# Improving Students' Mathematical Reasoning Abilities Through Social Cognitive Learning Using GeoGebra

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Abstract-There have been many studies on technology-supported learning based on cognitive theory in the literature. However, little is known about GeoGebra-assisted social cognitive learning in supporting students' reasoning abilities for online learning during the COVID-19 pandemic. This study aims to examine and analyze the differences in the improvement of students' mathematical reasoning abilities who follow GeoGebra-assisted social cognitive learning (Geo-SCL) and GeoGebra-assisted problem-based learning (Geo-PBL). This study used a quantitative method with a quasi-experimental nonequivalent pretest post-test control-group design. The sample consisted of 70 students from XI SMA Negeri 8 in Bandung, Indonesia. Before and after therapy, research data were collected using a mathematical reasoning test consisting of 5 essay questions. Paired sample t-test analysis and independent t-test were used to answer the research hypothesis. The results of the study concluded that students who studied with Geo-SCL obtained a higher increase in mathematical reasoning abilities than students who studied with Geo-PBL, with the criteria for improving abilities in both classes were in the moderate category. Research findings related to the application of Geo-SCL can be an alternative learning model in online learning situations.

**Keywords**—mathematical reasoning ability, social cognitive learning, problem based learning, GeoGebra

### 1 Introduction

The emphasis of education today is on providing students with the skills needed to adapt to the challenges of the 21st century [1]. Likewise, in order to survive in today's modern society, mathematical reasoning skills are needed to help develop critical thinking so that students are able to effectively solve problems and interpret information in various contexts and understand Mathematics in a more meaningful way [2]–[4]. Therefore, it is imperative for teachers to create a learning climate that can accommodate students in building mathematical reasoning abilities.

The ability to reason mathematically is the foundation for learning mathematics [5]. Mathematical reasoning ability is closely related to logical, analytical, and critical thinking patterns [6]. Mathematical reasoning ability is intimately tied to logical,

analytical, and critical thinking patterns. This opinion follows the opinion [7]–[9] that mathematical reasoning ability is a cognitive activity process to draw conclusions or produce a statement that has been verified. Someone with poor mathematical reasoning abilities will always struggle to solve difficulties because of their incapacity to link information to conclude [10]–[12]. Therefore, every student needs to develop mathematical reasoning abilities [6].

Scientific reasoning is highly emphasized in educational standards. in the PISA test, mathematical reasoning ability is also one of the abilities tested [13]. On the other hand, the mathematics scores of Indonesian students are low in the results of the PISA survey [14]. This condition is reinforced by the results of TIMSS which reports that Indonesia ranks 5th from the bottom [15]–[18], this indicates that the objectives of mathematics have not been fully achieved. The low math score is related to students' reasoning abilities, because one of the goals of mathematics subjects stated by [19] is that students can use reasoning on patterns and features, as well as mathematical operations, to make generalizations, compile evidence, and explain mathematical ideas and claims. This fact emphasizes that efforts to improve mathematical reasoning abilities in the classroom are a concern in the teaching and learning of mathematics.

Currently, the situation of online learning as a medium in continuing the teaching and learning process during the Covid-19 period [20]. This situation requires adjustment of the learning process compared to before, so that the learning paradigm in a situation like this is an absolute must to change. One learning theory that can support this situation is social cognitive theory.

Social cognition theory is a theory that states that most human learning takes place in a social setting. Humans learn knowledge, rules, skills, techniques, beliefs, and attitudes through observing other people. Individuals also analyze the utility and appropriateness of behaviors emerging from the modeled behavior reinforcement, and then they act based on their views about their skills and the expected effects of their activities [21], [22].

According to [21], more learning occurs without real reinforcement. In his research, it turns out that people can learn new responses by looking at other people's responses, even learning continues to occur without participating in doing what they have learned, and the model he observes also does not get from his behavior. Learning through observation is much more efficient than learning through direct experience. Through observation, people can obtain innumerable responses, which may be followed by connection or reinforcement. Modeling is at the heart of observational learning. Imitation or imitation should not be used to replace the word modeling because modeling entails more than just imitating or repeating what the model person (others) does; modeling entails adding and or subtracting observed behavior, generalizing multiple observations at once, and involving cognitive processes. So, it is suspected that social cognitive theory can accommodate online learning processes such as in the current COVID-19 situation.

