions posed at the Methodology of the study section of the paper, we can conclude that there was a significant change in students' scores that can be attributed to the application of the computational experiment via electronic worksheets. However there was not a significant shift on students' beliefs regarding Physics and Mathematics, a result that can be attributed to the fact that students in Mathematics and Science Higher Education have established beliefs for Physics and Mathematics, generally favourable ones, since they have chosen that direction in their studies after their Entry Examinations, but also since they have come in contact with a series of courses that have reinforced those beliefs.

The application of the computational experiment via electronic worksheets in Mathematica can therefore be characterized as successful, providing to Higher Education's teachers a powerful tool in order to introduce and successfully implement discovery learning activities in their courses combined with the powerful method of the computational experiment. The students in that context can develop both Mathematics and Science knowledge and skills more effectively, come in contact with the possibilities cognitive tools have to offer, but also come in contact with the Computational Science, a domain that has much to offer in Mathematics and Science Education.

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