Computer Adaptive Practice for a Foundation Mathematics Course

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Abstract—The Computer Adaptive Practice Quiz (CAP Quiz) is a Moodle plugin that uses an Elo rating system to facilitate mastery of mathematical skills. The CAP Quiz presents students with one question at a time and selects each question to give students an estimated 0.75 chance of getting the question correct. By working through questions until their score reaches a desired threshold, students achieve mastery of the required mathematical skills while being exposed to questions at an appropriate level. The CAP Quiz can be used with STACK questions to assess mathematical skills and has been especially useful for foundational skills such as fraction arithmetic and factorising. Students entering tertiary foundation mathematics courses often arrive with misconceptions or knowledge gaps in foundational topics. I will outline how we have used the CAP Quiz to set up a co-requisite approach to achieving mastery in some identified core skills. I will also discuss the setup process for CAP Quiz, some initial reflection on the success of the co-requisite model, and some future developments and research questions.

Keywords—adaptive learning, mastery learning, computer adaptive practice, Elo rating, computer enhanced learning, foundations and bridging

1 Introduction

Foundation mathematics courses play an increasingly important role in allowing students to access tertiary STEM studies, regardless of prerequisite gaps or disadvantage in their prior studies. Students entering foundation courses arrive with varied mathematical backgrounds as well as misconceptions and low self-confidence. The challenge of teaching a foundation mathematics course that acts as a prerequisite for further study is to cater to these varied backgrounds while still providing adequate prerequisite content for progression to further courses.

This paper outlines the use Computer Adaptive Practice (CAP) as part of a foundation mathematics course at a New Zealand university. The adaptive functionality of the CAP Quiz and its mastery learning emphasis is used to provide support for students as they fill in required background skills, while still improving student confidence. CAP Quiz is an open-source Moodle plugin that facilitates mastery learning by exposing students to progressively more difficult questions as they improve and master easier content.

Similarly, if students need extra support they receive easier questions. While this implementation is focused on mathematics content, the Moodle plugin would work with any Moodle question type.

In this paper I briefly discuss the overall course design and the role of CAP Quiz within this design on a few essential mathematics skills. I outline the implementation of the CAP Quiz including the various settings available and how these fit into the course design. I then reflect on the implications and limitations of these settings.

1.1 Self-efficacy

The design and use of CAP Quiz is based in self-efficacy and increasing student confidence and motivation. This is achieved by exposing students to repeated practice of questions where they have a fairly high chance of succeeding, then slowly increasing the difficulty of the questions until a predetermined competency threshold is met.

Self-efficacy is "people's judgements of their capabilities to organize and execute courses of action required to attain designated types of performances" [1]. That is, the belief that one can succeed at a specific task. Unsurprisingly, low self-efficacy is a barrier for many foundation mathematics students who arrive at university having little recent experience of success in mathematics, and in some cases having received messaging from past teachers and mentors that they are "not a maths person". However, research has found that not only are self-efficacy and motivation predictors of success in tertiary mathematics, but there is opportunity to improve these greatly during first-year tertiary studies [2].

Experiencing success is known to be one of the main factors that improves self-efficacy as it reinforces the belief that one can succeed at mathematics [3], [4]. This success becomes more manageable if the task is broken down into smaller components, for example 'Can I succeed at this question?' rather than 'Can I succeed at the whole course?' Another aspect of providing opportunities for students to succeed is to set tasks or questions at an appropriate level of difficulty. Providing 'easy wins' for students can make the overall task seem more manageable [5], [6]. Furthermore, success at a larger task such as acknowledging a 'mastery' grade above 70 or 80% provides meaningful feedback and encouragement for students who have many experiences of obtaining 50% or less on such tasks.