The simple reasoning process can be developed through the observation process. Students can learn through one's way of thinking in solving problems. The development of the reasoning process can be done by being aware of situations that can motivate students to actively observe the model. The model in this case can be in the form of teachers, colleagues, or teaching materials [21]. Presenting a model that attracts

attention and motivates students can focus students on the observation process. Listening to the teacher delivering the material, peers expressing opinions in solving problems or observing problem solving schemes presented in teaching materials, can help students understand the learning behavior displayed. In this situation, cognitive factors play a role in observing, weighing and absorbing the learning behavior displayed by the model. So it is hoped that the social cognitive learning model can accommodate students' mathematical reasoning processes in online learning.

Learning reinforcement can be assisted by using a computer [23]. Rapid developments in the world of technology make a positive contribution in the process of conveying mathematical concepts [24]–[28]. Research findings state that computer-assisted teaching materials can expand mathematical knowledge in exploring subject matter [20], [24], [26], [29], [30]. GeoGebra is one of the many application software that can help in visually understanding a concept in mathematics [20], [31]. GeoGebra has a feature that can run on a computer or mobile device, so this makes GeoGebra more flexible than similar applications [26]. GeoGebra software simplifies the depiction of complicated geometric ideas and aids in the development of students' mathematical reasoning abilities [29]. GeoGebra is a mathematics learning media that helps enter data from mathematical functions into the form of images that are able to provide understanding to students [32], [33]. GeoGebra includes a device that has visuals that link mathematical functions based on demands, as well as fundamental functions of instructions in input that make it simpler for students to develop reasoning, so students may generalize data and produce conclusions based on the usefulness of current procedures [34]–[36].

Referring to the explanation above, previous research that applies social cognitive learning in mathematics is still minimal. Recent research conducted by [37] in the field of Education, reports on the application of social cognitive concepts in its setting in the online learning period. The results obtained that social cognitive theory which consists of behavior, cognitive and situational factors described through social, cognitive and teacher presence, as a case study framework on online and distance learning, provides an alternative how to condition learning that can be built in situations online during the covid-19 pandemic. Meanwhile, other applications are found in the business sector [38]–[41], in the criminal sector [42], and in the information and management sector [43], [44]. Based on the search results, the researchers concluded that social cognitive learning is still not widely applied in the field of mathematics education. In addition, researchers want to include the application of GeoGebra to strengthen the application of social cognitive theory. The GeoGebra program, which can be operated on mobile phones or computers, will make it simpler for students to access and use this application during the learning process, so that students will be more motivated and engaged in the continuous learning process. In addition, researchers will use a GeoGebra-assisted problem-based learning in the control group to assess the effectiveness of the social cognitive learning. The learning process takes place online in both the experimental and control classes.

Based on the description above, this study aims to examine and analyze the differences in the improvement of students' mathematical reasoning abilities who study with GeoGebra-assisted social cognitive learning (Geo-SCL) and GeoGebra-assisted problem-based learning (Geo-PBL). The research questions posed are (1) What is the description of the criteria for improving students' mathematical reasoning

abilities who study with GeoGebra-assisted social cognitive learning (Geo-SCL) and GeoGebra-assisted problem-based learning (Geo-PBL)? (2) Is there a difference in the effect of the implementation of Geo-SCL and Geo-PBL on the acquisition of mathematical reasoning abilities? (3) is there a difference in increasing reasoning ability between students who study with Geo-SCL and Geo-PBL?

## 2 GeoGebra

GeoGebra was created by Markus Hohenwarter in 2001. Unlike most other software, GeoGebra can be downloaded for free from the internet: <u>www.geogebra.org</u>. GeoGebra works on a very wide spectrum of computer systems that have Java programs. GeoGebra complements existing computer programs for learning algebra, such as Derive, Maple, MuPad and computer programs for learning geometry, such as Geometry Sketchpad or CABRI Geometry's.

Unlike other programs that are used specifically to teach and study geometry or algebra separately, GeoGebra is designed to teach and study algebra and geometry simultaneously. GeoGebra is a dynamic geometry application that requires pupils to work with points, vectors, segments, lines, and conic sections. Equations and coordinates, on the other hand, may be directly inserted. Algebraically specified functions can therefore be dynamically adjusted [20], [34]–[36].

