

## PAPER

# Enhancing Students' Linear Algebra I Learning Using Assessment Through STACK

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## ABSTRACT

E-learning is seen as a new philosophy of education that encompasses all existing forms of education, including full-time education. Currently, there is a wide variety of software for developing interactive content. The aim of this study is to improve students' learning of Linear Algebra I using STACK (the system for teaching and assessment using a computer algebra kernel) at Bahir Dar University. The experience of using STACK questions has shown that their application helps students of all forms of education in learning Linear Algebra I and greatly facilitates the learning process of students, allowing them to master the contents of Linear Algebra I. In a mastery quiz, students try different algebra content repeatedly until they understand the concept. After students have mastered the content, they take a test quiz. Moreover, STACK helps teachers in a scoring (cumulative) system of knowledge assessment, makes the learning outcomes more visible and convenient for analysis. In addition, it is noted in the research that the STACK tasks allow students to review analytical solutions to complex types of problems and organize hints that help them to solve tasks. The approach saves the teacher's time to check solutions at any time and give students individual options for tasks. As part of the study, the STACK tasks were analysed, which revealed a greater number of advantages of learning and assessment compared to its disadvantages and revealed the possibility of its application in the educational process. It is recommended that educational policy makers must integrate STACK into a curriculum of Mathematics.

## KEYWORDS

Linear Algebra I, STACK, learning, assessment

## 1 INTRODUCTION

The use of digital technologies in teaching, learning and assessment is becoming more common today [1–3]. This is due to a variety of benefits that come with it, such as: B. Fostering collaboration, preparing students for the future, and providing a more engaging learning environment. STACK is a software consisting of a number of independent modules that work together to promote the implementation of an application. The components, which can include an operating system, architectural

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layers, protocols, runtime environments, databases, and function calls, are stacked on top of each other in a hierarchy.

STACK is a computer-aided assessment plug-in for the learner management systems Moodle and ILIAS. It adds challenging assessments in mathematics and related disciplines, with an emphasis on formative assessment underpinned by computer algebra. Note that learners must enter their solution as a math expression, rather than selecting an option as in a multiple-choice question. Using the Maxima open-source computer algebra system, STACK allows question authors to create randomly generated math questions within structured templates, review student answers submitted within that question, and provide instant feedback, taking into account the errors made.

Software stacks can be simple or complex, depending on the desired application functionality, and can encompass components and services from an enterprise's on-premises resources, from third-party providers (e.g., SaaS providers), or from a cloud provider. There is no fundamental condition for the modules and services that must be combined into a software stack, other than that their characteristics and functions underpin the development, deployment, and functioning of an application. Depending on the desired application, this could be at the least: an operating system, a database, tools for promotion, a programming language and the application. Other components that can be a portion of a more complicated software stack encompass abstracted physical resources and virtualization, scheduling and orchestration, databases, computers, networks, security, user interface, and more.

STACK is one of the most effective data structures for assessing arithmetic expressions in programming languages. An arithmetic expression consists of operands and operators.

## 2 LITERATURE REVIEW

Assessment guided learning is one of the key messages of educational science. While a variety of new teaching concepts have entered the classroom in recent years with the advancement of technology-based learning tools, little emphasis has been placed on redesigning student assessment performance to reflect the opportunities these new tools offer.

Research on STACK has been widely published by authors from different contexts and shows its use in different educational settings around the world. For instance, according to Back [4], although STACK was developed to enable student-provided answers in computer-based assessment of mathematics, the system also provides different types of possibilities for various types of multiple-choice questions. When it comes to formative assessments, immediate, high-quality, and detailed feedback is vital as it improves and strengthens student learning [5–6]. Satisfaction with technology-enabled services improves continued use [7]. Technology integration and teacher behaviour are positively related to student engagement [8]. Technology-driven assessment can optimize higher education teachers' decisions in an intelligent learning environment [9].

## 3 METHODOLOGY

The second year undergraduate Mathematics programme students of College of Science at Bahir Dar University were selected for the STACK enabled integrated teaching. The study was done for one semester, during October 2019 to December 2019.

The class comprised of 29 students. The students were comprised of 20.69% female and 79.31% male. The students were taught the course linear algebra I through STACK-enabled integrated teaching.

Figures 1 through 6 show a random sample of a student whose work originates from a computer-based assessment plug-in for the STACK learning management system. Figure 1 shows the number of attempts, starting time, state of the work (whether it is completed or not), the time of completion, the amount of time taken, marks and grades.

The screenshot shows a student's assessment record for 'Math 2nd Bekele Mindesil'. The record includes: Attempts (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13), Started on (Friday, 1 November 2019, 10:50 PM), State (Finished), Completed on (Sunday, 3 November 2019, 7:15 AM), Time taken (1 day 8 hours), Marks (5.00/7.00), and Grade (7.14 out of 10.00 (71%)).

