AuthOMath: Combining the Strengths of STACK and GeoGebra for School and Academic Mathematics

https://doi.org/10.3991/ijet.v18i03.36535

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Abstract—GeoGebra has its strength in creating multimodal dynamic and interactive math applets and is widely used in secondary school math teaching. STACK is particularly strong in generating randomized tasks with adaptive feedback and is mostly used in university-level math teaching. The Erasmus+ project AuthOMath (2022–2024) aims to combine the strengths of both systems in an authoring tool with a transformative digitization potential in mathematics teaching and learning.

Keywords—mathematics, interactivity, multimodality, feedback, assessment

1 The transformative potential of digitization in math learning

The potential of digitization in mathematics teaching and learning has been fully explored in educative materials employing technical innovations that are specific to digital media and tools. As a simple example: traditionally, teaching geometric concepts is often accompanied by figures drawn on paper or on a blackboard. For representing a parallelogram, the figure ▱ commonly first comes to mind. Yet the square □ also is a parallelogram which, though a special one, adds to a completer picture of what a parallelogram is. Furthermore, a parallelogram is a mathematical concept determined by a single definition. Nevertheless, a collection of static figures hardly reflects that. Hence, a parallelogram should be understood best as a single geometric object that can be manifested in a variety of figures. This can be achieved when a definition-based, interactive construction within a Dynamic Geometry Software (DGS) is used. Presented to a learner (Figure 1), it allows exploration of the various shapes which the underlying definition allows, thus building a dynamic and manifold mental visualization of "what a parallelogram is".

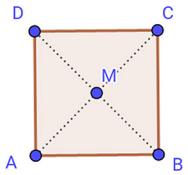

Fig. 1. This figure is not a square. Drag the vertices to see what else it can look like, and explain why it really is a parallelogram

Interactivity and dynamization are two specific aspects of the digital mathematics software enabling a transformative change in teaching and learning mathematics. Multimodality is another aspect, which means representing the same object in many interconnected forms. Yet another aspect is the randomization of verbal, pictorial, algebraic, and other representation forms of mathematics content, including providing answer-based automatic feedback.

Interactivity, dynamization, and multimodality could be considered strengths of DGS such as GeoGebra, while STACK is particularly strong in randomization and adaptive feedback design. Bringing both together for supporting the transformative change in mathematics teaching and learning is one of the aims of the Erasmus+ Project AuthOMath (2022–2024). This project also seeks to offer necessary didactical foundations for authors to be able to design effective digital mathematical tasks. Doing so, the project aims to use the specific affordances of both the STACK and GeoGebra authoring systems to initiate a genuine didactic reflection, being one of the key competencies to develop in teacher education.

