

## PAPER

# An Interdisciplinary Educational Proposal in Junior High School: The Fractal Geometry in Science, Computer Science and Art Lessons

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## ABSTRACT

The purpose of this study is to provide an interdisciplinary approach to fractals within the traditional school curriculum. The proposed activities are expected to help teachers to provide a comprehensive and engaging learning experience for students that fosters deeper understanding, creativity, and connections within the sciences. Fractals are complex geometric shapes that are self-similar, and therefore exhibit similar patterns at every scale. They are created by repeating a simple process over and over. Fractals differ from traditional geometric shapes because they are non-regular, but are very common in nature, such as clouds, mountains, trees and snowflakes. Also fractals are impressive mathematical creations and can contribute a lot to the understanding of Junior High School mathematics because they could be fun and at the same time an exciting way to introduce many areas of mathematics and physics. By connecting fractals to different mathematical concepts and applications, Junior High School students can develop their problem-solving skills and gain a deeper appreciation for the beauty and complexity of mathematics.

## KEYWORDS

fractals, interdisciplinary education, science, art lesson, computer science, junior high school education

## 1 INTRODUCTION

In this paper we intend to describe an interdisciplinary educational proposal for students of Junior High School. It is very important for the students in that age group to learn how to work through interdisciplinary projects. For this purpose, we have prepared several activities using fractal geometry, both in the nature and sciences, such as mathematics and physics. Finally, we combined fractal geometry with the art lesson and the computer science. For this purpose, the acquisition of specific

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skills is necessary, which is achieved at the end of each lesson. We consider it necessary for students to have gained experience through interdisciplinary teaching proposals, so that they develop the necessary skills and strategies as well as their critical thinking that they will use throughout their education. The present study is divided in two parts. In the first part, there is a brief description and presentation of fractals, while in the second part, the interdisciplinary educational proposal based on fractals in Junior High School is analyzed.

## 2 AIMS OF THE STUDY

The interdisciplinary approach has become an important and challenging technique in the modern curriculum. The interdisciplinary approach synthesizes more than one discipline and creates teams of teachers and students that enrich the overall educational experience. Student education has suffered because of the inferior pedagogy of traditional methodologies that concentrate specifically on only one discipline. The interdisciplinary approach provides many benefits that develop into much needed lifelong learning skills that are essential to a student's future learning [1]. Interdisciplinary education in sciences refers to an educational approach that integrates knowledge and methods from different scientific disciplines to provide a comprehensive and holistic understanding of natural phenomena.

The benefits of interdisciplinary education in sciences are numerous. The interdisciplinary education provides a more comprehensive understanding of natural phenomena by drawing on the knowledge and methods of multiple scientific disciplines. This helps to provide a more complete picture of complex natural systems and processes. Furthermore, interdisciplinary education encourages creative problem-solving by providing a broader perspective and allowing for the integration of diverse ideas and approaches. This can lead to innovative solutions of complex problems.

The interdisciplinary education requires effective communication between teachers from different disciplines. This can help students develop strong communication skills and learn how to effectively communicate complex scientific concepts to diverse audiences. Also it encourages critical thinking by challenging students to consider multiple perspectives and weigh the strengths and weaknesses of different approaches. This can help them develop a more nuanced and sophisticated understanding of scientific concepts. In the end, interdisciplinary education provides students with the opportunity to engage in collaborative research projects that draw on the methods and expertise of multiple disciplines. This can help students develop a broad range of research skills and techniques that can be applied in a variety of contexts. Overall, interdisciplinary education in sciences can provide students with a more comprehensive and holistic understanding of the natural world, while also promoting creative problem-solving, effective communication, critical thinking, and research skills. The following objectives are planned to be developed in this study:

1. Junior High School students will develop skills and strategies that can serve as an example for other interdisciplinary educational projects.
2. Students will enhance their cognitive abilities.
3. Students will get to know a new learning model, different from today's traditional education.

The purpose of this didactic proposal is for Junior High School children to come into contact with a different approach to the science lessons. It will give them the

opportunity to work in groups and cooperative to experiment, imagine, investigate and finally understand complex concepts of mathematics.

Presenting fractals to Junior High School students will serve to make mathematics more interesting. Instead of studying mathematical topics in a traditional fashion, the study of fractals helps students become part of a recent mathematical discovery. Students will be able to explore fractals and connect many aspects of mathematics. They will also see the link between mathematics and nature [2]. This work is a continuation of our research in new technologies in the area of teaching and learning and how these can be implemented in high school [3].