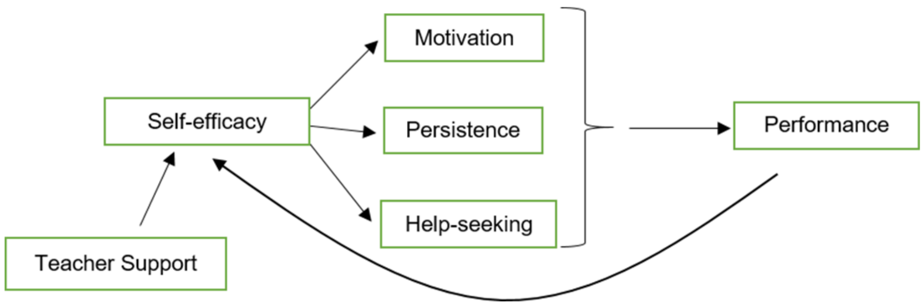

Fig. 1. Summary of self-efficacy model

1.2 Co-requisite model

Many mathematics foundation and bridging students have difficulty with essential algebraic skills. This can manifest as persistent errors throughout course content and prevent a deeper understanding of the material. For example, these errors can include misuse of notation such as negatives or the equals sign, or incorrectly remembered algorithms for fraction arithmetic. By identifying and addressing the underlying algebraic misconceptions, rather than just correcting the individual error, we can equip students to develop these skills and transition to an algebraic way of thinking [7], [8].

The design of a foundations course should identify students' common mistakes and provide support and remedies for the misconceptions underlying these mistakes. Co-requisite models, mastery learning, and repeated practice are all remedies that have been shown to be useful in addressing misconceptions. Co-requisite models for foundational skills have been shown to benefit students [9], [10] and emphasise that these are skills to be improved and mastered over time rather than attempted in an isolated unit. Repeated, targeted practice is also a useful tool against specific misconceptions [11], [12]. In identifying a few crucial skills and misconceptions, we can require a mastery level of understanding on this content, even if mastery thresholds aren't appropriate for all of the course content.

The CAP Quiz is a useful tool for a foundation mathematics course. The nature of the CAP Quiz (outlined below) means that each individual quiz is best suited to the mastery of one particular skill. It can therefore be applied to specific skills to provide self-paced, corequisite support while also building student self-efficacy. This can be useful to build student self-efficacy for skills that may have negative associations for students (e.g. fraction arithmetic) and by setting mastery learning requirements on skills that are fundamental to progressing in the course (e.g. factorisation). The remainder of this paper outlines how the CAP Quiz has been used as part of a foundation mathematics course at a New Zealand university, as well as the implications of various settings within the CAP Quiz setup.

2 What is the CAP Quiz?

Computer Adaptive Practice (CAP) is introduced by [13] and the CAP Quiz Moodle plugin (https://moodle.org/plugins/mod_capquiz) was developed by Schaathun et al. at NTNU in Ålesund to facilitate repeated practice and gamification of mathematics skills. CAP Quiz can be implemented using any Moodle question type, including STACK which is what we rely on in this implementation (https://stack-assessment.org). STACK is a Moodle question type that relies on a computer algebra system where students input algebraic and numeric answers (not just multiple choice), as shown in Figure 2.

While attempting the CAP Quiz, a student is presented with one question at a time which is automatically graded. Based on the outcome (correct/incorrect) the student's score is updated and the student makes progress towards achieving 'stars' until they reach a predetermined mastery threshold, represented by a certain number of stars. In Figure 2, there are three stars available and the threshold requirement is to obtain all three of these stars. The first star has already been obtained, and 6% of the progress required for the second star. Upon clicking 'Next', the student is then shown a new question.

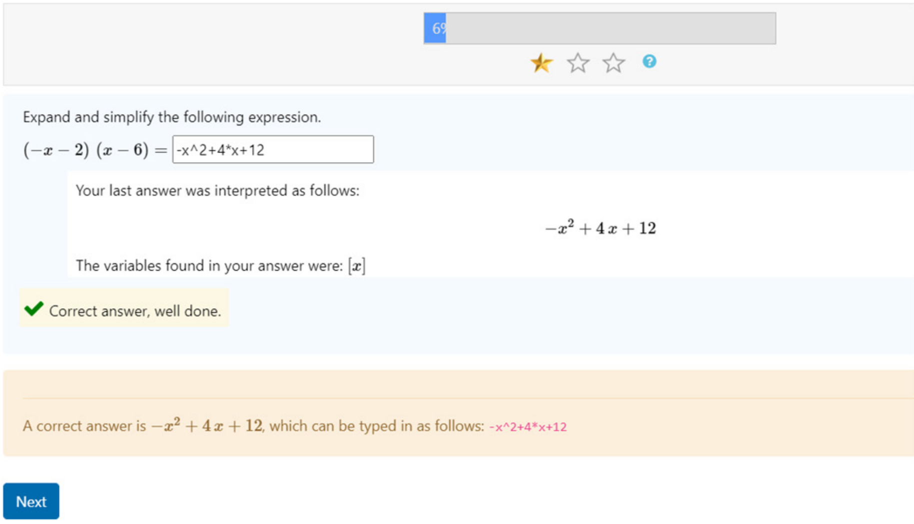

Fig. 2. CAP Quiz example question

The Elo rating system underpins the grading scheme of the CAP Quiz [13], [14], where students compete against a question bank. Elo rating systems originated for chess tournaments, so that opponents can be sensibly ranked against each other. [14] provides an overview of Elo rating systems as applied to adaptive learning:

> *The basic principle of the system is simple: each player is assigned a rating, this rating is updated after each match, the update is proportional to the surprisingness of the match result. If a strong player beats a weak player, the results is not surprising and the update is small whereas if the opposite happens, the update is large.* [14]

The CAP Quiz considers students and questions as 'players' and facilitates a 'tournament' where the student plays a series of matches against questions from a question bank. Each question is chosen so that students have a good probability of 'winning' against the question. If a student gets an easy question correct then they gain only a small increase in their rating (this is not surprising), whereas if they correctly answer a difficult question then they gain a larger increase in their rating. This difficulty rating is determined by comparing the Elo score of the student against that of the question. Moreover, as the quiz selects a question for the student to attempt, it compares the relative rankings of the student and question and selects one where there is a 75% chance of the student 'winning'. This probability can be adjusted by the instructor in quiz setup, but for the purposes of this paper we assume 75% is the desired probability [13], [15], [16].

The rating θ_j of student j is updated according to the outcome $S_j \in \{0,1\}$ of playing against a question:

$$\widehat{\theta}_j = \theta_j + K(S_j - E(S_j)) \tag{1}$$

Where K is a scaling factor determined by the instructor in the quiz settings and $E(S_j)$ is the expected outcome of this match. If K is large then this gives more volatility in the student rating. The question rating is also updated based on student results, but this can be set with a fairly low scaling factor.

The probability of student j winning against question k is given as follows:

$$E(S_j) = \frac{1}{1+10^{(\theta_k-\theta_j)/400}}. \tag{2}$$

Where θ_j and θ_k are the current ratings of the student and question, respectively. This means a quiz with success rating set to X will draw a question of difficulty:

$$\theta_k = \theta_j + 400\log_{10}\left(\frac{1}{X}-1\right). \tag{3}$$

In practice, with $X = 0.75$, the quiz will choose a question with a rating about 191 less than the current student rating. There is some variability in this because the CAP Quiz first chooses the closest N questions to the desired rating, excluding the questions that have most recently been selected, then randomly chooses one of those.

Students enter the quiz with standard rating, e.g. 1200 and questions are added to the question bank with a pre-set difficulty rating. Questions with rating approx. 1010 will be the starting questions that students first see, so should be written accordingly. Questions with a lower score are only attempted by a student if they answer these initial questions incorrectly (or if the quiz is forced to choose them because there are fewer than N questions with rating close to 1010).

Thresholds are set for each star, as well as the total number of stars for the quiz, and the number of stars required for a 'passing grade' on the quiz. In the example shown in Figure 2, the mastery threshold requires students to obtain all three of the available stars.

3 Implementation of the CAP Quiz

Here I describe an implementation of the CAP Quiz in a foundation mathematics course at a New Zealand university. This course acts as a prerequisite for entry into first year calculus courses for students who did not complete this prerequisite content in high school. The course usually has about 700 students per year (spread over two semesters), and includes students with a wide range of previous mathematics experience. Some students only narrowly failed prerequisite content in school whereas other students did not study mathematics for the last few years of their schooling. This course has no assumed prerequisite and has previously had low levels of student engagement and pass rates as low as 40%. The CAP Quiz implementation was part of a larger project to redesign the course for student success, targeting student self-efficacy and engagement.

The CAP Quiz was used in this redesign to set a mastery threshold on three foundational skills: fraction arithmetic, factorisation, and rules of exponents. Supporting content is presented in videos and notes that are made available to students for self-paced study. The overall course redesign implements a large weekly Moodle quiz using STACK questions. This allows students to practice the weekly content, and incorporates interleaving and revision [17], [18]. This weekly quiz runs in parallel to the CAP Quiz content so that students can continue to progress with new content while simultaneously improving the foundational skills listed above.