GeoGebra is an excellent resource for both instructors and students. GeoGebra, unlike commercial software, may be installed on home computers and utilized by students and teachers at any time and from any location. GeoGebra provides an efficient way to build a creative and dynamic learning environment in which students may experiment with various mathematical subjects. GeoGebra introduces a new dynamically linked learning environment [26], [33].

The dynamic meaning in the above statement means that users with GeoGebra can generate interactive mathematical applications. In addition, GeoGebra can be used and copied for free and is included in open-source software that can cause anyone to improve or change the program for the better [32]. Multi-platform means GeoGebra is available for various types of computers such as tablets, PCs and various other computer systems. Based on some of the comments made above about GeoGebra, the researchers believe that it is an excellent learning media for assisting in the learning process, particularly in the form of seeing an item.

# **3** Research methods

#### 3.1 Research design

This study used a quantitative method with a quasi-experimental nonequivalent pretest post-test control group design [45]. The study included both the independent and dependent variables. A learning model is the independent variable, while mathematical reasoning ability is the dependent variable [46], [47].

The study included two groups: the experimental group and the control group. In the experimental class, a Geo-SCL was applied, while in the control class a Geo-PBL was applied. Researchers first introduce the functions and features of GeoGebra. The study gave the test twice to both classes. The pre-test is the first test given before treatment, while Post-test is the second test given after treatment. The purpose of giving these two tests is to determine the increase in mathematical reasoning ability that occurs in both classes. The research took place online using the zoom meeting application for the learning process. Google Classroom, Google Forms and WhatsApp were used to organize the collection of assignments and test results for mathematical reasoning abilities. The selection of Google Classroom, Google Form and WhatsApp applications with the consideration that these applications are familiar to students and can help organize and communicate in the learning process [48]–[51].

### 3.2 Sample

The research sample amounted to 70 students of class XI SMA Negeri 8 Bandung, Indonesia. The experimental class consisted of 35 students consisting of 19 male students and 16 female students, while the students in the control class consisted of 35 students consisting of 17 male students and 18 female students. The sampling technique used is purposive sampling. The researcher communicates with the mathematics teacher in selecting the two classes that have equal or balanced abilities and is strengthened by an independent *t* test. The independent *t*-test resulted in a score of t(68) = 0.837with p > 0.05, indicating that the abilities of the experimental and control classes are the same or equal. The determination of ability equality seeks to guarantee that any progress is the consequence of inequalities in the availability of learning models.

#### 3.3 Research instruments

Data collection uses a mathematical reasoning ability test. The exam consists of 5 essay-style questions. The exam is structured to measure students' mathematical reasoning abilities. The aspects that are considered in measuring mathematical reasoning ability consist of (1) Memorized Reasoning, (2) Algorithmic Reasoning, (3) Novelty, (4) Plausible, and (5) Mathematical foundation [52], [53]. Before being used as a data collection tool, the mathematical reasoning ability test instrument was validated and tested for reliability in an effort to ensure that the data obtained during the research process was valid and reliable. Validity test through content validity and empirical validity procedures. Content validity is an assessment by experts. This study involved five experts, namely two professors and three doctors in math education. The results of the content validity test showed that the test instrument was content valid.

Empirical validity is in the form of statistical instrument testing using product moment correlation analysis, which is comparing test scores with certain criteria that are used as benchmarks outside the test in question. Before being used, the mathematical reasoning ability test instrument was tested on 37 students who had studied the limit function. Empirical validity was carried out by correlating the scores obtained by students on the mathematical reasoning ability test with the average daily test scores

obtained by students in the previous semester. The valid coefficient on the instrument is 0.462 > 0.325, so the mathematical reasoning ability test is valid. While the reliability test using alpha-cronbach, the test results obtained a score of 0.495, so that the mathematical reasoning ability test is reliable in the medium category.

### 3.4 Research procedure

This study is divided into two stages: preparation and implementation. The following is a brief description of the research procedure.