## 2.1 Fractals and self-similarity

In school geometry, we learn about lines, circles, squares, cubes, cylinders and spheres. In nature, however, other types of shapes prevail around us: clouds, lightning, ice crystals, sponges and coastlines, which present a complexity that is not at all similar to the simple geometric objects of “classical” geometry.

The last mathematical challenge of our century is the study of the above figures. That is, to understand how random arrangements are created, such as clouds or the composition of a snowflake. For this reason, fractal geometry was created to describe shapes and patterns found in nature. The creator of fractal geometry is Benoit Mandelbrot. The term fractal was proposed by Mandelbrot himself and comes from the Latin word “fractus”, which means fragmented or broken [4]. With the term fractal, both in mathematics and physics as well as in other sciences, we described a geometric shape that is repeated identically to an infinite degree of magnification. That is, a geometric shape that can be enlarged many times and part of it will continue to be present in its original form. This special feature of fractals is called self-similarity [5].

The world around us is full of fractals, full of natural objects that strongly show self-similarity [6]. Typical examples are broccoli, fern leaf, trees, etc. (Figure 1). Fractal structure can even be found in the human body, such as in the lungs, neurons or the branches of blood vessels. Of course, it is important to mention that the fractals we find in nature do not show infinite detail at magnification like the fractals resulting from mathematical relationships.



Fig. 1. Examples of fractals

Broadly speaking, mathematical and natural fractals are shapes whose roughness and fragmentation neither tend to vanish, nor fluctuate up and down, but remain essentially unchanged as one zooms in continually and examination is refined. Hence, the structure of every piece holds the key to the whole structure. An alternative term is self-similarity, which expresses that each part is a linear geometric

reduction of the whole, with the same reduction ratios in all directions. Figure 2 illustrates a standard strictly self-similar fractal, called Sierpinski gasket [7].

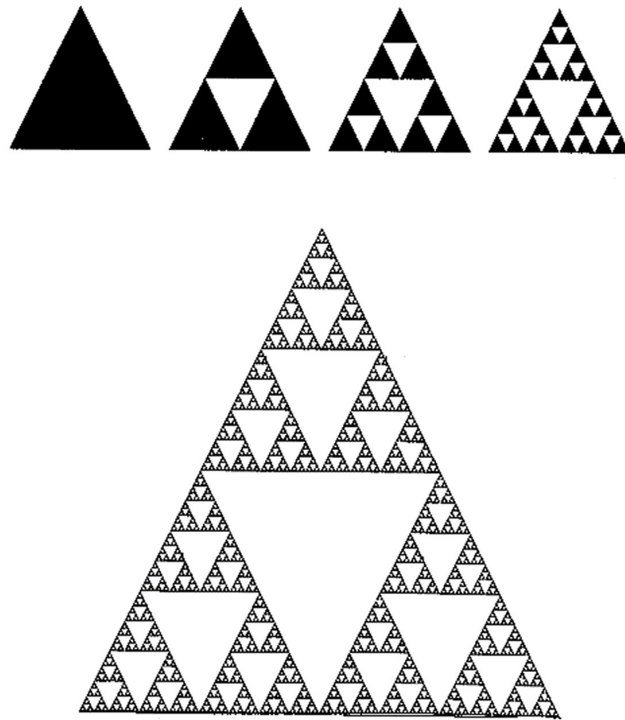


Fig. 2. The Sierpinski triangle

Yet, in the Junior High School mathematics curriculum, discussions about shapes continue to focus exclusively on objects with classical geometric shapes. This behavior is not unexpected, given the “newness” of fractal geometry. However, it seems that, as with classical geometry, some basic concepts of fractal geometry should be introduced to young children, including shape, self-similarity, and measurement [8].

**Pedagogical utilization of fractals.** Fractals can be a fascinating subject to teach in the classroom, because they can be a great way to introduce math and physics concepts to children in Junior High School. Integrating fractals into the classroom can help students develop an appreciation for the beauty and complexity of mathematics, encourage critical thinking and problem-solving skills as students learn how to create and analyze fractal patterns. Fractals are impressive mathematical creations and can speak volumes about the understanding of Junior High School mathematics because, in addition to the others discussed below, they could be a fun and exciting way to introduce many areas of study in Junior High School mathematics.

Here are some ways fractals can be used in a Junior High School math class.

**Geometry:** Fractals are geometric shapes that exhibit self-similarity, that is, they repeat themselves by creating smaller copies of themselves. The study of fractals can deepen students’ understanding of concepts such as symmetry, and similarity.

