Model of Adaptive System for Mathematical Training of Students within eLearning Environment

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Abstract—The article discusses issues related to the design, development and implementation of the model of adaptive system for mathematical training of students within eLearning environment. It proposes a systematic set of scientific, theoretical and methodological provisions and approaches that determine the design of the model. The research objective of the paper is designing, developing and implementing a model of the adaptive e-learning system for students to improve their mathematical skills. The article gives the definition of the adaptive system for mathematical training of students. It describes the algorithm of adaptive learning of students within the system in the form of a sequence of operations and actions focused on achieving the learning outcomes, including diagnostics and detailing of personal student's characteristics; designing a student model based on personal student's characteristics and preferences; differentiation and selection of educational technologies, resources and teaching methods depending on personal student's characteristics and preferences; formation and provision of an individual learning path to a student; assessment and analysis of student' performance. In the work the authors used theoretical, empirical and mathematical research methods. The article gives the results of a pedagogical experiment on the use of the adaptive system for mathematical training of students within eLearning environment in the educational process of the higher educational institution, which proves a positive dynamics in the level of mathematical preparedness of students studying within the proposed adaptive system.

Keywords—mathematics education, e-learning, adaptive learning, eLearning environment, model of learning, student, HEI

1 Introduction

The dynamic processes taking place in the modern world, the rapid development of new technologies, and, as a result, an increase in the flow of information require from each student, as well as a specialist having education, a willingness to constantly improve their knowledge and skills. The outlined transition to a knowledge society as a new phase in the development of a modern high-tech society requires changes in various fields, primarily, in the field of education.

Today, it is clearly observed that students of the modern world have become more intolerant of the manifestations of the established clichés in education, lack of flexibility in educational effects [1]; they became to pragmatically assess the emerging circumstances of life and the importance of knowledge and competencies, give priority to building personal preferences in organizing their future life and educational prospects. At the same time, intellectual operations of thinking (understanding, concretization, abstraction, generalization, modeling, analogy, associations, etc.), which underlie the formation of universal educational actions of students, for various objective and subjective reasons ceased to develop effectively in school education [2, 3]. And this process loses the role of mathematics education as one of the most effective tools for personal development and mastering social experience, including against the background of examples how it is important to apply mathematics to real life, science and the development of productive resources.

The relevance of this research is determined by the significant changes taking place in all spheres of human social practice caused by digital processes characterizing the development of the world community. The importance of digitalization of education is caused by the need to adapt the system of vocational education and training to the demands of the digital economy and digital society, the formation of which are global trends of the modern era [4, 5]. When planning to achieve the goals set out in the designated documents, it is necessary to take into account that the process of digitalization of education has two sides:

– *firstly*, the formation of a rich eLearning environment as a set of software and hardware, psychological and pedagogical, information and communication, organizational and managerial systems that ensure the educational process aimed at achieving learning outcomes by students taking into account the requirements of the educational standards [5];
– *secondly*, a deep modernization of the educational process designed to prepare a person for life in a digital society and professional activity in a digital economy.

The research topic is extremely relevant, because today the history of adaptive digital learning environments, which began more than thirty years ago, is making another stage of its development. The COVID-19 pandemic has significantly changed the higher education system [6]. At the beginning of 2020, students from all over the world began to study online: traditional lectures and seminars moved to a new mode, previously unknown for many of them. During a short period of time, online technologies became widespread, while the use of adaptive systems in the educational process became the most popular direction.

Many scientists were engaged in research within the frameworks of the adaptive learning systems. Thus, Awadh A. Y. Al-Qahtani and S. Higgins [7] said that such systems contribute to the development of motivation and involvement of students in learning, S. Oxman, W. Wong [8] focused on improving students' learning outcomes, and D.P. Danilaev and Y.E. Polskiy [9] – on preparing students to enter the labor market.

The adaptive learning is a holistic multifactor process based on the principles of differentiation and individualization. One of the fundamental works in the field of the adaptive learning is the study of E.Z. Vlasova [10] who defines the main components of adaptive learning: orientation to interdisciplinary training, reflection of the specifics of professional activity, simultaneous management and self-organization of students in combination with reflection.

The founder of the applying the adaptation in learning based on automated programs is considered to be G. Pask [11]. The author defined the adaptive learning as a process, the course of which is continuously and promptly adjusted to the individual student's cognitive characteristics. The management of such a training is considered as a decision-making process in which the choice of the learning path significantly depends on the initial data or on the learning background. Back in the last century, specialist in the field of building the learning machines, academician A.I. Berg spoke about the undoubted benefits and fruitful widespread use of computers in the learning system, which "... will allow implementing a system of so-called adaptive learning, providing ... a perfect methodology of the educational process" [12, p. 375].