**Preparation stage.** The preparation stage includes field studies and literature studies to read the phenomenon of the research problem to be studied. After the problem is formulated, the preparation of research instruments is carried out. The preparation of the instrument passed the validity and reliability procedures. The validity test uses content validity and empirical validity. The content validity test asks for expert views on the instrument in measuring mathematical reasoning abilities. Then a number of revisions were made based on input from experts, and the instrument was tested on students outside of the research participants. The instrument's test attempts to assess its empirical validity and reliability. After the instrument is declared valid and reliable, the research can be carried out.

**Implementation stage.** The research implementation phase includes determining research participants. At this stage the management of research-related licensing is submitted to the relevant parties. Researchers conducted observations and discussed with mathematics teachers in determining research participants. After acquiring research volunteers, the researcher administered a pre-test before to implementing the learning therapy, followed by a post-test at the conclusion of the session.

#### 3.5 Data analysis

Analysis of research data using quantitative statistical tests. Prior to performing statistical tests, normality and homogeneity checks were done. The Kolmogorov-Smirnov test was employed for normality, whereas the Levene test was utilized for homogeneity. Normalized gain was utilized to assess the improvement in students' mathematical reasoning ability in both classes [54]. The normalized Gain formula and criteria follow the scheme in Table 1 as follows.

Normalized Gain  $(g) = \frac{\text{Posttest score} - \text{Pretest score}}{\text{Ideal maximum score} - \text{Pretest score}}$ 

Normalized Gain Score (g)	Interpretation
$g \ge 0.70$	High
$0.30 \le g < 0.70$	Medium
g < 0.30	Low

Table 1. Criteria for normalized gain scores (g)

The research questions posed are descriptive and hypothetical. The first research question is descriptive, where descriptive statistical analysis is used as a tool to explain the description of the criteria for increasing mathematical reasoning abilities in both classes, which include the mean, standard deviation, and skewness. The criteria for increasing mathematical reasoning ability in both classes refer to Table 1. While the second and third research questions are hypothetical, where statistical paired sample t-test and independent t-test are used to answer the second and third research questions. The analysis process is assisted by using SPSS 25 software. The analysis procedure begins with analyzing pre-test, post-test, and N-gain in determining the data is normal and homogeneous. Then proceed with the paired sample t-test and independent t-test [55].

# 4 Finding and discussion

### 4.1 Finding

The purpose of this study was to determine the difference in the improvement of students' mathematical reasoning abilities who studied with Geo-SCL and Geo-PBL. Based on these objectives, three questions were derived to be able to achieve the intended research objectives. The following describes the results of descriptive statistical analysis related to increasing mathematical reasoning abilities based on the model. Descriptive statistical analysis was used to answer the first research question. The results obtained are presented in Table 2 as follows.

Observed Aspects			Statistic	Std. Error
N-gain_Geo_SCL	Mean		.6551	.02792
	95% Confidence Interval	Lower Bound	.5984	
	for Mean	Upper Bound	.7119	
	5% Trimmed Mean	÷	.6584	
	Median	.6800		
	Variance		.027	
	Std. Deviation		.16516	
	Minimum		.33	
	Maximum		.93	
	Range	.60		
	Interquartile Range	.18		
	Skewness	280	.398	
	Kurtosis	459	.778	
N-gain_Geo_PBL	Mean	.5454	.02442	
	95% Confidence Interval for Mean	Lower Bound	.4958	
		Upper Bound	.5950	
	5% Trimmed Mean	5% Trimmed Mean		
	Median	.5400		
	Variance		.021	
	Std. Deviation	.14445		
	Minimum	.17		
	Maximum	.87		
	Range	.70		
	Interquartile Range	.18		
	Skewness	051	.398	
	Kurtosis		.586	.778

 Table 2. Descriptive statistical analysis of increasing mathematical reasoning abilities

 based on the model

Based on the average values, standard deviation and skewness in Table 2, it is known that the increase in mathematical reasoning abilities of students who study with Geo-SCL (N-gain mean = 0.65) is higher than the increase in mathematical reasoning abilities of students who study with Geo-PBL (N-gain mean = 0.54). The improvement criteria experienced in both classes are in the medium category. When viewed from the distribution of the improvement data in both classes, the increase in the mathematical reasoning ability of students who studied with Geo-SCL (s = 0.165) was more diverse than the increase in the mathematical reasoning ability of students who studied on the skewness score, both classes got a negative score (Geo-SCL skewness score = -0.28 and Geo-PBL = -0.05), so the graphs of increasing mathematical reasoning abilities of the two classes tended to be negative, meaning that the score of increasing mathematical reasoning abilities tend to cluster at high scores.

