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iJET International Journal of Emerging Technologies in Learning

iJET | eISSN: 1863-0383 | Vol. 18 No. 14 (2023) | OPEN ACCESS

https://doi.org/10.3991/ijet.v18i14.40341

PAPER

Elements of Algorithmic Thinking in the Teaching of School Geometry through the Application of Geometric Problems

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ABSTRACT

Algorithmic thinking and the creation of algorithms have traditionally been associated with mathematics. It is based on the general perception of an algorithm as a logically unambiguous and precise prescription for performing a certain set of operations, through which we reach a result in real time in a finite number of steps. There are well-known examples from history, such as the division algorithm used by ancient Babylonian mathematicians, Eratosthenes algorithm for finding prime numbers, Euclid's algorithm for finding the greatest common divisor of two numbers, and cryptographic algorithm for coding and breaking, invented by Arabic mathematicians in the 9th century. Although the usage of algorithms and the development of algorithmic thinking currently fall within the domain of computer science, algorithms still play a role in mathematics and its teaching today. Contemporary mathematics, and especially its teaching in schools of all grades, prefers specific algorithms in arithmetic, algebra, and calculus. For example, operations with numbers, modifications of algebraic expressions, and derivation of functions. Teaching geometry in schools involves solving a variety of problems, many of which are presented as word problems. Algorithmization of school geometric tasks is therefore hardly visible and possible at first glance. However, there are ways to solve examples of a certain kind and to establish a characteristic and common algorithmic procedure for them. Algorithmic thinking in geometry and the application of algorithms in the teaching of thematic parts of school geometry are specific issue that we deal with in this study. We will focus on a detailed analysis of the possibilities of developing algorithmic thinking in school geometry and the algorithmization of geometric tasks.

KEYWORDS

algorithm, mathematics, approaches, historical view, calculate, construction tasks

1 INTRODUCTION

By developing algorithmic thinking, we contribute significantly to several key competences, particularly, problem solving, computer and mathematical literacy, etc.

Vallo, D., Rumanová, L., Bočková, V. (2023). Elements of Algorithmic Thinking in the Teaching of School Geometry through the Application of Geometric Problems. *International Journal of Emerging Technologies in Learning (iJET)*, 18(14), pp. 229–243. https://doi.org/10.3991/ijet.v18i14.40341

Article submitted 2023-04-04. Resubmitted 2023-05-07. Final acceptance 2023-05-09. Final version published as submitted by the authors.

Algorithmic thinking is an individual's ability to construct new algorithms with the goal of solving a given problem [1]. Within the teaching process, students can analyze problem, design a solution algorithm to solve the problem, write down the algorithm in an understandable formal form and verify their correctness.

Algorithmic thinking should not be confined solely to the realm of computer science, as it finds applications in everyday life as well [2]. In mathematics, algorithmic thinking is developed through constructional tasks, while other subjects employs various sets of instructions for carrying out work. It is advisable to devote space to algorithmization itself using visualizations and experiential methods. This approach is also acceptable for students who may have weaker abstract thinking skills or a less developed mathematical background.

In the Encyclopedia of Education and Information Technologies [3], the authors emphasize the importance of introducing algorithmic thinking at an early stage, and guiding students towards achieving mastery in the subject [4].

Algorithmic thinking is a concept that has just entered our lives, yet it has always existed throughout human history. In order to understand algorithmic thinking, one has to examine its roots in the past [5]. Therefore, our paper will delve into historical tasks related to algorithmic thinking. We also emphasize the use of algorithmic thinking in mathematics education, where attention is based on thinking and performing the necessary actions in accordance with a clearly defined objectives.

2 THE BEGINNINGS OF ALGORITHMIC THINKING IN GEOMETRY: A SHORT HISTORICAL INTRODUCTION

Geometry, since its inception, has been associated with experimentation and the necessity for practical application. The quest for precision within this scientific discipline began to take shape in the 6th century BC.

Euclid, in his pioneering work titled Elements [6,7] (Gr. Stoicheia), synthesized and expanded upon the existing knowledge of geometry. This book became a commonly used geometry textbook for the next two thousand millennia. During its creation, the arithmetic-algebraic interpretation of mathematical entities was not yet well developed, and thus, the geometric method naturally prevailed within the book.