Today, the return of researchers to the ideas of adaptive learning has become possible and necessary, on the one hand, due to the objective needs of a modern society for training specialists ready to solve professional tasks in dynamically changing conditions, and, on the other hand, with the emergence of a new interdisciplinary theoretical and methodological base for research, taking into account achievements in the field of teaching and psychology, mathematics and computer science, management theory and artificial intelligence.

In terms of the management theory, the adaptive learning system is characterized by adaptation to specific conditions of the educational process, depending on certain necessary standards aimed at perfecting and improving the quality of learning.

At the level of higher education of V.I. Vasilyev and T.N. Tyagunova [13] present the adaptive learning system as a set of targeted measures aimed at improving the level and quality of education, but not reducible separately to either forms of learning, forms of knowledge control, or forms of organizing the efficient individual work of students, etc.

The implementation of adaptive properties should be based on the specific features of a particular academic subject, which determine not only the conceptual methodological and content aspects, but also the nature of future professional activity.

Thus, researcher T.L. Anisova [14] describes a model of the adaptive system for mathematical training in the higher technical educational institution using teaching methods that include summarising tables, reference schemes, animated learning tasks. When designing the adaptive system for future mathematics teachers, T.E. Chikina [15] uses building a workbook for practical classes, technological maps and diagnostic materials designed for prompt control and monitoring of students' progress. E.V. Smirnova [16] proposed the adaptive system for mathematical training of first-year students, which is based on the author's Mathcad express cards, the organization of student work in pairs and microgroups at an individual pace, three-level teaching monitoring, educational counseling. O.N. Berishvili [17] considers the adaptive system for mathematical training of engineers in the higher agricultural educational institution based on a set of teaching tools that stimulate the development of abilities to use optimal decision-making methods in future professional activity that are adequate to the strategy of agricultural production development in market conditions.

In our work, the *adaptive system for mathematical training of students* of higher educational institutions is understood as an open dynamic multicomponent learning system aimed at satisfying the professional, personal and social needs of students in mathematical knowledge, skills, and experiences; characterized by adaptation to the individual characteristics and preferences of students based on modeling the stylistic characteristics of a student, taking into account physiological characteristics, imitation of a profession-oriented learning environment [2].

Thus, the purpose of the research presented in this article is designing, developing and implementing a model of the adaptive system for mathematical training of students within eLearning environment.

1.1 Theoretical framework

The theoretical and methodological basis of the paper was a set of methodological approaches that reflect the initial research positions and provide a certain level of theorization of provisions erected. Here is our position on the correlation level of theoretical and methodological approaches in the model development:

- *general-science basis*, including the universally recognized principles of knowledge and categories of science, the worldview approaches to the educational process, the erected theoretical provisions and the transformation of reality, is a system approach, which allows representing the entire process of mathematical training in the adaptive system form, defining and characterizing its components and subjects, identifying interrelationships and interdependencies, and applying the learning methods and means in a complex and balanced manner. Also, systematic approach provisions are applicable to the functioning of the eLearning environment, which is considered as an open self-organizing system in close connection with the development of the pedagogical system;
- *theoretical and methodological strategy* is a set of the following approaches that determine the structural and content characteristics of the adaptive system for mathematical training of students:
  - *synergistic approach* that reflects in our research the design of the adaptive mechanisms of the system adaptation to internal or external changes that lead it to a non-equilibrium; considers fluctuations (random deviations and agitations) of the personality adaptation process due to its disequilibrium and the amplitude of deviations in the system adaptation process increasing its disequilibrium and bringing it closer to the bifurcation points (branching);
  - *cybernetic approach* that determines the functioning and optimal management of learning in the adaptive system for mathematical training of students. The general scheme of the management mechanism for the educational process in the conditions of the eLearning environment includes student model, training strategy and scenario; promotes integrity and efficiency of students' mastering the basic training programs, improving the quality and effectiveness of implementing mathematical training in the electronic form;
  - *environmental approach* that characterizes the organization of the educational process in the eLearning environment of the higher education institution, which provides access to curriculum and working programs of the subjects, provides for various forms of classes, procedures for assessing learning outcomes, formation

of an electronic portfolio, as well as interaction between the participants of the educational process;
  - *person-centered approach* that allowed building the educational process for each student taking into account their individual abilities and characteristics (psychological, pedagogical, and physiological), profession-oriented preferences and motives;
- practice-oriented tactics that specifies the research problem in terms of a particular set of methods and procedures specific to the scientific field includes:
  - *competency-based approach* that orientates the design of the adaptive system for mathematical training within eLearning environment of higher educational institutions to the model of a competent specialist with a significant shift towards student-centered learning; allows the transition from reproduction of mathematical knowledge to its practical application, provides the opportunity to implement mathematical knowledge in the future professional activities;
  - *context approach* that is connected to organising training within the eLearning environment, which allows access to profession-oriented and instrumental environments and services that simulate and imitate future professional activities; non-linearity of the educational process, which facilitates personification of educational activities and organising the multilevel learning; provision of competence-oriented courses with an interdisciplinary focus;
  - *technological approach* that presents the process of developing the adaptive system for mathematical training of students within eLeaning environment as a hierarchical and ordered aggregate of technological procedures for designing the educational process, guaranteeing the achievement of the planned result (both at theoretical and practical levels);
  - *axiological approach* that promotes the development of students' intellectual, attitudinal, aesthetic, moral values, the development of their intellectual, creative and moral potential, which manifests in the ability to freely navigate complex professional and social situations, to perceive and implement innovations.