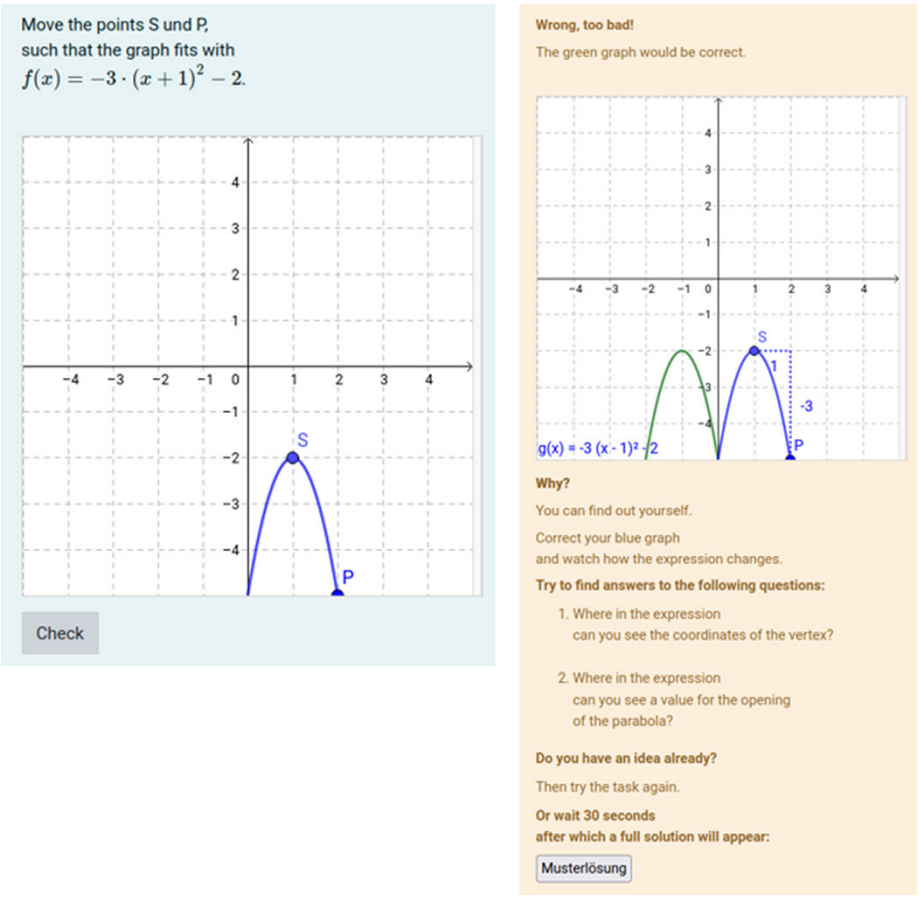

Fig. 2. STACK question with a randomized interactive GeoGebra applet (left) and feedback to a wrong answer (right) with another interactive GeoGebra applet

2 AuthOMath and its objectives

AuthOMath – Authoring Online Material with Multimodal, Dynamic and Interactive Applets and Automated Feedback for Learning Math – is an Erasmus+ project coordinated at the Pädagogische Hochschule Heidelberg (Germany), and supported by three partners: Universidad de Cantabria (Spain), Johannes Kepler Universität Linz (Austria), and University of Edinburgh (UK). The project's three main objectives are:

2.1 Extending STACK for authoring questions with GeoGebra applets

The project will provide a Moodle-based authoring tool for randomized interactive and dynamic multimodal mathematical tasks with automatic adaptive feedback. For this, STACK will be extended to enable GeoGebra applets in math questions which interact with STACK variables. This means that applets can be randomized like generic STACK elements and that a users' interactions with an applet are transferred to STACK for evaluation. In Figure 2, both the task area and the feedback contain a GeoGebra applet. The second also is interactive to initiate learning activities and foster the users' knowledge and understanding. The applets were created with external programming tools which provided the necessary Javascript code for their implementations [1]. The project's aim here is to reduce coding to a minimum.

2.2 Developing didactic fundamentals for creating interactive, multimodal math tasks with adaptive feedback

The project also will provide guidelines for designing and using effective digital learning material in mathematics education, based in part on these interactive tools. These guidelines will firstly address the necessary didactic fundamentals for creating material. Information about coding STACK questions is of secondary concern.

With this objective, AuthOMath initiates necessary didactical reflection processes that emerge from turning digital innovations into useful learning tools. For example, the feedback feature of STACK needs a differentiated analysis of the mathematical concepts and procedures addressed in a problem, and of the variety of solving strategies that learners could follow. Diagnostic competencies will be also required to identify different correct solutions as well as to anticipate possible user mistakes, for example, careless actions or systemic misconceptions of the objects involved [3]. For each answer, suitable feedback needs to be provided, which draws from the many possible forms of feedback [2] to address the necessary basis for understanding [4].

These guidelines are intended for use in pre-service and in-service teacher education. For designing digital tasks with adaptive feedback, teacher students could develop their diagnostic competencies by doing research by first reflecting on the mathematical and didactical fundamentals of each task, followed by small empirical evaluations of their tasks with pupils. The in-service teachers could contribute to differentiated feedback on existing tasks by drawing from their extensive experiences from classroom teaching, thus improving materials and thus meeting requirements of their classroom teaching.

2.3 Providing an online platform for public access

Eventually, the project results will be accessible via a Moodle-based Open Educational Resources (OER) learning platform containing both the didactical concept for the design of multimodal interactive mathematics tasks and adaptive feedback and a STACK plugin for creating STACK questions with GeoGebra applets. Results will form a complete educational package that can be used in all phases of teacher education and in-service teaching.

3 Acknowledgements

The project is co-funded by the European Union under the Erasmus+ Programme, No. 2021-1-DE01-KA220-HED-000032031. This publication reflects only the authors' view. The Commission is not responsible for any use that may be made of this publication.

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Article submitted 2022-10-11. Resubmitted 2022-11-19. Final acceptance 2022-11-21. Final version published as submitted by the authors.