The next analysis is related to the normality test of the data and the homogeneity of the data on each reasoning ability score obtained from the two classes as a requirement before the paired sample t-test and independent t-test are carried out. This test is a series of answers to the second and third research questions. The results of the data normality test using the Kolmogorov-Smirnov test are presented in Table 3.

	Madal	Kolmo	gorov-Smir	nov <sup>a</sup>
	Model	Statistic	df	Sig.
Pre-Test	Geo-SCL	.142	35	.073
	Geo-PBL	.139	35	.085
Post-Test	Geo-SCL	.144	35	.065
	Geo-PBL	.141	35	.076
N-gain	Geo-SCL	.110	35	.200*
	Geo-PBL	.085	35	.200*

Table 3. Normality test of data pre-test, post-test, N-gain in both classes

Notes: \* This is a lower bound of the true significance. \* Lilliefors Significance Correction.

The results in Table 3 show that the significant level (sig.) of the pre-test, post-test and N-gain scores in both classes was well above 0.05. That is, all the data, both pre-test, post-test and N-gain scores from each class came from a population that is normally distributed. While the homogeneity test used the Levene test which is presented in Table 4.

	Levene Statistic	df1	df2	Sig.
Pre-Test	.859	1	68	.357
Post-Test	.173	1	68	.679
N-gain	.836	1	68	.364

Table 4. Uji homogenitas data pre-test, post-test, dan N-gain pada kedua kelas

Table 4 shows that the significant level of pre-test, post-test, and N-gain scores in both classes is well above 0.05. This condition explains that the pre-test, post-test, and N-gain scores from both classes have homogeneous variances. Because the statistical requirements related to the normality test and data homogeneity are met, the paired sample t-test and independent t-test can be performed.

The results of the analysis using SPSS software, obtained the output of the Paired sample t-test in Table 5, Table 6 and Table 7 as follows.

Table 5. Paired samples statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Pre_Geo_SCL	24.5714	35	13.30477	2.24892
	Post_Geo_SCL	73.5714	35	13.03679	2.20362
Pair 2	Pre_Geo_PBL	32.4286	35	11.39991	1.92694
	Post_Geo_PBL	68.7143	35	12.50546	2.11381

		Paired Di	fferences			Sig. (2-tailed)
		Mean	Std. Deviation	t	df	
Pair 1	Pre_Geo_SCL-Post_Geo_SCL	-49.00000	14.02728	-20.666	34	.000
Pair 2	Pre_Geo_PBL-Post_Geo_PBL	-36.28571	9.42016	-22.788	34	.000

Table 6. Paired sample t-test in both classes

		Ν	Correlation	Sig.
Pair 1	Pre_Geo_SCL & Post_ Geo_SCL	35	.433	.009
Pair 2	Pre_Geo_PBL & Post_Geo_PBL	35	.693	.000

Table 6 shows the results of Pair 1, namely the comparison of Pretest-Posttest mathematical reasoning abilities of students who study with Geo-SCL obtained a significant level of 0.00, which is smaller than the  $\alpha = 0.05$ . Thus, the implementation of Geo-SCL significantly affects the acquisition of students' mathematical reasoning abilities. While the results of Table 7 show the relationship between pre-test and post-test scores. Results In Pair 1, a correlation score of 0.43 was obtained with a significance level of 0.009, which is far below 0.05, meaning that there is a significant positive relationship between the pre-test and post-test reasoning abilities of students who study with Geo-SCL.

Other analysis results from Table 6 show Pair 2, namely the comparison of Pretest-Posttest mathematical reasoning abilities of students who study with Geo-PBL obtained a significant level of 0.000, which is smaller than the  $\alpha = 0.05$ . Thus, the implementation of Geo-PBL significantly affects the acquisition of students' mathematical reasoning abilities. While the results of Table 7 in Pair 2 obtained a correlation score of 0.69 with a significant level of 0.00, which is far below 0.05, meaning that there is a significant positive relationship between pre-test and post-test reasoning abilities of students who study with Geo-PBL.