2 Materials and methods

2.1 Participants

Experimental study consisted of 165 students of the majors Applied Mathematics and Computer Science (AM&CS), and Psychological and Pedagogic Education (PPE). Students were divided into experimental (82 students: 64 AM&CS students and 18 PPE students) and control (83 students: 66 AM&CS students and 17 PPE students) groups. Homogeneity and equilibrity of groups has been confirmed by the results of control assessment of experimental and control groups. The experiment was carried out within the framework of training in the subjects "Optimization Methods" (for AM&CS) and "Operations Research" (for PPE) during the spring term of the 2020–2021 academic year. Students of the experimental groups (EG) studied within an adaptive system for mathematical training, while students of the control groups (CG) studied outside of it, but had a possibility to use electronic educational mathematical resources and auxiliary integrated mathematical packages of applied programmes during studying.

2.2 Methods

The research used a complex of mutually complementary methods of studying:

– *theoretical* (methods of theoretical and methodological analysis (comparative analysis, retrospective, simulative and taxonomy), method of systemizing the existing experience and research findings);
– *empirical* (observation, poll, dialogue, survey, interviews, tests, expert assessment, pedagogical experiment);
– *mathematical* (correlative and dispersive analyses, methods of mathematical statistics, mathematical simulation).

In the creation of the adaptive system for mathematical training of students within eLearning environment a significant role was assigned to the methods for determining the content and hierarchical structure, parameters and tools for designing knowledge bases; for using semantic net of related frames of the fractal structure of knowledge.

3 Results

Modeling as one of the methods of scientific research is widely used in the educational systems and allows uniting theoretical and empirical aspects combining building the logical structures and scientific abstractions with experimental data. A *model* means an artificially created object in the form of a diagram, which is similar to the object or phenomenon under study, displays and reproduces in a simpler and more primitive form the structure, characteristics, correlations and relationships between the elements of this object [18]. The model allows giving an accurate definition and description of the components of a system or a process, schematically depicting the relationships (both internal and external) between the components; it is an effective tool for visualized and comparative study of its components.

The purpose of the offered model of the adaptive system for mathematical training of students within eLearning environment is to increase the level of students' mathematical competence based on the analysis of their individual characteristics and learning outcomes. We define student' mathematical competence as an integrative characteristic of a personality; it is characterized by a system of knowledge in mathematics, readiness and ability to apply them in professional and social activities. The process of forming mathematical competence among contemporary students of higher educational institutions assumes:

1) forming mathematical knowledge, skills and abilities according to the basic educational program of a higher educational institution;
2) forming the ability and readiness to apply mathematical knowledge and skills in professional activity;

3) forming the capacity to use modern means of information and communication technologies in the process of mathematical modeling and designing of the tasks of professional activity.

In the process of developing the model, the idea of the system unity principle was used as a basis, suggesting that the essence of any pedagogical system, regardless of its level, is determined by the same correlated and interdependent components in it, without which it cannot function. In our understanding, studying within the adaptive eLearning environment is represented by a sequence of operations and actions aimed at achieving the learning outcomes, consisting of the following process stages:

1. Diagnostics and detailing of student's individual characteristics.
2. Designing a student model based on the individual characteristics and preferences.
3. Differentiation and selection of educational technologies, resources and instructional practices depending on student's individual characteristics and preferences.
4. Forming and providing an individual learning path adapted to the particular student.
5. Assessment and analysis of students' learning outcomes.

Let us consider in detail the technology of studying in the adaptive system for mathematical training of students within eLearning environment of higher educational institution based on the implementing of organizational and pedagogical processes.

3.1 Diagnostics of students' individual characteristics and features

The idea of mathematical training of university students within the adaptive system is based on the assumption that the ability of each particular student is revealed as much as possible under best suited conditions. Organization of the process of mathematical training of students in the developed adaptive system within the eLearning environment provides an opportunity to build cognitive activity of students in accordance with their individual characteristics and given requirements, which is achieved by a clear correlation of goals and stages of activity, methods and tools, learning content and educational technologies. For this purpose students' training is organised in accordance with their individual features and required capabilities taking into account different factors: psychological and pedagogical, physiological, profession-oriented ones.

There is a list of questionnaires for diagnostics and taking into account students' psychological and pedagogical characteristics and individual features. A student is required to complete them for further data entry and building student individual model. It is possible to complete all the questionnaires for diagnostics at the initial entry, which is considered as optimal, or do it gradually during the educational process within the adaptive system.