**Algebra:** Fractals exhibit interesting properties that can be studied using ratios and proportions. Also, Junior High School students can investigate how the length, area, and perimeter of fractals change as the scale changes.

**Computer Science:** Fractals are easily and impressively created using simple algorithms and also they are an ideal example for exploring the intersection of mathematics and computer science.

**Arts lessons:** Fractals can be used to create beautiful and intricate works of art. Junior High School students can explore the use of fractals in art and design and create their own fractal-inspired artwork.

### 3 STATE OF THE ART

The implementation of this didactic approach is best carried out in the IT laboratory, so that the students make use of the computers, but also the teacher makes use of the interactive whiteboard that is present in most school laboratories. It is also important that the students do not need to have a high cognitive level in mathematics, nor any special prerequisite knowledge. After all, this was the goal from the beginning, because the purpose of this particular teaching approach is to attract as many students as possible, motivate them to observe and experiment with new concepts and produce new knowledge through a combination of their own discovery and creation. The classroom orchestration will be in small groups so that they can exchange ideas, communicate, reflect and finally express themselves using mathematical concepts. In this way, students cultivate scientific dialogue practices.

**General data.** Teaching Approach: Enriched lecture, collaborative learning, learning by doing, exploratory instruction.

**Goals.** The student should be able to

1. Design simple fractal tree.
2. Distinguish fractal patterns.
3. Understand the self-similarity of a fractal.
4. Change parameters in a function, process, or technique.

*Teaching Means:* Whiteboard, computer, projector, papers and pencils.  
The teaching approach that we propose includes the following stages.

- i) Brief historical overview.
- ii) Fractal search in our surrounding world and in nature.
- iii) Fractal construction in two ways, either statically, i.e. with paper, pencil and classical geometry tools, or with the use of a computer and appropriately selected software.
- iv) Investigation and study of fractals and their properties.
- v) Encourage children to explore the artistic side of fractals by creating their own fractal designs
- vi) Search for real problems with fractals

**Course #1: Mathematics and Physics.** At the beginning of the course, it is a good idea to give a brief historical overview of how fractal geometry began. In other words, a simple introduction to what fractals are and what was the need that led us to study them. At this point, students will be confronted with the concept of self-similarity and we could draw many examples from the world around us to make this concept more understandable to children. Then, the students will be ready to construct their own fractal objects, at first using paper and pencils, but also later with the use of special software and the help of the computer. This process of creating in

both ways is fun and helps students develop higher-level math and reasoning skills that the traditional teaching in mathematics can't provide.

Junior High School students can use a simple algorithm to create a fractal tree. The algorithm involves drawing a straight segment of a certain length  $L$  and then branching it into two smaller straight segments of length  $L/2$  with also a certain angle. The process is then repeated for each of the two smaller straight sections, creating a tree-like pattern with rich branches. Students can experiment with different values of the initial length of the line segment and the angle used for the branches to create trees of different sizes and shapes. They can also explore how the tree changes as the number of iterations increases.

A good approach to creating a fractal tree is to make Leonardo's tree. The well-known Leonardo Da Vinci noticed that the branches of trees when they grow and develop by creating new branches follow a specific mathematical sequence (see Figure 3).

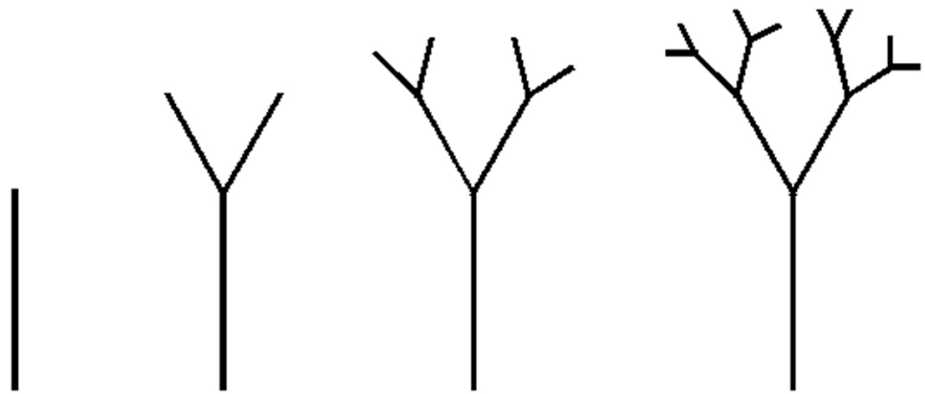


Fig. 3. The fractal tree

“All the branches of a tree at every stage of its height when put together are equal in thickness to the trunk.”