Persian mathematician, astronomer, and geographer Mohammad ibn Musa al-Khwarizmi (780–850), Latinized as Algoritmi, introduced the Indian numerals in the decimal number system to the Western world in his work The Compendious Book on Calculation by Completion and Balancing (Latin translation from the 12th Century). He also resented the first systematic solution of linear and quadratic equations in Arabic. Actually, the word algebra is derived from the word al-jabr, one operation he used to solve quadratic equations. Al-Khwarizmi's contribution to mathematics was famous. The term algorithm stems from Algoritmi, the Latin form of his name [8].

2.1 Pythagorean triples

Thales of Miletus (about 624 BC–546 BC), as along with Pythagoras of Samos (about 580 BC–496 BC) and his students, were among the earliest scholars who formulated and established geometric knowledge through logical deduction, drawing upon planimetry and stereometry. Pythagorean arithmetic modeled some of its results using figural numbers. For example, according to [9], it is quite likely that the Pythagoreans invented the construction of an infinite the technique of shape psephophoria (the arrangement of pebbles into specific shapes), which allowed for the construction of an infinite number of Pythagorean triples. The algorithmic procedure, as depicted in Figure 1, is as follows:

1. Pick up pebbles whose number represents an odd square number, e.g. $9 = 3^2$;
2. Store the pebbles in the gnomon (in the shape of the letter L);
3. Complete the gnomon with a small 4×4 square (per square);
4. The gnomon and the small square give a new large square (with dimensions of 5×5).

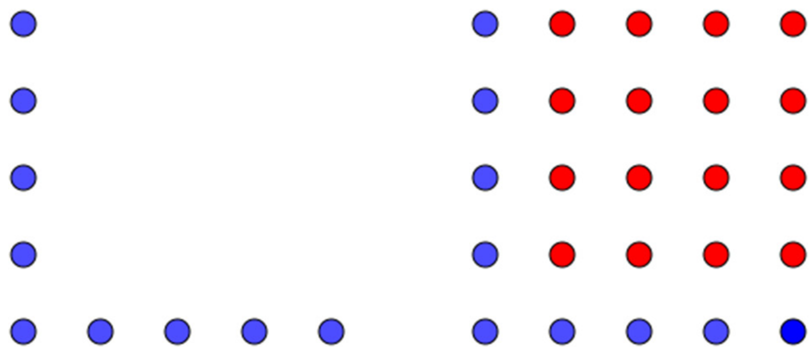

Fig. 1. Pythagoreans' method of shape psephophoria

Geometric substitution of numbers by subject models and the ability to solve arithmetic laid the foundations of algebra with a geometric model [10]. In modern algebra notation, we would state that the gnomon as (1), the small square as (2), the large square as (3) and holds (4):

$$u^2 = 2m + 1 \tag{1}$$

$$m^2 \tag{2}$$

$$(m + 1)^2 \tag{3}$$

$$2m + 1 + m^2 = (m + 1)^2 \tag{4}$$

This gives us the primitive Pythagorean triple (u, m, $m + 1$), in our illustrative case (3,4,5).

For several centuries, the tradition of geometric-graphic solutions continued without a perceived necessity for algorithmization, although the Pythagorean arithmetical philosophical-mystical worldview, based on natural numbers, would have required it.

2.2 Euclidean algorithm

Euclid (about 365 BC–300 BC) also contributed to the development of algorithmization in mathematics. He introduced algorithm for computing greatest common divisor. It is based on the fact that, given two positive integers a and b such that $a > b$, the common divisors of a and b are the same as the common divisors of $a - b$ and b [11].

The phenomenon of divisibility is present in many phenomena from the real world, in rhythm, in periodic events, as well as in geometric patterns. Euclid's algorithm can therefore, also be implemented geometrically and could be formulated as follows:

1. Determine the shorter side of the rectangle;
2. Divide the given rectangle into unit squares;
3. Tessellate the rectangle with successively smaller squares until you cover the entire shorter side of the rectangle;
4. Discard options that extend beyond the original rectangle;
5. The side of the largest square that satisfies full coverage without overlap is the greatest common divisor.

In Figure 2, a basic rectangle with dimensions 8×12 is depicted. This rectangle can be divided into ninety-six unit squares, twenty-four squares with dimensions 2×2, and six squares with dimensions 4×4. However, it is observed that the rectangle cannot be evenly divided into squares with dimensions of 3×3, and there are no other possibilities. The greatest common divisor of numbers eight and 12 is four.