For defining *psychological and pedagogical factors* connected with the peculiarities of students' stylistic characteristics, we used the following models of learning styles within the adaptive system:

1) Kolb's Experiential Learning Model [19], which defines education as a way of forming knowledge through the transformation of experience where the learning style consists of a way of gathering information and a way of reaction to academic

information and its assessment. Based on the combination of each of the two ways and components there are four main students' style types: Divergents, Assimilators, Convergents and Accommodators;

2) Honey & Mumford Learning Styles Model [20], based on the definition of students' style typologies depending on the major style of activity: Activists, Reflectors, Theorists and Pragmatists;
3) Gregorc's Mind Style Model [21], according to which a learning style is defined depending on the way of cognition (concrete or abstract) and a learning strategy (sequential or random). Based on that there are four main basic learning styles: Concrete Sequential, Abstract Sequential, Abstract Random and Concrete Random;
4) VARK Learning Style Model [22], according to which the educational process is based on individual psychological characteristics of the cognitive structure of a person, disposition toward the use of the ways of student's interaction with the learning material based on the perceptual canals; there are Visual Learners, Auditory Learners, Digital Learners and Kinaesthetic Learners;
5) Felder-Silverman Teaching Style Model [23] built on students' preferences depending on the way of collecting and processing information based on four factors with two opposing meanings, there are: Visual and Verbal Learners, Active and Reflective Learners, Sensing and Intuitive Learners, Sequential and Global Learners;
6) Whole Brain Model [24], based on the analysis of individual (right and left brain hemi-spheres) and organisational (conceptual, empiric) preferences in thinking, according to which there is the division of students into Theorists, Organisers, Humanitarians and Innovators.

The analysis of style models showed that each of them is characterised by an individual combination of criteria and some of which are repeated. For example, the Concrete-Abstract characteristic of a student corresponds to both Kolb's and Gregorc's models, the Active-Reflective one corresponds to Kolb's and Felder-Silverman models. Sequential element appears in Gregorc's and Felder-Silverman models, Visual element – in VARK and Felder-Silverman models, etc. Different factors consideration depends on models application goals and tasks. The necessity of a few models usage is due to the fact that none of these models separately cannot personalise the educational environmental to the maximum, individualisation in this case is produced only partly because either not all possible criteria are defined or they are defined not at most.

After psychological and pedagogical factors analysis, the adaptive system comes to *physiologic factors* diagnostics, which allow considering peculiar features of students' psychological and physical development. For his purpose a student has an opportunity to choose a category and mention the nosology kind for disable people. In particular, the instructional design and adaptation algorithm design of the adaptive system for mathematical training of students within eLearning environment for people with limited health capacities take into account the peculiar features of the leaning content and educational tools application; eLearning interface design in different forms for general public users with disabilities (Braille types, sound reproduction, enlarge text, etc), extra module introduction on psychology and pedagogical follow-up. The system includes the educational modules on Math subjects "Operations Research" and "Methods of Optimisation" for hearing and visually impaired students.

Profession-oriented factors are directed to the conflict resolution between abstract and isolated math knowledge gained at higher educational institutions and their application in the future occupation. Professional direction includes math competence formation and development, scientific outlook formation, intellectual development, etc. Profession-oriented factors consideration in the adaptive system for mathematical training of students assumed a range of actions:

1) revealing of math training peculiar features of a future specialist with professional activity consideration specificity;
2) analysing professional competences of a graduate student of the corresponding major/training program, and their interrelation with the subjects directed to the student's math competence formation;
3) determining the math content volume that is necessary and enough to solve tasks of professional activity;
4) designing a complex of examples and learning materials related to solving the tasks from future professional activity;
5) selecting and providing the learning material to activate the use of math knowledge by students in solving the applied and profession-oriented tasks.

Profession-oriented direction is defined on the basis of student survey about the major and training program, and then the analysis and assessment of the competence passport and matrix both in general and on the definite subjects.

The gained data on student's individual criteria (psychological and pedagogical, physiological and profession-oriented) after diagnostics are filled in the database for the further arranging of a student model.

3.2 Designing a student model based on student's individual characteristics and preferences

The model (profile, image) of the student includes a set of facts, characteristics and personality traits based on psychological, pedagogical, physiological, and profession-oriented factors, as well as the data on all students' learning outcomes. Moreover, the following data on student's learning outcomes were used for the model design: date and time of access to course elements; number of hits for resources of each kind (theoretical materials, practical assignments, tests) and their duration; number of messages on course the forums and chat rooms; total number of access procedures to the course for certain calendar periods; student's evaluations (interim, final); timeliness of the learning schedule (number of days elapsed after deadlines for an assignment). This variable list represents information about students and their activity on each mathematical course.