The question is 'Question 1' (Partially correct, Mark: 1.00 out of 2.00). It asks to consider the system of equations  $M\mathbf{x} = \mathbf{b}$  where  $M$  is a  $4 \times 4$  matrix. The augmented matrix  $[M|\mathbf{b}]$  is given as:

$$N = \begin{bmatrix} -1 & -1 & -2 & 2 & -1 \\ 0 & -5 & -5 & 0 & -15 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(The line separating the coefficient matrix from the vector of constants has been omitted.)

(a) Perform elementary row operations on  $N$  until it reaches reduced row echelon form.

The student's answer is shown as a matrix:

$$\begin{bmatrix} 1 & 0 & 1 & -2 & -2 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The system interpreted the answer as:

$$\begin{bmatrix} 1 & 0 & 1 & -2 & -2 \\ 0 & 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Fig. 1.

Figures 2–6 show whether the student’s work is correct or not. If the student’s answers are not correct, STACK will explain each step so that the student can learn from their mistakes.

The screenshot shows 'Question 2' (Correct, Mark: 1.00 out of 1.00). The question asks for the augmented matrix of the system of linear equations  $A\mathbf{x} = \mathbf{b}$ , which is reduced to the following form:

$$\begin{bmatrix} 1 & -2 & 3 & -1 & 3 \\ 0 & 0 & 1 & -5 & 5 \\ 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(The line separating the coefficient matrix from the vector of constants has been omitted.)

How many solutions does this system have?

Select one:

- a. Infinitely many solutions. ✓
- b. No solutions.
- c. One unique solution.

The student's answer is correct. The explanation provided is:

Your answer is correct.  
 From the third row we know that  $x_4 = 7$ .  
 Then using this in the second row we know that  $x_3 - 5 \cdot 7 = 5$  so  $x_3 = 40$ .  
 This leaves us with the first row which reduces to  $x_1 - 2 \cdot x_2 + 3 \cdot 40 - 7 = 3$ .  
 We can choose  $x_2$  to be any value and find the value of  $x_1$  which gives a solution. Therefore there are infinitely many solutions.  
 The correct answer is: Infinitely many solutions.

Fig. 2.

**Question 3**  
Correct  
Mark 1.00 out of 1.00  
Flag question  
Edit question

Select the matrices that are in Reduced Row Echelon Form.

Select one or more:

$\begin{pmatrix} 1 & -3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

✓ This is in Reduced Row Echelon Form.

$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$\begin{pmatrix} 3 & 1 & -2 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

✓ This is in Reduced Row Echelon Form.

Your answer is correct.

The correct answers are:  $\begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & -3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

Fig. 3.

**Question 4**  
Correct  
Mark 1.00 out of 1.00  
Flag question  
Edit question

Tidy STACK question tool | Question tests & deployed variants

Write the following matrix in Reduced Row Echelon form.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 8 & 4 \\ 0 & 1 & 4 & 8 \end{bmatrix}$$

1	0	0	-52
0	1	0	40
0	0	1	-8

Your last answer was interpreted as follows:

$$\begin{bmatrix} 1 & 0 & 0 & -52 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & -8 \end{bmatrix}$$

✓ Correct answer, well done.

We need to perform row operations so that the matrix is in reduced row echelon form. A matrix is in reduced row echelon form if

- all nonzero rows are above any rows of all zeroes
- the leading coefficient of a nonzero row is always strictly to the right of the leading coefficient of the row above it
- the leading coefficient of a nonzero row is 1
- the leading coefficient is the only nonzero entry in its column.

In our case this gives

$$\begin{bmatrix} 1 & 0 & 0 & -52 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & -8 \end{bmatrix}$$

Fig. 4.

**Question 5**  
Correct  
Mark 1.00 out of 1.00  
Flag question  
Edit question

Tidy STACK question tool | Question tests & deployed variants

Find the augmented matrix of the system represented by

$$x_1 + 6x_2 + x_3 = 3$$

$$x_1 - 3x_3 = 1$$

$$7x_1 + 2x_2 - x_3 = 0$$

Input values for the vector of constants in the last column of the matrix below.

1	6	1	3
1	0	-3	1
7	2	-1	0

Your last answer was interpreted as follows:

$$\begin{bmatrix} 1 & 6 & 1 & 3 \\ 1 & 0 & -3 & 1 \\ 7 & 2 & -1 & 0 \end{bmatrix}$$

✔ Correct answer, well done.

In this question we are looking for a matrix  $M$  to write the equation system in the form  $Mx = b$ . The augmented matrix is of the form  $[M|b]$ . We really just need to read of the coefficients carefully, not forgetting to add in any zeros when a variable does not appear in the equation. In this case we get

$$\begin{bmatrix} 1 & 6 & 1 & 3 \\ 1 & 0 & -3 & 1 \\ 7 & 2 & -1 & 0 \end{bmatrix}$$

Fig. 5.