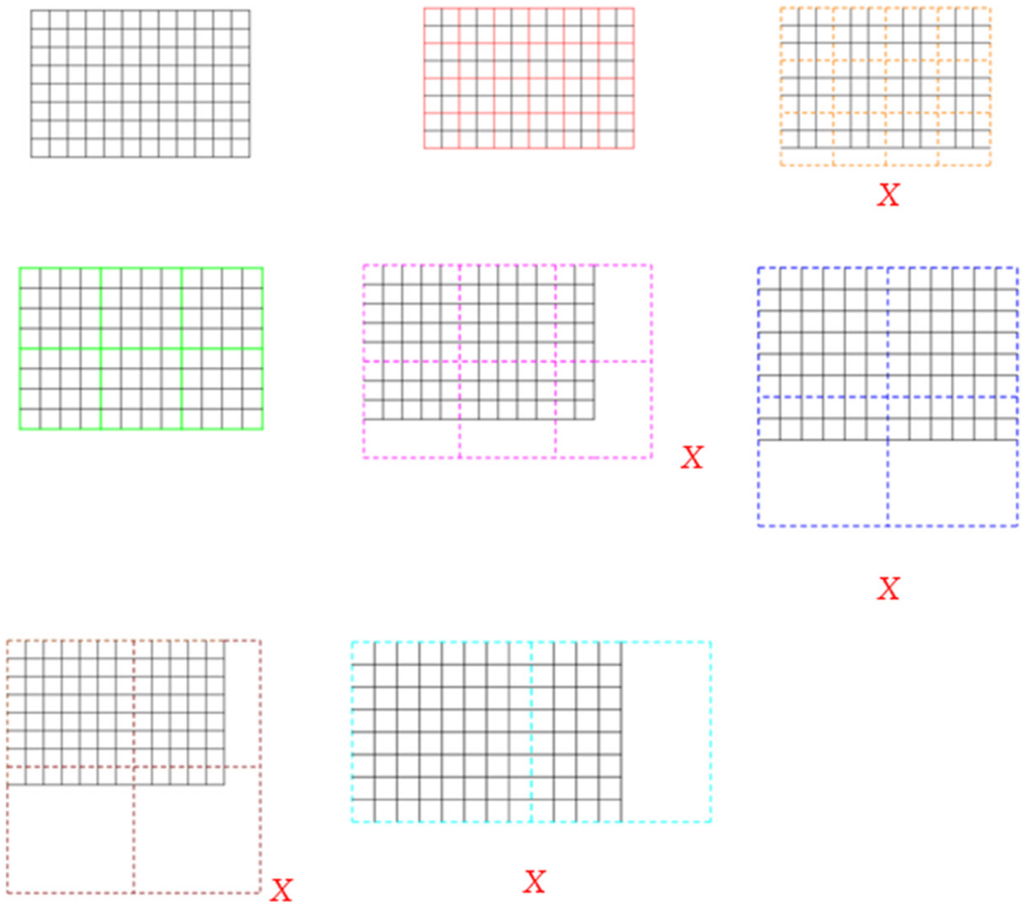

Fig. 2. Geometric interpretation of the Euclidean algorithm

2.3 Quadratic equation

By the 3rd century AD, with the influence of the works of Diophantus of Alexandria (around 250 AD), geometry and mathematics were partially algebraized [12]. However, the geometric language created by the Pythagoreans in the 6th century BC, with its rhetorical form, lasted until the Middle Ages. An example is the solution of a quadratic equation (5) provided in the textbook of the Persian mathematician al-Kwarizmi (in about 780–850), whose name gave rise to the term algorithm [13].

$$x^2 + 10x = 39 \tag{5}$$

"If someone says: the square and ten of its roots equals thirty-nine dirhams, so that means that if you add to any square what is equal to ten roots, you get thirty-nine. The rule is as follows: Halve (the number of) roots, you get five in this problem, multiply it by the same number, it will be twenty-five. Add that to thirty-nine, it's sixty-four. Take the root of that, it will be eight. Subtract half (the number) of the roots from that, i.e. five, three to go. That's the square root you were looking for, and the square will be nine."

At the time of the creation of the aforementioned algorithm, the arithmetic-algebraic interpretation of mathematical entities had not yet fully and well developed. At that time, the rhetorical form naturally dominated, which was the only guarantor of the creation and comprehensible interpretation of the mathematics at that time [11].