The set of data and facts of a student model are necessary for designing an effective mechanism for adaptation and implementation of the supervised training algorithm. At the same time, it is important to emphasize that quality training supervision is possible if there is full information on actors and nature of the educational process (both at the beginning stage and during its implementation). During student model design in the adaptive system, the following functional principles were taken into account:

principle of validity (taking into account students' individual characteristics which are substantial for achieving the learning objectives); principle of adequacy(ensuring consistency of a model with a student based on many and varied parameters); principle of dynamicity (a model is constantly specified and updated as far as data about a student is added and specified).

3.3 Differentiation and selection of educational technologies, learning tools and methods depending on student's individual characteristics and preferences

The methodological content of the instructional design of the adaptive system is focused on the development of the model of students' personality development at the higher educational institution, which allows them to easily adapt and become independent in the modern information society and future professional life. In order to differentiate ways of presenting the content of mathematical subjects in the adaptive system, we used methods of ontological modeling, which allowed forming the unified glossaries for the subject field, creating concept hierarchy, classifying training elements to optimize their searching when designing individual training paths / routes.

In accordance with semantic and taxonomic properties of ontological modeling, we provide the content structure of learning / knowledge model of the adaptive system for mathematical training of students of higher educational institutions. (Table 1, Figure 1).

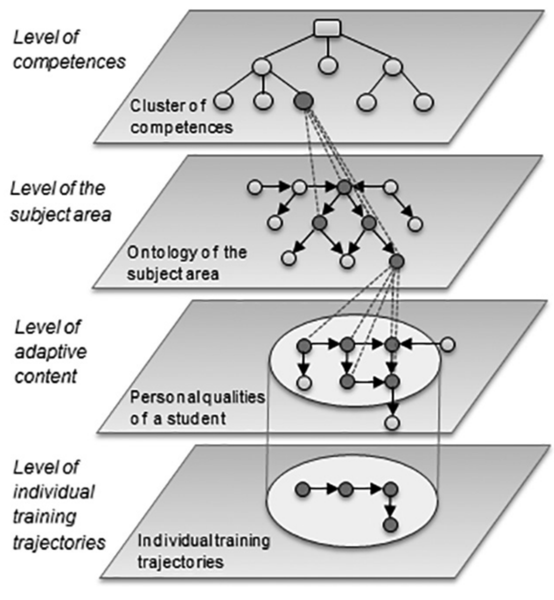

Fig. 1. Level presentation of the learning content in the adaptive system for mathematical training of students

Table 1. Results and list of actions at different levels

Level of	Result	Actions
Level of competences	Cluster of competences	– identifying and determining the list of students' competences that need to be formed within the system of mathematical training; – building a cluster of students' competencies formed in the process of mathematical training in the system / subject / module;
Level of the subject area	Ontology of the subject area	– formalizing knowledge of the subject area of each academic subject (name; term of study; number of academic hours; goals and objectives; formed competencies, etc.); – decomposition of the learning material into sections / topics / training elements; – matching each training element with the competence to be formed;
Level of adaptive content	Models of adaptive content Training elements designed taking into account students' personal characteristics	– determining a set of criteria to adapt learning material to students' personal style characteristics, individual qualities and abilities; – designing and developing each training element in different variations: 1) *type of presenting the learning materials*: text description, graphic representation (illustrations, diagrams, models), video (video lectures, video practicum, online seminar), audio (audio lectures, audio dictionary, audio reference); 2) *difficulty level*: elementary, intermediate, advanced; 3) *volume of the learning material*: in brief, detailed study; 4) *strategy of presenting the learning material*: serial presentation of small fragments, complete presentation of the training element; 5) *forms of organizing the educational activity*: theoretical learning, laboratory and practical work, preparation for a test, an exam, research practice, comprehensive study of the course; 6) *learning pace*: slow, normal, accelerated; 7) *instructional techniques*: providing methodological recommendations and instructions, creating case studies (including simulations, games, etc), organizing group/ individual work.
Level of individual training trajectories	Individual training trajectories / paths	– determining a set of students' individual learning trajectories / paths based on their personal qualities, abilities and preferences; – designing individual learning trajectories / paths from the relevant training elements and rules

The level structure of the content of mathematical training allowed structuring the subjects in accordance with the educational standards of different majors/training programs, dividing them into training elements taking into account students' style, individual qualities and abilities, preparing the structure of the database and knowledge base to form the individual learning paths.

To transfer students to the adaptive learning system, the learning material was initially processed depending on the above criteria: each training element was designed in text (text, hypertext), graphic (in the form of diagrams and models), audio, visual (graphics, illustrations, videos) variants; they were divided into fragments of the required volume and the difficulty level; then they were grouped to organize various forms of educational activities, etc. This process was laborious, but it showed its effectiveness in improving the quality of mathematical training of university students.