**Question 6**  
Incorrect  
Mark 0.00 out of 1.00  
Flag question  
Edit question

Tidy STACK question tool | Question tests & deployed variants

Consider a system of 7 linear equations in 4 variables.

Only one of the following statements is true. Select the true statement.

- (No answer given)
- The system does not have a unique solution.
- The system will have no solutions.
- The system could have a unique solution.
- The system will have infinitely many solutions.
- The system will have at least one solution.

✘ Incorrect answer.

When there are more variables than equations we cannot have a unique solution. However, this could either lead to no solutions or infinitely many solutions.

If there are at least as many equations as variables, then it is possible to have a unique solution, but it is also possible to have none or infinitely many solutions.

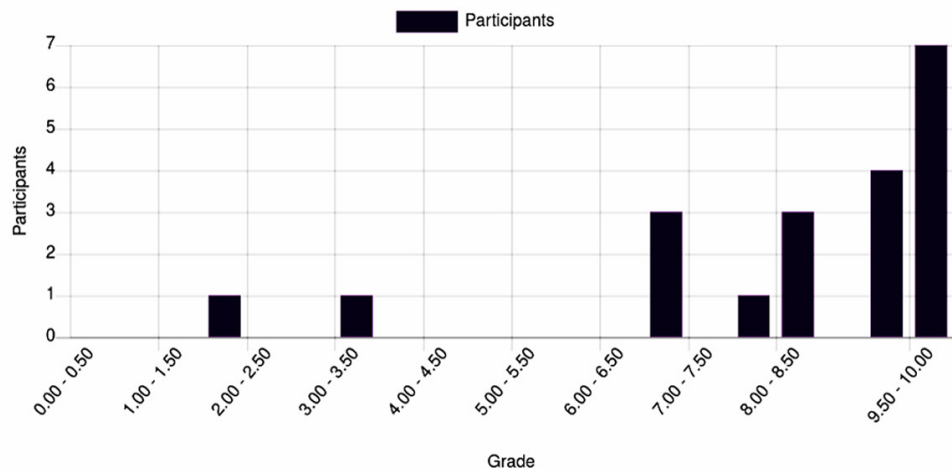
A correct answer is:

- The system could have a unique solution.

Fig. 6.

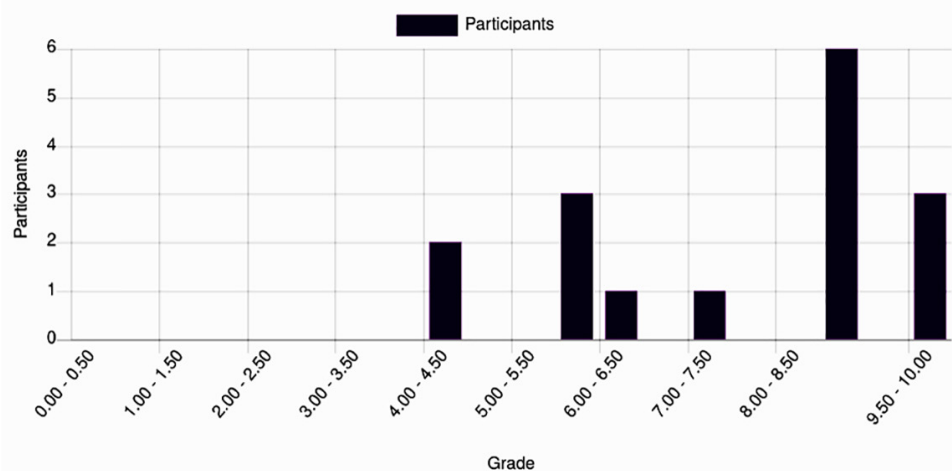
The following bar charts, Mastery quiz 1, Mastery quiz 2, Mastery quiz 3, Mastery quiz 4, show the total number of participants/students in each grade range.