The next analysis is in the form of hypothesis testing related to the difference in improvement between mathematical reasoning abilities between students who study with Geo-SCL and students who study with Geo-PBL using the independent t test which is presented in Table 8.

		Levene's Test for Equality of Variances		t-test for Equality of Means		r Equality Aeans
		F	Sig.	t	df	Sig. (2-tailed)
N-gain	Equal variances assumed	.836	.364	2.97	68	.004
	Equal variances not assumed			2.97	66.73	.004

Table 8. Comparative test of N-gain in both classes

Based on Table 8, the test results show that t(68) = 2.97 with a significant level (Sig. (2-tailed)) of 0.004, which is far below 0.05. so it can be concluded that there is a significant difference in the increase in mathematical reasoning ability between students who study with Geo-SCL and students who study with Geo-PBL. Based on Table 2, it can be

seen that the increase in the mathematical reasoning ability of students who study with Geo-SCL (an increase of 0.65) is higher than the increase of mathematical reasoning ability of students who study with Geo-PBL (an increase of 0.55).

#### 4.2 Discussion

Online learning has become a medium for delivering material during the COVID-19 period. Using various tools, students and teachers can communicate remotely, avoiding the need to meet in person in the classroom. Some applications have their respective advantages, requiring several applications to support the entire online learning process. For example, the zoom meeting application is used for face-to-face and virtual delivery of material. In contrast, the Google Classroom and Google Form applications manage assignments and record student attendance. The application of GeoGebra helps students to recognize and understand the concept of limit functions more efficiently. GeoGebra can accommodate the representation of mathematical concepts in multimode or multi-representation. In GeoGebra, there are windows for Algebraic representation (analytic), Visual representation (graphics), and numerical representation in the form of spreadsheets. Besides that, GeoGebra is also very easy to use. Its use is the point and click, no programming. Students can use this feature to see the concepts of limit functions from various angles and take advantage of their strengths to understand them better. All of these interactions occur 'live' on the monitor screen, so students do not have to wait long to find out whether the response they give is right or wrong. This learning situation can motivate students when studying in the Covid-19 situation. The application of the learning series can maintain students' emotions even though they cannot interact directly with teachers and other fellow students.

The results of the descriptive analysis showed that the increase in students' mathematical reasoning abilities in the two learning classes was in the medium category. This condition explains that the Geo-SCL or Geo-PBL learning model can facilitate students in improving their mathematical reasoning abilities. Each model has unique characteristics in guiding the learning process. The use of GeoGebra helps in understanding the concept of the limit of a function. The application of Geo-SCL and Geo-PBL showed differences in the acquisition of students' mathematical reasoning abilities.

The analysis results in Table 6 explain the differences in the acquisition of students' mathematical reasoning abilities before and after applying the Geo-SCL and Geo-PBL models. Table 5 shows the differences in students' mathematical reasoning abilities before and after applying the two models. The results showed that the post-test value of mathematical reasoning ability after applying Geo-SCL (M = 73.57) and Geo-PBL (M = 68.71) was higher than the pre-test value before applying Geo-SCL (M = 24.57) and Geo-PBL (M = 32.42). The correlation analysis between pre-test and post-test scores in Table 7 explains a significant positive relationship between pre-test and post-test students' reasoning abilities in each class Geo-SCL (r = 0.43) and Geo-PBL. (r = 0.69).

Improving mathematical reasoning abilities can be done by training students to be able to convey their ideas or ideas towards a concept or case [56]. Mathematical reasoning ability can be built simply by observing. When the teacher delivers the material, or when students present their findings, other students can focus on listening and observing the process of conveying ideas carried out by the teacher or student [57]. In this

situation, social cognitive theory explains that the learning process can occur simply through the process of observing or observing. In social cognitive learning theory it is explained that the learning process occurs through the interaction between behavior, personal and situational factors [21], [37], [58].

