Visual Tools for Quantum Mechanics Education

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***Abstract*—We present the project Visual Quantum Mechanics, which uses computer-generated visualizations and animations to redefine content and quality of quantum-mechanical education at all levels. Main target group have been students of theoretical physics at universities, but more recently, we have developed learning objects for use at high schools. We describe the reasons for a visual approach to quantum mechanics and some specific methods for the visualization of quantum-mechanical objects.**

***Index Terms*—Visualization, Quantum Mechanics.**

I. Introduction

This paper reports on the project "Visual Quantum Mechanics", which is a systematic effort to explore and to teach quantum mechanics with the help of computer-generated animations and visualizations [1]. Quantum mechanics describes reality in a rather abstract way with the help of mathematical objects like wave functions and linear operators. From these objects, statistical predictions about physical systems can be derived with the help of certain interpretation rules. Thus, students are confronted with a situation where the connection between the mathematical description and the observed physical reality is rather indirect. Wave functions cannot really be "seen" in the usual meaning of the word. There is no other way to create pictures of wave functions than by means of computer graphics.

In quantum mechanics, the scales of space, time, and energy are rather exotic and completely removed from everyday experience. Therefore, when learning the theory or when doing exercises, students have no feeling of what is right and what is wrong. Here the animations and visualizations play an essential role. They provide students with a certain phenomenological background of quantum mechanics, and generate an intuitive feeling for the behavior of wave functions that cannot easily be acquired by studying the abstract theory alone. Moreover, movies have proven to be a great motivation for students, because the desire to understand the interesting phenomena visible in the movies is a great driving force when it comes to learn the abstract theory.equal. Use automatic hyphenation and check spelling. Digitize or paste down figures.

II. An Example: Visualization of Wave Functions

The main problem with the visualization of quantum phenomena is that wave functions define rather high-dimensional data at each point of space-time. In nonrelativistic quantum mechanics, the wave functions that form the solutions of the one-particle Schrödinger equation are complex-valued functions of space and time, and in relativistic quantum mechanics, the solutions of the Dirac equation have four complex dimensions and there is no hope to visualize all this information in a single picture.

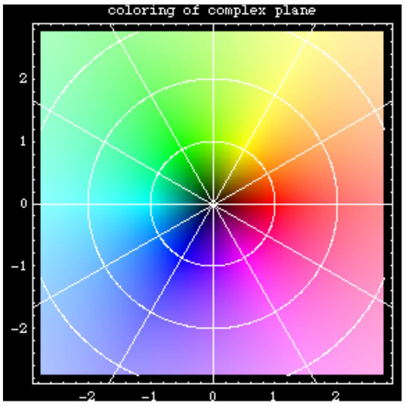

Figure 1. Color map for complex numbers.

In general, for the purpose of visualization, one has to extract some interesting part of the information encoded in the wave function. But in case of the nonrelativistic single-particle problem, the wave function can be visualized without loss of information by making use of colors. Colors form a three-dimensional manifold (with dimensions red, green, and blue, or, alternatively hue, saturation, and brightness) and one can encode three-dimensional data-values with colors. As complex-valued wave functions provide data with two real dimensions, we can make use of the colors on the surface of the color

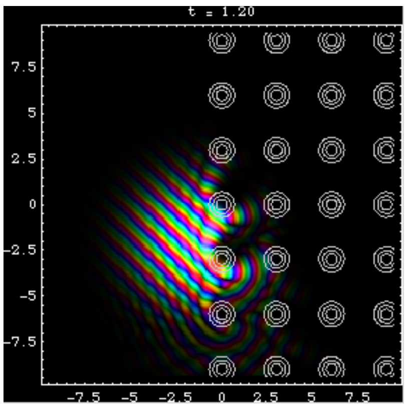

Figure 2. Image of a complex-valued wave function describing particle in the process of entering a crystal while being partly reflected at its surface..

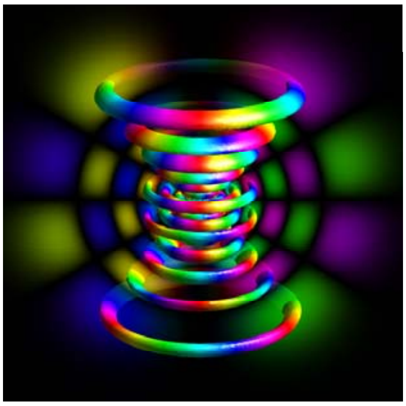

Figure 3. Hydrogen wave function.

Figure 4. Superposition state with spin.

sphere, where colors occur with the maximal saturation and are therefore easily distinguished.

I have been using the color map in Fig. 1 for the visualization of wave functions since 1994. This color map describes the phase of a complex number by the hue (which, like the phase may be considered a periodic quantity) and the absolute value by the brightness.

In Fig. 2 we show a wave function of a particle entering a crystal. At each point in space, the color describes the complex number associated to that point. The image is a snapshot from an animation that shows the emergence of an interference pattern in the reflected part of the wave function. It shows not only the position probability density, but also the distribution of momenta (wavelength) within the wave packet (wave-particle dualism).

III. The Project Visual Quantum Mechanics

Two books about visual quantum mechanics have been written that were based on the idea of presenting movies of quantum processes as a phenomenological background to the theory. Together, they contain about 700 learning objects built around short video sequences on an accompanying CD-ROM, and special software for the visualization of quantum phenomena in two and three dimensions.

Teaching with the help of animations implies changes in the approach to quantum mechanics and a shift in emphasis for the development of the theory. For example, while conventional books emphasize stationary states, the main focus of our presentation are time-dependent phenomena. The software not only contains learning objects for self-study and as a supplement of conventional teaching, but also provides tools for explorative learning and creating animations. The first book, dealing mainly with one-and two-dimensional quantum systems, has won the European Academic Software Award EASA 2000 with a special mention of the jury for outstanding innovation in the field. The second book has earned the EASA again in the year 2004. It deals mainly with three-dimensional systems, particles with spin, relativistic quantum mechanics, and quantum computing. Figure 3 shows, as an example, an electronic state of the hydrogen atom and Fig. 4 visualizes in addition the spin of the electron [3].

In quantum mechanics, animations can be used at all levels of education. Recently, we have created learning objects representing an approach to quantum mechanics that is suitable for high school education. One main point was to explain the necessity of complex-valued wave functions for the description of interference (on an elementary level, this means to explain the necessity of colors in the animations of quantum phenomena). The units have been tested and evaluated in practice with pupils from the last pre-university grade at the gymnasium BORG Graz. Moreover, efforts are under way to make these learning objects available for web-based training in the framework of adaptive, metadata-driven, learner-centered tutoring systems.

Acknowledgment

The preferred spelling of the word “acknowledgment” in America is without an “e” after the “g.” Try to avoid the stilted expression, “One of us (R.B.G.) thanks ...” Instead, try “R.B.G. thanks ...” Put sponsor acknowledgments in the unnumbered footnote on the first page.

References

[1] The project’s web site is at http://vqm.uni-graz.at (June 2006)

[2] B. Thaller: Visual Quantum Mechanics, Springer New York 2000, ISBN 0-387-98929-3

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[4] Web site (in german) of the project “Quantenmechanik-visualisiert und animiert” is at http://vqm.uni-graqz.at/qms/ (June 2006).

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