### Overall number of students achieving grade ranges



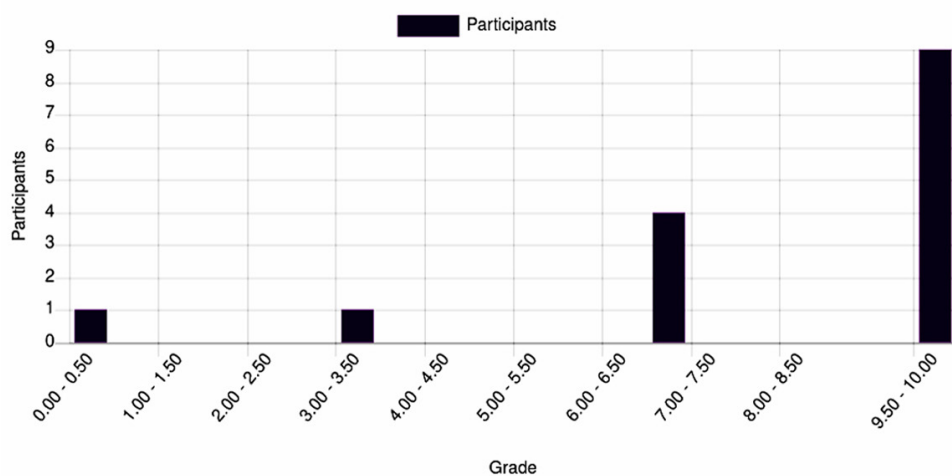
Mastery quiz. 1.

### Overall number of students achieving grade ranges



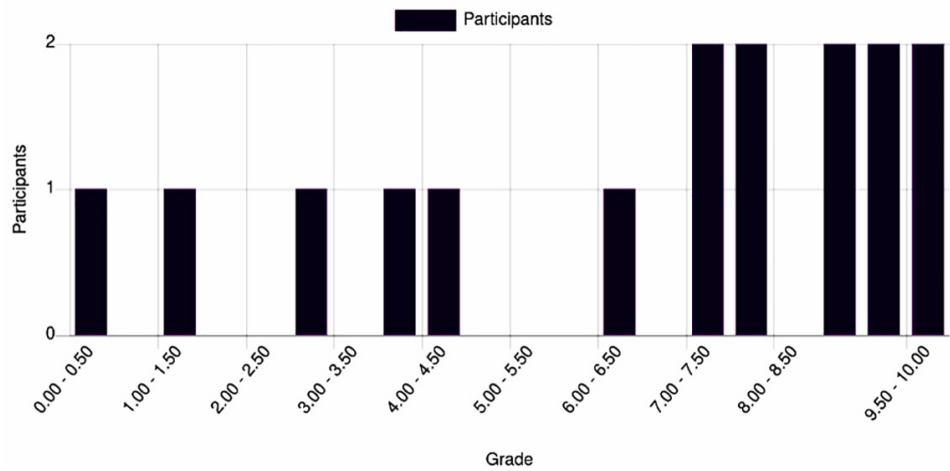
Mastery quiz. 2.

### Overall number of students achieving grade ranges



Mastery quiz. 3.

## Overall number of students achieving grade ranges



Mastery quiz. 4.

### 3.1 Results and discussions

The first half of this section provides a quantitative analysis of students' results, looking at the results, number of attempts and attempt times of the STACK quizzes, and exam results.

### 3.2 Quantitative analysis

In this study, a comparison was made between students' final exam scores and their STACK quiz scores, the number of times they tried the STACK quizzes, and the total time they spent on the STACK quizzes.

### 3.3 Qualitative analysis

The researcher conducted interviews with FGDs to better understand how students use STACK and to find possible explanations for the results. Findings were grouped into three main thematic areas: challenges in using STACK, value of STACK, and potential of STACK. For each topic, we discuss the students' answers and present some quotes that are representative or of particular interest.

### 3.4 Challenges of using STACK

According to interviewees, lack of access to devices and internet connection was one of the main reasons why some students stopped engaging with the quizzes. The feedback from the interviews was confirmed by all responses in the FGDs, although to a small extent they argued that few students faced this challenge. They estimated that around 20% of the students had problems with access. Some could not complete their tasks on time for certain reasons, because they did not have a smartphone or laptop or even internet packages to access the tasks.

### 3.5 Value of STACK

A key aspect of STACK that was beneficial to all students on the course was its ability to provide instant feedback and randomize questions for practice. This corresponds to 15 out of 20 of the answers from the interviews and 21 out of 25 of those in the FGDs. According to 15 out of 20 in the interview, most students who did not do well on the continuous assessment used Mastery Quizzes (because they received instant feedback) to practice and prepare for the final exam.

### 3.6 Potential of STACK

To find out how learners perceive the use of STACK in formative assessment, 18 out of 20 of the respondents recommended continuous assessment conducted using weekly STACK tests contributed a lot to their learning. Six students had concerns, namely access to the equipment as some students could not complete the tests on time. Of the FGDs, 20 out of 25 of the students recommended that STACK should be integrated into all mathematics courses because of its capabilities to provide instant feedback, point out errors, guide the learning process and allow them to practice their math skills. In both the interviews and the 3 FGDs, students found using STACK very motivating and indicated that it kept them engaged in learning and practising math. These responses suggest that this could be a possible explanation for why, despite the challenges of using STACK, some students managed to think outside the box and see the potential to help them.

## 4 CONFLICT OF INTEREST

There is no conflict of interest.

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