According to [21], more learning occurs without real reinforcement. In his research, it turns out that people can learn new responses by seeing other people's responses, even learning continues to occur without participating in doing what they have learned, and the model he observes also does not get reinforsement from his behavior. Learning through observation is much more efficient than learning through direct experience. Through observation, people can obtain innumerable responses, which may be followed by connection or reinforcement. Modeling is at the heart of observational learning. Imitation or imitation should not be used to replace the word modeling because modeling entails more than just imitating or repeating what the model person (others) does; modeling entails adding and or subtracting observed behavior, generalizing multiple observations at once, and involving cognitive processes.

The simple reasoning process can be developed through the observation process. Students can learn through one's way of thinking in solving problems. The development of the reasoning process can be done by observing situations that can motivate students to actively observe the model [59]. The model in this case can be in the form of teachers, colleagues, or teaching materials [21]. Presenting a model that attracts attention and motivates students can focus students on the observation process, such as the visualization displayed in the GeoGebra application. Listening to the teacher delivering the material, peers expressing opinions in solving problems and observing problem solving schemes presented in teaching materials, helps students understand the learning behavior displayed during the learning process [60]. In this situation, cognitive factors play a role in observing, weighing and absorbing the learning behavior displayed by the model.

It is different from the learning conditions in the class that learns with Geo-PBL. In this class, students are faced with problems related to the concepts to be studied. Contextual problems are used to stimulate and attract the attention of students to be able to focus on the topic of the material being studied. The successful application of problem-based learning reported by [61]–[63] concluded that mathematical reasoning abilities can be developed through this model. The initial ability of students is very important in the process of applying this model, this is because students must be able to learn, and reason about the problems given at the beginning of learning.

The following analysis in Table 8 is related to the difference in the increase in mathematical reasoning ability between the two classes, showing a significant difference in the increase in mathematical reasoning ability between students who study with Geo-SCL and Geo-PBL. Students who use the GeoGebra-assisted social cognitive learning model improve their mathematical reasoning abilities more than students who use the GeoGebra-assisted problem-based learning model. This conclusion backs up the findings of [37], which explains that social cognitive learning theory can be implemented in the online learning process during the Covid-19 epidemic. This finding is consistent with the previous study [20], [24], [26], [29], [31], [64], which found that GeoGebra can be integrated into mathematics learning and aid in the process of grasping mathematical ideas. On the other hand, the results of the study report that the problem-based learning model is not bad. This is based on the results obtained, namely there is a significant increase in the application of the GeoGebra-assisted problem-based learning

model. This finding also corroborates the results of research conducted by [65], [66], which in its findings reports that digital games and problem-based learning can help in improving level thinking skills, increase motivation and motivate students to be interested in the lesson.

### 4.3 Implications

Online learning is the primary medium to keep the learning process going during the Covid-19 period, which requires much support. The researcher realizes that students' emotional factors can be one of the inhibiting factors for the learning process, especially in online learning. This opinion is reinforced by [67], which states that students' emotions in online learning show a dynamic transition from anxiety and curiosity to emotional stability and then back to anxiety. This condition is a phenomenon that interacts with changing sources in the research context, so the selection of a suitable learning model and the use of various media that can help strengthen the delivery of information to students is essential to be prepared. Simply learning can occur through observation of models (in this case, teachers, GeoGebra, teaching materials), which can slightly ease the burden on teachers and students. Teachers must become models and present alternative models that can make students focus on learning in online learning situations. The selection and use of various supporting applications that can organize the delivery of learning information to students can be a researcher's effort to maintain students' emotional stability so that they can stay focused on learning. The implications of the results of this study can be used as consideration for teachers in carrying out mathematics learning, especially during the covid-19 pandemic.

### 5 Conclusion

The results showed that the criteria for increasing mathematical reasoning abilities in both classes were in the medium category. The implementation of Geo-SCL and Geo-PBL respectively has a significant effect on students' mathematical reasoning abilities, where students who study with Geo-SCL get a higher improvement than students who study with Geo-PBL. The research has limitations, namely the research subjects are only in class XI high school students. So it is necessary to do research again in applying the social cognitive learning model at other school levels. In addition, the aspect of mathematical ability is only limited to students' mathematical reasoning abilities, so that measurements on higher mathematical aspects need to be followed up. Based on the research findings and the limitations of this study, further research can develop a social cognitive learning model as an alternative learning model in online learning situations.

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