Today we can solve equations symbolically. As the author gradually describes the solution procedure, we are also follow the geometric essence of the corresponding algorithm.

First, a unit with content (6) is constructed and geometrically completed into a square. This gives the equation (7), which has solutions (8) and (9). The root of the equation (9) however, cannot be found geometrically.

$$x^2 + 10x \tag{6}$$

$$(x+5)^2 = 64 = 8^2 \tag{7}$$

$$x_1 = 3 \tag{8}$$

$$x_2 = -13 \tag{9}$$

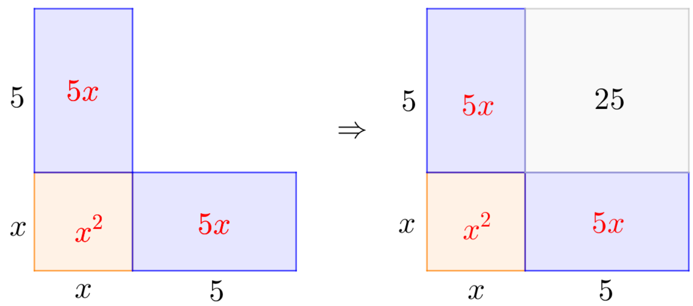

Fig. 3. Geometric interpretation of the Al-Khorezmi algorithm for the solution of a quadratic equation – part 1

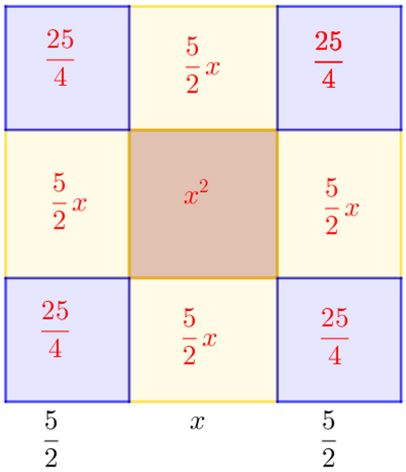

Fig. 4. Geometric interpretation of the Al-Khorezmi algorithm for the solution of a quadratic equation – part 2

According to [14] the broader cultural characteristics of a society always mirrored in its mathematical thinking. It points to the fact that the Greek approach (shown in Figure 3) is clear, simple, and functional, while the Arabic interpretation of solving the same problem (as depicted in Figure 4) exudes a sense of rhythm and symmetry.

3 ALGORITHMIC THINKING IN SCHOOL GEOMETRY

The content of school geometry is very broad; it goes through many thematic units. Teaching oriented only on theory in the form of acquiring definitions and geometric theorems is insufficient, and it is necessary to solve many different geometric tasks.

In the literature, it is specified in more detail that these are determination tasks with geometric content. They are construction tasks in which the construction is perceived as having a sufficiently general meaning, (e.g., calculating the roots of an algebraic equation, investigating sets of points of a given property, searching for optimal algorithms, etc.) [15].

Among the basic types of determination tasks with geometric content, the following are included:

- Computational tasks
- Proof tasks
- Construction tasks

We note that proof tasks in school geometry are a special type of geometric task. We believe that they cannot be solved algorithmically because their solutions require a creative approach to the problem of mathematical content and methods, in which heuristic strategies and approaches are mainly used [16,17].

3.1 Algorithmic thinking in solutions of tasks to calculate

Computational tasks might require priority inclusion in arithmetic and algebra, but they are applied to geometric material. Algorithms learned from arithmetic and algebra are used for various calculations: perimeters and contents of plane figures, surfaces and volumes of bodies, calculations of distances between two points, or the size of angles. As a rule, geometry provides the formulas and geometric knowledge necessary for solving.

Overall, the calculation tasks are an integrated approach to teaching arithmetic, algebraic, and geometric content. The problem is solved collectively when we consider the given and searched elements as variables. We gradually modify and derive the necessary relationships and formulas. The searched object is one or more sessions. The solution to a specific task is then the result after substituting the values for the independent variables and calculating the corresponding values for the dependent variable.

In earlier literature [18], this approach was referred to as "solution according to formulas." Today we use the more fitting term "solution according to the algorithm" or "solving the problem by the algebraic geometric method." To illustrate this, let's consider an example. [19,20].

Golden section. Cut the given segment AB into two pieces of different lengths in such a way that the ratio of the whole segment to that of the longer segment is equal to the ratio of the longer segment to that of the shorter segment.