The nature of the CAP Quiz, where a student plays a tournament against a question bank, means that a quiz is best suited to mastery of one particular skill rather than combining multiple skills in the one quiz. This course therefore contains three separate CAP Quizzes, one for each of the three skills mentioned above. The student then progresses through increasingly difficult questions from the question bank. For example, Table 1 shows a selection of questions from the Factorisation quiz along with an approximate rating.

Table 1. Factorisation CAP Quiz: Sample questions, approximate question and student ratings

Sample Question	Question Rating	Student Rating
Expand $3(x + 2)$	830	1021
Factorise $4x + 8$	900	1091
Factorise $6x + 10$	1050	1241
Fully factorise $3x^2 + 12x$	1100	1291
1 Star	*1109*	*1300*
Factorise $6x + 10y$	1110	1301
Expand and simplify $2x(3x + 5y)$	1163	1354
Expand and simplify $(3 + x)(3 – x)$	1180	1371
Expand and simplify $(x + 4)(x + 5)$	1211	1402
Fully factorise $2(x + 1) + x(x + 1)$	1220	1411
2 Stars	*1259*	*1450*
Factorise $x^2 + 5x + 6$	1277	1468
Expand and simplify $(2x + 3)(2x – 3)$	1278	1469
Factorise $x^2 + 6x +9$	1284	1475
Factorise $x^2 – 9$	1300	1491
Factorise $9x^2 – 25$	1345	1536
Factorise $2x^2 + 20x + 42$	1400	1591
3 Stars	*1409*	*1600*

Students were awarded 5% towards their final grade upon successful completion of the three CAP Quizzes. This required obtaining three stars in each of the three quizzes (9 stars in total) and any student who had less than 9 stars receive 0 of this 5%. Students were encouraged to complete the quizzes within the first three weeks of semester, though the quiz remained open until the end of lectures. Students start with a default rating of 1200, then rating thresholds for each of the stars were set at 1300, 1450 and 1600. Table 1 highlights how this compares to the question ratings given that students are presented with questions approximately 191 points below their current rating (Equation 3). This allows students to quickly obtain the first star, but the third star is only obtained after a significant amount of practice.

The student K-factor was set to 28 and the question K-factor was set to 8. In practice, this meant that a students who got most questions correct completed 100–140 questions before obtaining three stars, though some students were observed to need as few as 70 questions or as many as 200. Student rating increased by 6–8 points for each correct answer and decreased by about 20–30 points for each incorrect answer (see Equation 1). The low K-factor for the questions allows a question rating to be updated in response to student results, but assumes some degree of accuracy in the initial rating set by the instructor when setting up the question bank. However, if many students find a question more difficult than its rating suggests then this rating is gradually updated to reflect this. The question ratings in Table 1 are recorded after cohort of about 700 students have attempted the quiz, so the question ratings have been updated according to student achievement.

The final aspect of the implementation in the overall course design is to add a visual progress bar for the CAP Quizzes. It is presented alongside a similar progress icon for the weekly homework so students have a quick visualisation of their overall progress through the course. For example, the student progress depicted in Figure 3 shows that this student still needs to complete the third CAP Quiz, and that they need to complete or revise the week 4 weekly quiz.

