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PAPER

Looking into Students' Cognitive Processes in an Online Collaborative Learning Environment

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ABSTRACT

The primary aim of this study was to examine how students' cognitive interactions in an online collaborative environment influence the formation and recognition processes of unconscious cognitive mechanisms appearing in the learning of mathematical concepts at the university level. The data analyzed was collected from a population of undergraduate students. The implemented methodology was supported by a qualitative approach. The study occurred in two phases: in the first phase, students answered three questionnaires, and in the second phase, stimulated recall interviews involving the verbalization of the cognitive processes were conducted retrospectively through the online collaborative environment provided by Zoom. The selection of pairs of students participating in these interviews was made after analyzing their responses, considering their richness in terms of differences and variations. Through the study, rich insights were gained into the cognitive processes examined. The analysis showed metacognitive processes through which students' unconscious misconceptions were examined through self-reflection on previously acquired inadequate schemas. Cognitive and metacognitive interactions during the interviews progressively led to an adequate understanding of mathematical concepts. The findings confirm the results of a similar study that states stimulated recall data gathered in collaborative environments could be useful in revealing relatively higher-level cognitive and metacognitive processes.

KEYWORDS

stimulated recall interview, metacognition, mathematics in higher education, technology in higher education

1 INTRODUCTION

A large part of the cognitive mechanisms that develop when students are learning a new mathematical concept take place without them being able to consciously explain what is happening or how it is happening. Cognitive neuroscience experiments in recent years have confirmed that, given our nature, it is not possible to

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focus our attention on everything we perceive in our environment through our senses, so most of our experience in the world is not conscious [1].

The cognitive dimension of this unconscious knowledge includes mental models, beliefs, values, schemes, and perceptions that influence the ways of thinking and acting, reflecting the image that one has of something that has been formed implicitly and unconsciously without having paid conscious attention.

Unconscious knowledge is a primary phenomenon that can be, to a certain extent, describable but which is not reducible to more elementary components and which has the appearance of self-evident and self-consistent knowledge, such as the perception of a color or the experience of emotion [2].

An unconscious representation of a concept is, therefore, a conception in which the incompleteness or vagueness of information is masked by a special mechanism that produces the sensation of immediacy, coherence, and trust. Due to the imperative need for implicit certainty as an absolute component of our mental practice and because self-evidence is the ultimate criterion for certainty, as beings with a mind of dual nature (conscious and unconscious), we will always continue to elaborate seemingly self-evident representations and interpretations [3].

One of the most challenging problems in this context is related to the role of these unconscious mechanisms in learning, since evidence indicates that they can lead to the unconscious acquisition of knowledge and that these unconscious mechanisms are structurally and functionally more complex than conscious ones, freely influencing learning processes [4]. Therefore, to achieve an adequate understanding of a certain concept, these unconscious learning processes must be brought to light through conscious intention, which involves self-regulated learning processes such as metacognitive ones.

Metacognition has traditionally been conceptualized as one of the three main components of self-regulated learning theory [5]. Generally, metacognition is defined as the act of taking one’s cognitive mechanisms as the object of cognition and involves higher-order reflection processes that require conscious effort. Having well-developed metacognitive skills is associated with more effective learning.

These metacognitive skills are especially relevant regarding some of the unconscious cognitive mechanisms that develop in the learning of certain mathematical concepts at the university level, the so-called *tacit models* [6]. These models are representations of certain abstract notions, which develop themselves at an initial stage of the learning process to facilitate and stimulate the comprehension or resolution of a task and which continue to influence, unconsciously, the cognitive processes of the learner, representing obstacles in the adequate learning of a mathematical concept. The term “tacit” means that the student is not aware of that influence, or at least of the extent of it.

In this context, the stimulated recall methodology [7] in a collaborative learning environment can help students develop metacognitive mechanisms that promote awareness and the overcoming of difficulties created by these unconscious learning mechanisms [8]. Such stimuli are considered to allow them to relive original situations accurately [9], which can be used to explore cognitive aspects behind decisions and actions, for example, reflection processes and self-assessment in different contexts [10].

2 PURPOSE OF THE STUDY

Hence, it is natural to study how primitive unconscious patterns (as *image schemas* [11]) and different semiotic resources associated with them are intertwined and evolve in the students' discourse, giving rise to these models. Although these models have been characterized in the literature [12] as being unconscious, they are very difficult to study directly, and, consequently, there are no studies about their formation processes, and very little is known about the metacognitive and other self-regulated learning processes needed to face the cognitive challenge posed by them.

On the other hand, online collaborative learning implies that students actively participate and engage in cognitive interactions, which could lead to the development of awareness concerning unconscious structures [8]. However, there are no studies that have investigated how these interactions and the metacognitive resources derived from them in an online collaborative learning environment could help students overcome errors caused by these models in their reasoning.

Therefore, to fill the study gap, our goal is to examine the formation processes of these unconscious cognitive mechanisms appearing during the learning process of a given mathematical concept at the university level and how students' interactions and the metacognitive resources as self-regulated learning processes derived from them could help them overcome errors caused by these models in their reasoning in an online collaborative learning environment.

3 METHODS

The implemented methodology was supported by a qualitative approach. The participants were 32 undergraduate students from different majors at the Austral University of Chile. Data collection for the study occurred in two phases: in the first phase, students independently answered three questionnaires related to the study of different features of mathematical infinity; and in the second phase, stimulated recall interviews [6] were conducted in an online collaborative environment provided by Zoom.

The purpose of the initial task was to collect information and data about students' reasoning and other cognitive processes. This data was analyzed qualitatively and provided the framework for registered observations of unconscious patterns associated with the formation of each of the *tacit models* later used in the stimulated recall interviews conducted in the second phase. The Zoom platform supplied a natural online learning environment where students were able to interact and exchange arguments and ideas during the interviews and, at the same time, allowed us to record these interactions for further analysis.