3.4 Formation and providing an individual learning path adapted to a particular student

The parameters of the pedagogical scenario required the selection of training elements and the formation of the adaptive learning material and its digestion technology (learning pace, techniques, etc.) for each student. After determining the prevailing learning style and building a suitable learning scenario based on it, the students are offered an optimal presentation of the training course, formed taking into account their individual features and style characteristics.

Taking into account of psychological and pedagogical factors of adaptation. For example, the diagnostics and detailed elaboration of the individual data of the student I.I. Ivanov showed that he belongs to the Abstract Sequential type according to the Gregorc's learning style model. This type characterizes students with an analytical mindset, logical, objective, careful, with systematized and structured knowledge. According to the pedagogical scenario for I.I. Ivanov, *the learning material* will be offered *in the form* of a text description with a large predominance of graphics, video and audio, with a clear structure and logical sequence. At the same time, *the volume of the learning material* presupposes its detailed presentation, the presence of a list of additional material and a set of useful links for a deeper self-studying of the subject, since the representatives of the Abstract Sequential type thoroughly analyze the problem focusing on the logic. At the same time, *the strategy of presenting the learning material* assumes a complete presentation of the training element, not a consistent presentation of small fragments, because students of this type prefer and remember large volumes of text well. In this regard, the adaptive system for I.I. Ivanov will have a normal or slow *learning pace*. The requirements for *the difficulty level of the learning material* depend on the results of the assessment of the initial, continuous and midterm control and may relate to the initial, medium and high. The preferred *form of organizing the educational activities* in the adaptive system for this student will be a comprehensive training, including theoretical study, laboratory and practical work, research, preparation for the test and exam. But at the same time, the educational process will necessarily be accompanied by a system of methodological recommendations, instructions, and aids. When choosing study assignments or projects, he will be offered individual work.

Personal experience in conducting classes in mathematical subjects has shown that students of the Abstract Sequential type think conceptually, detail information, and thoroughly analyze it before making a decision. When preparing projects, they study a large amount of information, read a lot and memorize voluminous material, which influences the success of passing tests and exams. However, in the process of working together in a group with other students, they experience difficulties. They are critical of students of the Abstract Random type, they are inattentive in relation to the needs of other students. They also have difficulties with a creative approach when implementing project activities.

The implementation of individual learning paths for this type of students in the adaptive system made it possible to increase their level of satisfaction with the educational process (from 69.5 to 93.7%), the level of knowledge digestion (by 22.6%), and work in the eLearning environment allowed them to participate in group projects with optimal distribution and performance of roles (the best results were obtained when they played the role of analyst, expert, designer).

Taking into account the physiological factors of adaptation. The developed adaptive system for mathematical training of students assumes taking into account special features of their physical development that is especially important in conditions of inclusive education, because people with disabilities are provided with personalised educational programs of mathematics depending on special features of nosology. For example, students with visual impairment can get the learning material on the mathematical subjects using Braille (with subsequent output into Braille's display – output device designed to display textual information in the form of six-point symbols of Braille's alphabet). Moreover, there are possibilities to increase the font size, change the colour scheme and mode of high contrast.

Students with hearing disorder can get the learning material in the recorded video-format with the capacity to increase the volume to the maximum. In some video-lectures there are additional clips of the learning material on the sign language with the help of the sign-language interpreter.

In this regard the effective education of the people with the limited health capacities in the conditions of eLearning environment required the following special material and technical resources: audio equipment (acoustic amplifier and loudspeakers), video equipment (multimedia projector, television set) document camera, multimedia system; Braille computer technique, video-magnifiers, electronic magnifiers and etc.

Taking into account profession-oriented factors of adaptation. Sequential structure of presenting the learning material in Mathematics in the adaptive system for mathematical training of students looked the following way: getting acquaintance with general theoretical part of the mathematical subject; studying the methods of solving the tasks on the topic in question; analyzing the situations and setting the tasks of the professional direction; applying the learned methods for profession-oriented tasks according to student's major.

Meanwhile, profession-oriented tasks used in the adaptive system corresponded to the following requirements:

– they included description of the situation which occurs in the student's professional activity depending the major/training program;
– they contained unknown characteristics of some professional object or process which should be examined / calculated / found using means of Mathematics;
– task content and solution was based on the content of the professional subjects, therefore, providing interconnection between mathematical and professional subjects;

– task solution contributed to forming the professional competences as a basis of successful professional activity;
– task solution provided intellectual and professional development of a future specialist personality.

For example, when designing the mathematical model to consolidate the theoretical material, the students were presented the task based on the Verhulst's model description that illustrates the evolution of isolated population with limited growth of its size or modeling the prediction regarding goods selling at the given characteristics of the demand curve, acceleration rate and investments.

3.5 The assessment and the analysis of students' performance

According with step-type educational scheme, the control tasks were carried out by the students at the final point of a definite topic / unit / subject. The educational procedure implemented in the tasks includes not only direct but feedback information and the rules of carrying out the sequential operations. It means each of the educational steps of the task consists of three linked components: information, operation with feedback and control.