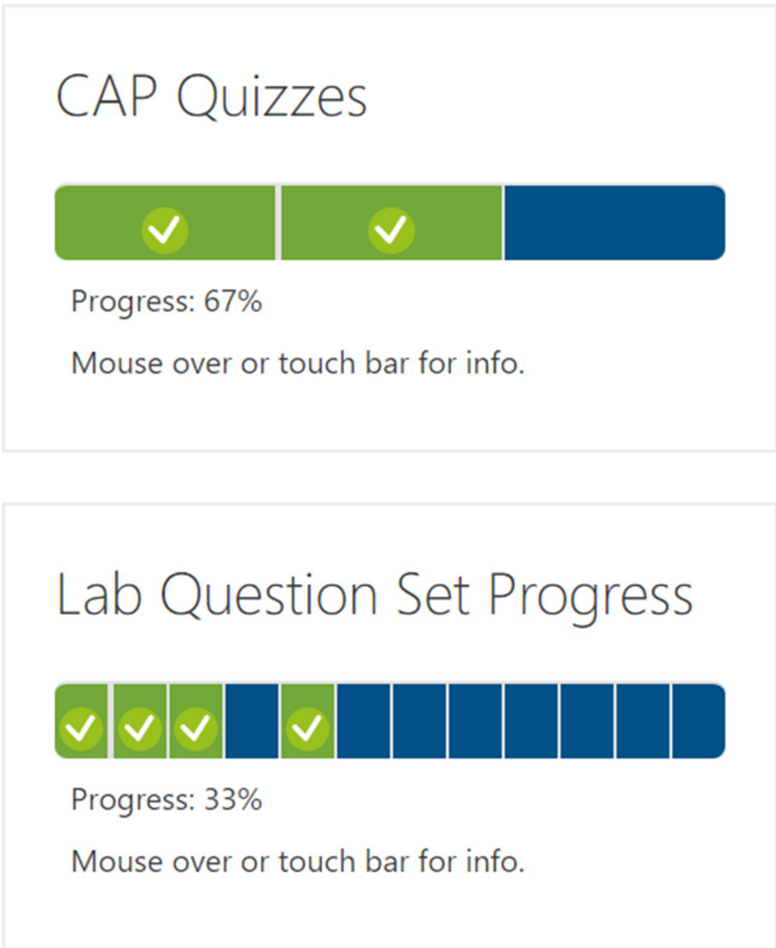

Fig. 3. Progress icons for sample student

4 Observations

Implementing the CAP Quiz for fraction arithmetic, factorisation and exponents has allowed students to practice these skills as much or as little as they need to reach the mastery threshold. It has also meant that this content is taught online so that this lecture time can be reallocated to other topics. Initial feedback from students indicated students with low self-efficacy/confidence seemed to appreciate the extra practice available and the scaffolding of the question difficulty. Some students were frustrated with the feeling of 'going backwards', however this seemed to be exacerbated for overconfident students who felt they made a 'small mistake'. So this is potentially a benefit of using automatically graded questions.

The process of setting up the CAP Quizzes within the overall course design still needs further development and fine-tuning. For example, some students have been observed struggling with fraction arithmetic even after completing the CAP Quiz, so future iterations will increase the mastery threshold and incorporate a wider variety of question types. Fractions in particular seems to be a deeply ingrained misconception for many students in this course, as well as a barrier to developing a conceptual understanding of topics ranging from rational functions, to trigonometry and differentiation. It seems to be a difficult problem to get the balance of requiring enough practice versus having a quiz that takes too long to complete and is demotivating for students. One potential adjustment could be to change the student K-factor to help with this, though the effect of this is more difficult to predict given that students would lose even more marks for a wrong answer. Losing too many marks per question could have an impact on student perception and motivation so this is an adjustment that should be taken with caution.

5 Further research

There are various questions around the ideal setup and uses for the CAP Quiz. In this foundation mathematics course I chose to apply the CAP Quiz to help students with mastery of fractions, factorising and exponents. However, it would be of interest to verify whether these are the optimal skills to be targeting. While they are all necessary skills, the goal is to target the particular skills that are barriers to success in other topics. For example the fractions topic was chosen because it has often been observed not only as an area with many misconceptions but also that a weak understanding of fractions subsequently hinders understanding of concepts such as rational expressions, limits, trigonometric functions and derivatives. However this is based on observation so it would be beneficial to have a more in-depth look at identifying these topics.

Another area of interest would be to further investigate student attitudes towards mastery thresholds, especially as they are only applied to selected skills. Do students see the value in building these skills to a mastery level? If not, how beneficial would it be to students to communicate this value to them, and how is this best communicated? It is likely that students convinced of the value of mastery learning on these topics would be less frustrated by penalties for incorrect answers and less likely to refer to calculators etc. Students would also be increasingly motivated by observing progress and achieving stars because they could see the benefits to their learning. Would this strengthen the connection from success to improved self-efficacy (as shown in Figure 1)?

There is also potential for the question rating data generated by the CAP Quiz to be useful for teacher development. Teacher support plays a large role in improving student self-efficacy, so there is benefit for teachers to better understand student perceptions of questions. For example, surprising question ratings could form the basis of useful reflection for teachers (and teaching assistants). We could find a question that we think should be θ_1 and but that adjusts to difficulty $\theta_2 > \theta_1$ and reflect on what made this question more difficult for students.