If we letting the length of the segment as $\boldsymbol{a}$ and the shorter segment as x, than holds true (10)

$$\frac{a}{a-x} = \frac{a-x}{x} \tag{10}$$

Evidently, $x \neq 0$ and $a - x \neq 0$. We derive (11,12)

$$ax = (a-x)^2 \tag{11}$$

$$x^2 - 3ax + a^2 = 0 \tag{12}$$

The roots of quadratic equation are in the form (13)

$$x_{1,2} = \frac{3a \pm \sqrt{9a^2 - 4a^2}}{2} = \frac{3 \pm \sqrt{5}}{2}a \tag{13}$$

It holds true that (14).

$$x_1 = \frac{3+\sqrt{5}}{2}a > a \tag{14}$$

It implies that the satisfying root is (15) and we obtain that (16).

$$x_2 = \frac{3-\sqrt{5}}{2}a \tag{15}$$

$$a - x_2 = a - \frac{3-\sqrt{5}}{2}a = a\frac{1+\sqrt{5}}{2} \tag{16}$$

The constructions of the segment of the length (15) is provided in Figure 5 by using Pythagorean Theorem and the constructions of the circle arcs. Some intriguing construction is in [21].

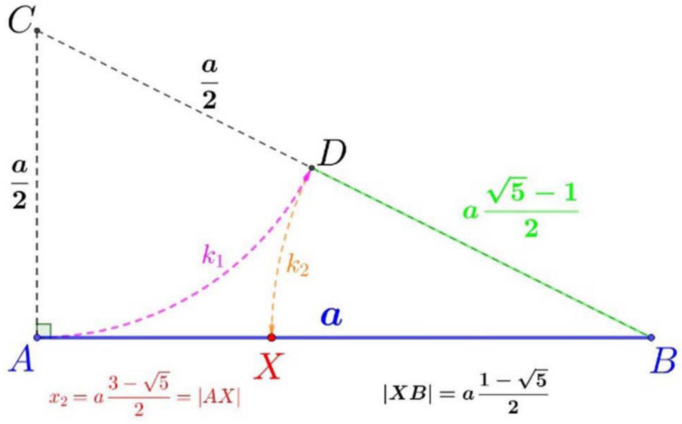

Fig. 5. Construction of the golden section based on the Pythagorean Theorem

In the book, author [18] further states that in secondary school, we should equally use algorithmic problem solving, in arithmetic, algebra, geometry, and trigonometry. This means that a system of algorithms should be developed for all areas of school mathematics, especially for structural geometry, and regulations should be formulated for solving certain basic simple tasks, which students would use as finished results just as they use, say for example, formulas in algebra and trigonometry. It is not appropriate to repeat these basic constructions every time.

However, the determination and selection of the aforementioned algorithms are subject to debate. The choice will largely depend on the curriculum and educational standards, which will heavily influence the decision-making process.

3.2 Algorithmic thinking in construction task

Construction tasks are typical for teaching geometry. These are tasks in which illustrative geometric shapes are constructed. Students draw them independently, while the construction is preceded by a logical consideration of the method of solution (analysis) and the careful drawing up of a construction plan, which outlines the construction procedure step by step.

Solving a construction task for a student means constructing a geometric figure according to valid sentences and customary rules. The student is required to logically deduce and supplement the given elements with others so that the desired formation can be constructed. The solution method itself must also be written down.

However, the method of solving the construction task shows elements of algorithmic thinking. We distinguish between an external algorithmic approach to the solution and an algorithm for solving the task itself.

The idea of an external algorithmic approach to solving a task follows a structured format and represents a kind of scheme that can appear formal at first glance. Through practice and years of proven methodologies for solving construction tasks, it is recommended that the solution of the task be divided into three phases. The solution phases namely analysis, construction, proof, and discussion [22,23] are integral stages in the solution process. These stages can be partially characterized in the form of instructions, as in the case of an algorithm. This can be illustrated in the form of instructions, as shown in Table 1.

Table 1. Instruction of solving the construction task as in the case of an algorithm

Stages of Construction Task	Algorithm
Analysis	Assume there is a solution. Sketch a picture with the desired figure. Mark the given data and the geometric figures to be constructed. Look for dependencies between the given and searched elements.
Construction	Carry out the construction itself. Write the procedure for constructing the desired geometric figure.
Proof	Find out whether the constructed unit meets the requirements of the assignment.
Discussion	If some elements are parameters, then specify the solvability conditions. Find out the number of satisfactory solutions.