To assess the performance in the adaptive system for mathematical training of students there was a program module tracking student's achievements at each stage by assessing the level of competence formation, collecting data and making analysis at the quantitative and qualitative levels. The content of the module includes:

1) defining the list of students' competences formed in the process of mathematical training according with the demands of modern educational standards;
2) designing the cluster of students' competences formed in the process of mathematical training;
3) developing competence passports, including the descriptions of formed competency levels: basic, upper; advanced;
4) drawing-up and presenting the competence matrix which clearly states the list of subjects that help to form the necessary skills;
5) forming the fund of assessment tools for the students' level of preparedness on the defined subjects, at the same time, the system of professional standard tasks in Mathematics should be elaborated to assess the development of the defined competences;
6) carrying out monitoring and assessment tasks (entrance, continuous, midterm, final);
7) collecting and analyzing the learning outcomes and formulating methodological recommendations and instructions.

The completed test gives the detailed result (mark, percent of completion, etc.) and a list of the topics to repeat. The students can view their results for all control tasks at any time and analyze the progress in studying the subjects. To analyze the full picture of the group progress or an individual student by the teacher the eLearning environment provides the grade book.

The adaptive system provides the possibility to export the data from the grade book into various document formats. The progress diagram shows a graphical representation of students' learning outcomes. In this way, the teacher can assess and analyse the data on student's progress and, if necessary, take corrective steps.

4 Discussion

Designing the adaptive system for mathematical training of students that considers their individual features and preferences on the basis of modelling student's stylistic characteristics, taking into account physiological aspects and professional orientation made possible to organize the cognitive activity of each student in accordance with the specified requirements, which was achieved through a clear balance between the goal and the stages of activity, methods and tools, learning content and educational technologies.

To test the effectiveness of the model of the adaptive system for mathematical training of students, we conducted a pedagogical experiment with students enrolled in the bachelor majors Applied Mathematics and Computer Science (EG_1 – 64 students, CG_1 – 66 students) and Psychological and Pedagogic Education (EG_2 – 18 students, CG_2 – 17 students). The quantitative distribution of EG students' results ($EG = EG_1 + EG_2$, 82 students) at the ascertaining stage of the experiment was as follows: low level of mathematical readiness revealed in 39.02% of students, medium – in 45.12% and high – in 15.85%; of CG students' results ($CG = CG_1 + CG_2$, 83 students): low – in 34.94%, medium – in 50.6% and high – in 14.46%.

The forming stage of the experiment involved organizing mathematical training of students in the experimental groups using the developed adaptive system within eLearning conditions of the university. Training was conducted in the subjects of "Methods of Optimization" and Operations Research".

The results of the CG and EG were analyzed on the basis of assessments, tests, case studies. The data we obtained indicated positive dynamics in the level of mathematical preparedness in both groups, but in the experimental group the dynamics is more evident (Figure 2).

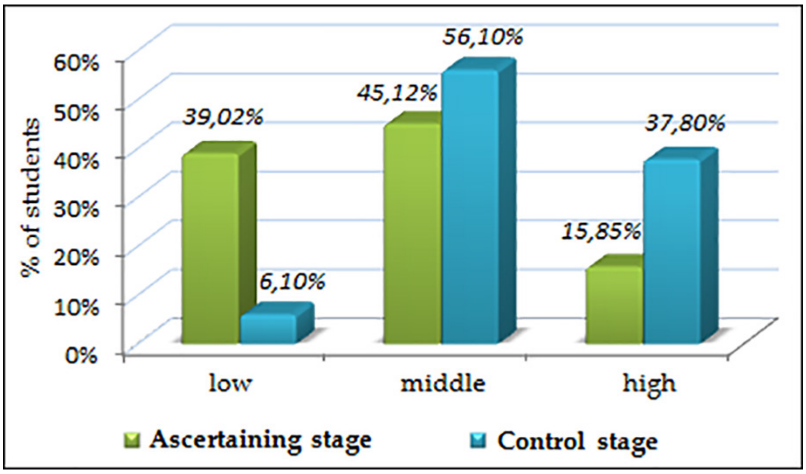

Fig. 2. Dynamics of the mathematical preparedness level of the EG students at the ascertaining and control stages of the experiment

Statistical testing of the hypothesis H_0 that the levels of mathematical preparedness of students in the EG and CG at the ascertaining and control stages are insignificant (homogeneous) was carried out by the χ^2 agreement criterion at the level of statistical significance $\alpha = 0.05$. The result was the value of Pearson's χ^2 statistic equal to 27.63 (when $\nu = 2$ $\chi^2_{\kappa p} = 6$) that allowed rejecting the null statistical hypothesis H_0 and accepting hypothesis H_1 as a plausible one, which shows a significant difference in the results of EG and CG and the increased level of mathematical preparedness of EG students.