—Leonardo Da Vinci

This is known as Leonardo Da Vinci's rule for branches. Fractal trees can have no randomness, so each branch is identical to the one before. The pattern consists of a line and two separate lines, known as the branches that continuously repeat until the shape of the tree begins to form. If randomness is added, then it could make the tree appear more realistic, although the original fractal tree uses a formula where the rotation only occurs around the origin. At the end of each segment, rotate right and left the same degree from the line and draw congruent shorter segments [9].

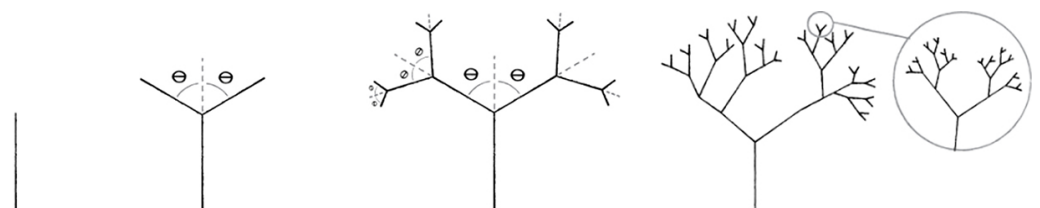


Fig. 4. Self – similarity

Each branch of the fractal tree is a replica of the base of the tree. At the end of the lesson the students will have the opportunity to experiment with their

creations by adding to them either rotation of the branches or their growth in a random way (Figure 4).

In the end of creating the tree, students with the help of the teacher can use it to explore different mathematical concepts, such as:

**Geometry:** Students can study the properties of the fractal tree, such as self-similarity and the relationship between the length of the branches and the total size of the tree. They can also explore how the angle of the branches affects the overall shape of the tree.

**Algebra:** Students can study the recursive nature of the algorithm and how it creates a tree-like structure through repeated iterations. Through explorations of this fractal, children are able to gain an appreciation of the beauty of the fractals displayed [10].

**Course #2: Computer science.** As we saw above, a classic example of a fractal shape is the binary tree. The specific shape starts from a vertical straight part that represents the main trunk of a tree. From the top of the trunk start two straight parts (branches), bearing the same length so as to form the same angle with the straight part. The process is repeated in the edges of each branch so as to maintain the ratio for the branches and for the angles to be equal. Geometrically, fractals have forms, or features, that repeat at different sizes over ranges of scales. Therefore, there appears to be a possibility to teach in different ways without using complicated mathematic forms.

Presentation of the concept of fractals by using simple examples like the Figure 5 below.



**Fig. 5.** Images of fractals

Then students are provided with part of a code and they are asked to type in a Python environment and 'run' it (Figure 6). They are given some time to write down how they conceptualize running the algorithm they have typed. Afterward they are asked to observe the fractal tree that they created. They are presented with the notion of fractal sets (6). The specific teaching process was implemented in Python, which is considered suitable for familiarizing learners with a programming language [8].

In the procedure of learning fractal geometry, observation of the construction of fractal images is the most important part as much as understanding mathematical ideas behind them. Students will understand mathematical concepts better if they could see the construction procedure of the fractal figures step by step [11].

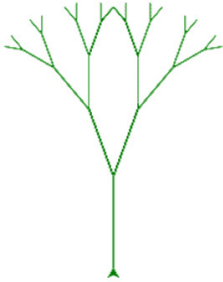
Input	Output
<pre> 1 import turtle 2 3 def fractal_tree(branch_len, t): 4     if branch_len &gt; 5: 5         t.forward(branch_len) 6         t.right(20) 7         fractal_tree(branch_len - 15, t) 8         t.left(40) 9         fractal_tree(branch_len - 15, t) 10        t.right(20) 11        t.backward(branch_len) 12 13 def main(): 14     screen = turtle.Screen() 15     screen.bgcolor("white") 16     t = turtle.Turtle() 17     t.speed(2) 18     t.color("green") 19 20     t.left(90) 21     t.up() 22     t.backward(100) 23     t.down() 24 25     fractal_tree(75, t) 26 27     screen.exitonclick() 28 29 if __name__ == "__main__": 30     main() </pre>	

Fig. 6. Python code

**Course #3: Art lesson.** Fractals are beautiful and can promote curiosity, have simple rules, can be easily modified, and create nearly uncontrollable temptation to explore different options to see what surprising patterns will emerge. Simple ideas lead to unexpected complexity and the exploration of fractal geometry appeals to all students of Junior High School [12].