It should be stressed that the mentioned approach is not an algorithm. This approach lacks some of the basic properties of an algorithm, such as element ability, determinability, and resultivity [24]. Instead, it can be classified as a heuristic approach [17].

Nevertheless, this approach provides insight into the multitude of varied geometric problems concerning plane figures. It allows to the categorization of these problems according to invariants and to focus on the choice of an appropriate solution method (method of geometric loci of points, application of planar transformations, algebraic method or analytical method by using a coordinate system).

However, behind the solution of each construction task lies a hidden direct algorithm for its solution. This is the construction phase, which includes the procedure of execution of individual, partial construction steps.

The algorithm for solving the problem according to Figure 5 [also transcribed with geometric symbolic language] is as follows:

1. Given the segment $a = AB$, $[AB, |AB| = a]$;
2. Through the point A construct the perpendicular segment AC of the length $\frac{a}{2}$, $[AC,\ AC \perp AB,\ |AC| = \frac{a}{2}]$;
3. Construct the arc k_1 with center C and radius $\frac{a}{2}$, $[k_1,\ k_1\left(C, \frac{a}{2}\right)]$;
4. The point of intersection of the arc with hypotenuse AC label as D, $[D, D \in AC \cap K_1]$;
5. Construct the arc with center B and radius BD, $[k_2, k_2(B, |BD|)]$;
6. Determine the point of intersection X of the last constructed arc with the segment AB, $[X, X \in k_2 \cap AB]$.

The successful resolution of structural problems depends upon the geometric knowledge, proficiency and the experience of the problem solver, particularly with respect to specific problem type. Algorithms are particularly useful for solving selected types of design tasks.

Triangle constructions are among the most frequently constructed structures in the teaching of geometry. These constructions can be solved by utilizing statements related to their similarity.

Thus, constructions based on the SSS (side – side –side), SAS (side – angle–side), or ASA (angle – side –angle) come into consideration. In Figure 6, an algorithm for constructing a triangle using the SAS theorem is presented in the form of a sequence of partial constructions [25].

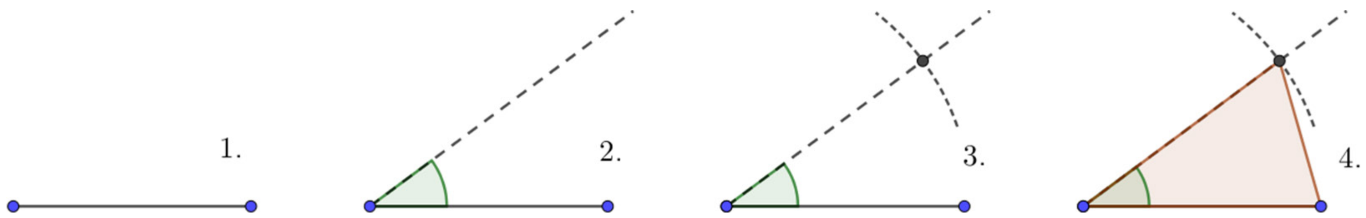

Fig. 6. Construction of triangle according to the sentences SAS

If we consider other elements of the triangle ABC, such as medians (m_a, m_b, m_c), altitudes (v_a, v_b, v_c), bisectors (u_α, u_β, u_γ), radius of circumcircle r and radius of incircle ρ, we get new possibilities. We can develop procedures -or algorithms for construction tasks involving a triangle, when three elements from the set are given [26]:

$$\{a, b, c, \alpha, \beta, \gamma, v_a, v_b, v_c, m_a, m_b, m_c, u_\alpha, u_\beta, u_\gamma, r, \rho\}$$

The described algorithms are for 98 types that can only be solved using a ruler and compass (Euclidean construction), whereas for remaining 52 types this is not possible.

For example, the construction and algorithm of the solution to construct a triangle *ABC* for the given medians m_a, m_b, m_c shown in Figure 7.