All students of the experimental and control groups of the major Applied Mathematics and Informatics and Psychological and Pedagogical Education passed the final control tasks. The comparative analysis allows making a conclusion on the positive dynamics of the level of mathematical preparedness in both groups, but in the experimental group the dynamics are more outspoken, including from a perspective of various mathematical tasks.

Thus, in terms of the tasks for the formation of conceptual mathematical abilities, the ability to present, explain, analyze and interpret the obtained mathematical results: the share of students completed more than 75% of tasks increased from 39.02% to 62.20% (from 40.96 to 44.58% in the control group); the share of students completed from 50 to 75% of tasks decreased from 43.90 to 34.15% (increased from 39.76 to 44.58% in the CG); the share of students completed less than 50% of tasks decreased from 17.07 to 3.66 % (from 19.28 to 10.84% in the CG).

The dynamics are similar in terms of the tasks for the formation of operational and algorithmic mathematical abilities aimed at strengthening of the algorithmic skills: the share of students completed more than 75% of tasks increased from 31.71% to 51.22% (from 27.71 to 36.14% in the control group); the share of students completed from 50 to 75% of tasks increased from 36.59 to 39.02% (from 38.55 to 46.99% in the CG); the share of students completed less than 50% of tasks decreased from 31.71 to 9.76% (from 33.73 to 16.87% in the CG).

In terms of the tasks of higher difficulty level: the share of students completed more than 75% of tasks increased from 6.10% to 37.80% (from 7.23 to 20.48% in the control group); the share of students completed from 50 to 75% of tasks increased from 31.71 to 46.34% (from 27.71 to 45.78% in the CG); the share of students completed less than 50% of tasks decreased from 57.32 to 15.85% (from 57.83 to 28.92% in the CG); the share of students who did not start solving the tasks of higher difficulty level decreased from 4.88 to 0% (from 7.23 to 4.82% in the CG).

In terms of the profession-oriented task solving the dynamics in the EG are more outspoken: the share of students completed more than 90% of profession-oriented tasks increased from 8.54% to 21.95% (from 10.84 to 13.25% in the control group); the share of students completed from 75 to 90% of profession-oriented tasks increased from 18.29 to 20.73% (from 14.46 to 16.87% in the CG); the share of students completed from 50 to 75% of profession-oriented tasks increased from 26.83 to 43.90% (from 28.92 to 31.33% in the CG); the share of students completed less than 50% of profession-oriented tasks decreased from 40.24 to 13.41% (from 42.17 to 36.14% in the CG); the share of students who did not start profession-oriented task solving decreased from 6.10 to 0% (from 3.61 to 2.41% in the CG).

5 Final remarks

In this paper we have presented a model of adaptive system for mathematical training of students within eLearning environment. The practical component is presented by the algorithm of learning in an adaptive electronic environment as a sequence of operations and actions aimed at achieving the set learning outcomes consisting of the following process stages: diagnostics and specification of student's personal characteristics (students' stylistic typologies using the Kolb, Honey&Mumford, Gregorc, VARK, Felder-Silverman, Herrmann learning models; physiological characteristics of the student – fatiguability level, physical conditions), professional orientation (major, training program); designing a student model based on the personal characteristics and preferences of a student; differentiation and selection of educational technologies, means and methods of education depending on student's personal characteristics and preferences; formation and provision to the student of an individual learning path adapted for a particular student; assessment and analysis of student's performance (correction, if necessary). The paper ends with the results of a pedagogical experiment on the use of the adaptive system for mathematical training of students within eLearning environment in the educational process of the university. The reliability and validity of the results obtained are provided by the initial consistent methodological and theoretical positions, the use of a wide range of approaches and methods of research, long-term experimental work, testing and introduction of the results obtained in the educational process of universities. The results of the research make a significant contribution to the development of the theory and methods of education in terms of improving the mathematical training of students of higher educational institutions, creating adaptive e-learning systems and designing the individual learning paths.

The results of the research show that the purposeful implementation of the adaptive system for mathematical training of students within eLearning environment of the university allowed students to raise the level of their mathematical preparedness and competence, motivation for learning mathematics, the ability to use a mathematical approach to advanced problem solving and decision-making on its basis.

Namely these achievements based on knowledge deepening that give a powerful motivational excitement to study the mathematical subjects and bring up to date the processes of self-organization of students' cognitive activity; as a result, the interest in the mathematics mastering increases, theoretical thinking develops (comparison, analogy, analysis, synthesis, etc.), the processes of dominance of the logical scheme of reasoning and the ability to generalize, analyze, synthesize and design are implemented.

This directly encourages the response to the challenges and contradictions in mathematical education that meet the needs of the modern production and technology development, the personal growth of each student, and understanding of the modern scientific world-view of the XXI century within the Fourth Industrial Revolution.

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