6 References

[1] A. Bandura, "Social foundations of thought and action," *Englewood Cliffs NJ*, vol. 1986, no. 23–28, 1986.
[2] S. Bengmark, H. Thunberg, and T. M. Winberg, "Success-factors in transition to university mathematics," *Int. J. Math. Educ. Sci. Technol.*, vol. 48, no. 7, pp. 988–1001, Nov. 2017, https://doi.org/10.1080/0020739X.2017.1310311
[3] E. M. Skaalvik, R. A. Federici, and R. M. Klassen, "Mathematics achievement and self-efficacy: Relations with motivation for mathematics," *Int. J. Educ. Res.*, vol. 72, pp. 129–136, Jan. 2015, https://doi.org/10.1016/j.ijer.2015.06.008
[4] T. Williams and K. Williams, "Self-efficacy and performance in mathematics: Reciprocal determinism in 33 nations," *J. Educ. Psychol.*, vol. 102, no. 2, pp. 453–466, May 2010, https://doi.org/10.1037/a0017271
[5] P. D. Eggen, D. P. Kauchak, and S. Garry, *Educational psychology: Windows on classrooms*. Pearson/Merrill/Prentice Hall, 2007.
[6] D. J. Lynch, "Motivational beliefs and learning strategies as predictors of academic performance in college physics," *Coll. Stud. J.*, vol. 44, no. 4, pp. 920–927, Dec. 2010.
[7] R. Cangelosi, S. Madrid, S. Cooper, J. Olson, and B. Hartter, "The negative sign and exponential expressions: Unveiling students' persistent errors and misconceptions," *J. Math. Behav.*, vol. 32, no. 1, pp. 69–82, Mar. 2013, https://doi.org/10.1016/j.jmathb.2012.10.002
[8] J. Vincent, C. Bardini, R. Pierce, and C. Pearn, "Misuse of the Equals Sign: An Entrenched Practice from Early Primary Years to Tertiary Mathematics," *Aust. Sr. Math. J.*, vol. 29, no. 2, pp. 31–39, 2015.
[9] P. W. Hillock, M. Jennings, A. Roberts, and V. Scharaschkin, "A mathematics support programme for first-year engineering students," *Int. J. Math. Educ. Sci. Technol.*, vol. 44, no. 7, pp. 1030–1044, 2013. https://doi.org/10.1080/0020739X.2013.823251
[10] A. W. Logue, D. Douglas, and M. Watanabe-Rose, "Corequisite Mathematics Remediation: Results Over Time and in Different Contexts," *Educ. Eval. Policy Anal.*, vol. 41, no. 3, pp. 294–315, Sep. 2019, https://doi.org/10.3102/0162373719848777
[11] C. Barton, *How I Wish I'd Taught Maths: Lessons Learned from Research, Conversations with Experts, and 12 Years of Mistakes*. John Catt Educational Limited, 2018. [Online]. Available: https://books.google.co.nz/books?id=aQb5swEACAAJ
[12] J. Hattie and H. Timperley, "The Power of Feedback," *Rev. Educ. Res.*, vol. 77, no. 1, pp. 81–112, Mar. 2007, https://doi.org/10.3102/003465430298487
[13] S. Klinkenberg, M. Straatemeier, and H. L. J. van der Maas, "Computer adaptive practice of Maths ability using a new item response model for on the fly ability and difficulty estimation," *Comput. Educ.*, vol. 57, no. 2, pp. 1813–1824, 2011, https://doi.org/10.1016/j.compedu.2011.02.003
[14] R. Pelánek, "Applications of the Elo rating system in adaptive educational systems," *Comput. Educ.*, vol. 98, pp. 169–179, 2016, https://doi.org/10.1016/j.compedu.2016.03.017

[15] T. J. Eggen and A. J. Verschoor, "Optimal testing with easy or difficult items in computerized adaptive testing," *Appl. Psychol. Meas.*, vol. 30, no. 5, pp. 379–393, 2006. https://doi.org/10.1177/0146621606288890
[16] S. Sampayo-Vargas, C. J. Cope, Z. He, and G. J. Byrne, "The effectiveness of adaptive difficulty adjustments on students' motivation and learning in an educational computer game," *Comput. Educ.*, vol. 69, pp. 452–462, Nov. 2013, https://doi.org/10.1016/j.compedu.2013.07.004
[17] J. M. Lang, *Small teaching: Everyday lessons from the science of learning*. John Wiley & Sons, 2021.
[18] D. Rohrer, R. F. Dedrick, and K. Burgess, "The benefit of interleaved mathematics practice is not limited to superficially similar kinds of problems," *Psychon. Bull. Rev.*, vol. 21, no. 5, pp. 1323–1330, Oct. 2014, https://doi.org/10.3758/s13423-014-0588-3

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