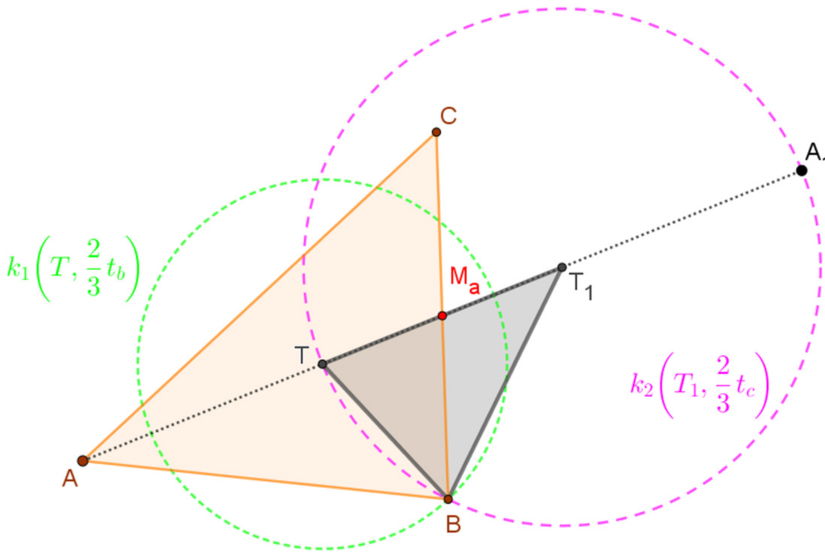

Fig. 7. Construction and algorithm of the solution to construct a triangle for the given medians m_a, m_b, m_c

Algorithm for this construction is:

1. TT_1B – auxiliary;

   Theorem *SSS*: $|TT_1| = \frac{2}{3}t_a$, $|TB| = \frac{2}{3}t_b$, $|T_1B| = \frac{2}{3}t_c$;

2. Completing triangle *ABC*

If we were to limit the choices of three specified elements to a set

$$\{a, b, c, \alpha, \beta, \gamma, v_a, v_b, v_c, m_a, m_b, m_c\},$$

then there is a simpler form of the algorithm, which is based on the construction of an auxiliary triangle.

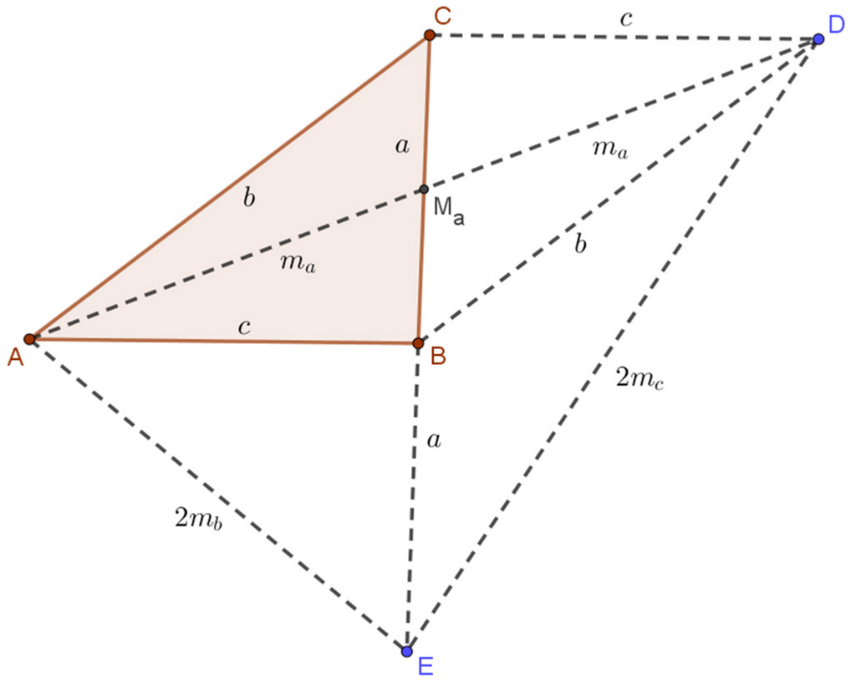

Fig. 8. Construction of the triangle based on the construction of an auxiliary triangle

Consider a triangle *ABC* and let the point *D* be the image of the *A* in central symmetry with the center M_a. The point *E*, in turn, is the image *C*, the vertex in central symmetry with the center in the vertex *B*. Figure 8 displays nine triangles.

In each of them, we determine the sides, heights, perpendiculars, and angles that can be expressed by simple dependence on the elements of the original triangle *ABC*. Table 2 provides further details [27].