Fractals can be used to show the students the art side of science. It is so interesting to see creativity with science just by the simple principle of self-similarity in many cases. Through the study of fractals, my students can see the math involved in art along with doing mathematical computations to get the principle behind the fractals. Fractals are a perfect combination of math and art. In our real life, in nature, we see many things that follow the principle of fractals but we do not necessarily notice them. Also, fractals are an interesting way of connecting mathematics to the world of art and realize that the process of iteration can produce beautiful fractals. Students in Junior High School can easily follow simple patterns to create fractal images on the paper [13].

It's obvious that geometry can be combined with art to enhance mathematical lessons for students. Educators must realize that visual representations of different mathematical concepts are a wonderful way to teach children geometry in a meaningful way [14]. The fundamental notion is that integrating the arts into one's teaching can help facilitate learning in the mathematics classroom, as the arts can recapture the wonder of learning mathematics. The connection between instruction and imagination is bridged and learning becomes play, and play becomes learning [15].

Students are already familiar with the concept of fractals, they already know that fractals are repeating geometric patterns that can be found in nature and created through mathematical algorithms. In this lesson, students will learn that many artists have also used fractals as inspiration for their artwork.

In the art lesson, students will have the opportunity to create a fractal tree, which is a typical example of how self-similarity is applied in nature as each branch of the tree is repeated using the shape of the original branch, which each time decreases in size but also rotates. Students can use the fractal tree as inspiration to create their own artwork, using different colors and shapes to highlight the same patterns of



the tree. The students also will be encouraged to experiment with different scales, rotations, and placements to create visually interesting patterns and to incorporate the principles of fractals into their artwork by repeating and scaling down shapes, creating self-similar patterns, or exploring symmetry and asymmetry. At the end of the lesson, the students will have the opportunity to share their artwork with the class to explain their creative choices and discuss the process of creating fractal-inspired pieces (Figure 7).

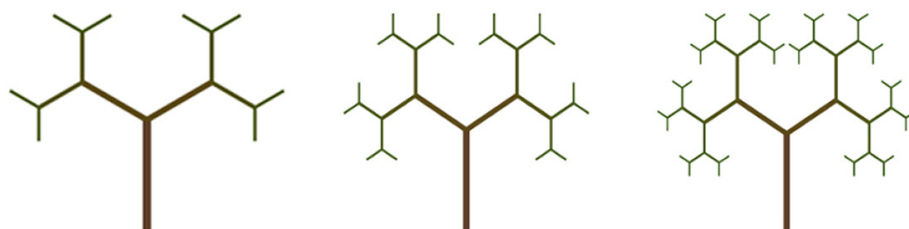


Fig. 7. Example of a fractal tree

By combining fractals and art in Junior High School, students gain a deeper understanding of how science can be applied creatively and they also can appreciate the elegance and complexity of both science and art. The students can understand that science is not limited only to definitions and equations and that art obeys the principles of science.

## 4 CONCLUSIONS

By presenting fractals in a fun and interactive way, students can develop an appreciation for the beauty and complexity of mathematics, as well as develop their critical thinking and problem-solving skills. We can claim that the study of fractals is capable of providing a rich and interdisciplinary study material that can deepen students' understanding of many different areas of mathematics. By connecting fractals to different mathematical concepts and applications, Junior High School students can develop their problem-solving skills and gain a deeper appreciation for the beauty and complexity of mathematics.

In summary, we could say that by creating and studying a fractal tree, both in the computer science class and in the art class, Junior High School students can gain knowledge about different mathematical concepts and applications, while developing their problem solving skills as well as discovering their creative skills. Finally, we can claim that the study of fractals is capable of providing a rich and interdisciplinary study material that can deepen students' understanding in many different areas of mathematics as well as other subjects in the school. Enriching teaching with the use of graphics opens up new potential, concerning teaching various lessons, since it is scientifically acknowledged that the use of pictures helps in keeping the students alert and makes the lesson more comprehensible for them [16].

There is too much content, not enough thinking, and too little fun in many of our courses. Inspiring and motivating students is critical, because once students are inspired and motivated, there are countless resources available to learn more about a subject. Inspiring and motivating students is far more important for long-term success than delivering information. Therefore, we must create a joy, an excitement, and a love for learning [8]. In future studies, we will apply the above principles in class and report our findings.

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