Table 2. Possibilities of constructing the triangle from three given elements

Triangle	Sides	Angles	Medians	Altitudes
ABC	a,b,c	α,β,γ	m_a,m_b,m_c	v_a,v_b,v_c
CBD	b,c,a	β,γ,α	m_b,m_c,m_a	v_c,v_b,v_a
BDE	$2m_c,a,b$	$\pi-\gamma,-,-$	$\frac{c}{2},-,-$	$-,v_a,v_b$
CDE	$2m_c,2a,c$	$\beta,-,-$	$-,b,-$	$-,v_a,2v_c$
ACE	$2a,2m_b,b$	$-,\gamma,-$	$-,c\ -$	$v_a,-,2v_b$
ADE	$2m_c,2m_b,2m_a$	$-,-,-$	$\frac{3}{2}c,\frac{3}{2}b,\frac{3}{2}a$	$-,-$
ABE	$a,2m_b,c$	$-,\pi-\beta,-$	$-,\frac{1}{2}b,-$	$v_a,-,v_c$

(Continued)

Table 2. Possibilities of constructing the triangle from three given elements *(continued)*

Triangle	Sides	Angles	Medians	Altitudes
ABD	$b, 2m_a, c$	$-, \pi-\alpha, -$	$-, \frac{1}{2}a, -$	$v_b, -, v_c$
ACD	$c, 2m_a, b$	$-, \pi-\alpha, -$	$-, \frac{1}{2}a, -$	$v_c, -, v_b$

Algorithm works as follows:

1. Take the specified three elements of the triangle ABC;
2. Look for these elements in the rows of the table (not necessarily in one column);
3. Construct an auxiliary triangle from the appropriate row;
4. Add to the triangle ABC.

For example, we can also use the auxiliary triangle ADE to the construction of Figure 7. Its sides are twice the length of the given medians m_a, m_b, m_c and the vertex B is its center of gravity.

4 DISCUSSION

Algorithmic thinking is an essential part of our daily life. It is considered as one of the most important computer thinking skills. Preparing students for life in the era of digital transformation is an important mission of modern education [28]. Algorithms, however, are not solely concerned with data and their interpretation, they are tools that can assist individuals in various domains. Even though most people perceive algorithms as a part of computer science and programming, we find a lot of its applications in mathematics, including geometry.

The study specially dealt with algorithmic thinking in the context of teaching school geometry. Embedding algorithmic thinking elements and practices into mathematics lessons is a complex and challenging process for teachers. Students use algorithmic thinking, for example, in solving computational and constructional geometric problems. Similarly, as mentioned in [29], geometric constructions can be viewed as a special type of algorithm that deal with geometric objects. In geometry lessons, students gradually develop proficiency in calculating the perimeter, content, and volume of various shapes or the construction phases of these shapes. These approaches exhibit some basic properties of the algorithm and can help students in solving the assigned tasks. A clear example of the algorithm can be found in the procedure for constructing the geometric figure. Those who become proficient in constructing triangles using the SSS condition, they can apply the same procedure to construct triangles based on the SAS, ASA, or other combinations of three different elements. Despite the fact that there are a lot of different construction tasks, we can identify recurring algorithms within the construction procedures themselves.

However, we must emphasize that these approaches do not lead students to formalism. On the contrary, the students have to use and apply them to a variety of geometric problems. They must engage in critical thinking and problem-solving to determine suitable solutions, with the mentioned algorithms serving as educational tools within the solution process. The ability to think and reason can develop rapidly when the students are challenged with creative problems [30].

It is essential to teach these algorithms in the first years of primary school and thus to develop their algorithmic thinking in geometry. Mastering given algorithms on simple tasks leads to easier use of the procedure on more difficult tasks. Algorithmic thinking competencies increase as students move into higher grades, and they are seen as imperative for all students from the beginning of their schooling.

We agree with the view that it is necessary to devote space to the algorithmization itself through visualizations and experiential methods. It is acceptable even for students who are weaker in abstract thinking and mathematical apparatus. As stated in [31], algorithms can be considered structures. As their research shows, the success of the test negatively correlates with the need for structure. Their findings lead to the need to introduce these algorithms into the teaching of mathematics, including geometry.

5 ACKNOWLEDGMENT

This work was supported by the Educational Grant Agency of the Ministry of Education, Science, Research and Sports of the Slovak Republic, No. KEGA 013UKF-4/2023.

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