The stimulated recall interview, as a type of introspective qualitative study methodology, involves verbalizing cognitive processes retrospectively [13] and was used in this case to induce students to engage and interact cognitively while reflecting on their reasoning processes revealed by their answers to the questions in the first phase.

The choice of students who participated, in pairs, in these interviews was made after analyzing their answers and considering their richness in terms of differences and variations. Pairs of students who had opposite answers in at least some of the questions were selected, with the intention that they used their explanatory statements to refute and resolve contradictions. Students were presented with written records of their responses on the Zoom screen and were asked to discuss their decision-making processes as they carried out their arguments and explanations. Verbal prompts were used when needed to develop such interactions and trigger, support, or provoke students' conjectures, encouraging them to reflect more deeply about their reasoning processes and enrich their cognitive processes.

Contradictory and opposite ideas were employed to prompt these useful interactions and online live discussion, and semi-structured questioning was used by the researcher to improve students' understanding of the concepts under study.

The data collected during the two phases was analyzed using one approach to qualitative content analysis: inductive content analysis [14], which involves analyzing data without preconceived categories and allowing flexibility to identify emerging patterns and concepts.

Let us highlight that the qualitative content analysis was carried out using two complementary lenses, represented by two phases of the analysis. The first phase constituted a macroanalysis, which revealed the general characteristics of the identified strategies developed by students and their interweaving throughout the discussion, highlighting broader patterns. The second phase, a microanalysis [15], described the process of formation of these *tacit models*, diving into their finest microstructure and showing us, in this case, how *image schemas* appeared and were combined, revealing how any of these combinations could lead to the formation of a given *tacit model*.

Specifically, to perform the macroanalysis of the data recorded during the stimulated recall interviews, the technique of repeatedly listening to the recordings was used to familiarize ourselves with the discussions. The recordings were then transcribed textually, using descriptive coding [16], which allowed us to examine each intervention. This coding was carried out considering phrases or words that could indicate the appearance of some of the *image schemas* already characterized in the literature.

On the other hand, to carry out the microanalysis of the transcribed data, one of the most important aspects considered was the mutual relationship established between different codes associated with the relevant *image schemas* at a specific moment (a synchronous analysis) and the evolution of these relationships over time (a diachronic analysis) [17], giving rise to the formation of these models.

However, it is important to highlight that, although the diachronic and synchronic articulation of the analysis made it possible to uncover the variety of connections and links between the different *image schemas* relevant in the formation processes of these *tacit models*, the results obtained were not susceptible to a cause-and-effect interpretation. Rather, they reflected the complexity and flexibility of the processes being examined, the evolution of these complex relationships, and how reciprocal interactions were structured [15]. In the context of the study of the appearance and evolution of these *image schemas*, it is not possible to find this type of cause-and-effect relationship since it is about the formation and evolution of unconscious cognitive structures, which, therefore, are not governed by the cognitive mechanisms typical of the conscious mind [2].

4 RESULTS AND DISCUSSION

The analysis of the material collected orally and in written form showed a wide range of cognitive processes. In particular, the inductive content analysis revealed how primitive cognitive structures such as *image schemas* and different semiotic resources associated with them were intertwined and evolved in students' discourse, giving rise to *tacit models* in their learning processes.

Let us recall that *image schemas* can be combined in different ways during the development of the cognitive mechanisms that entail conceptual construction processes: they can be merged, modifying the characteristics of the resulting *image schema* (intersection of schemas, in which each one contributes some components to the new *image schema*); can be grouped as a collection of *image schemas* that do not alter, per se, the properties of a certain spatiotemporal relationship or structure, (union of schemas that functions as a joint representation of a particular concept); or they can be grouped conceptually as a union or collection, in the same way than in the previous case, but also adding a sequential dimension [18].

It was found that the *image schema* known as "path/process" [19] is combined with its imperfective aspect [20] to form the *tacit model* known as *divergence* [12], which involves an iterative process ("path/process") that has no end (imperfective aspect). This was evident when students argued, for example, "the result of an infinite sum cannot be finite" because it is an "endless process" or when they affirmed "because the process always continues" or "it can always continue" or "because there is always a distance between them."

The analysis also showed the well-known *image schemas* "origin" and "destination" merged sequentially with the previous one, "path/process," to give rise to the *image schema* known as "origin-path-destination," considered by Lakoff and Nunez [19] as one of the most important used in the construction of mathematical concepts.

The above *image schema* ("origin-path-destination") was also combined with its imperfective aspect (has no final state) to form the *tacit model* known as *inexhaustibility* [12], as a continuous process in motion that has no end. The above could be observed when students argued that a process "never ends," for example, "because you can always continue adding something."

Furthermore, we noted that the "origin-path-destination" was joined with other *image schemas*, giving rise to other *tacit models* relevant in this case. Specifically, the *image schema* "origin-path-destiny" was combined with the *image schemas* "resistance/opposition" and "barrier" [21] to form the *tacit model* known as *undefined* [12], which appeared when students affirmed that "it is not possible to know" ("barrier") if the process ("origin-path-destination") will have an end or not ("resistance/opposition"), or "because it is not known what distances separate them" or "because the distances cannot even be measured," or "because we do not know the magnitude of infinity" or "because when a quantity is infinite, the result cannot be known exactly."

On the other hand, it was observed that the "origin-path-destination" was merged in a different way with the "barrier" schema, forming the *tacit model* known as *unreachable* [12], which becomes evident when students assume that the limit of a process ("origin-path-destination") cannot be reached ("barrier"), which was manifested when they expressed "we can get as close as we want to the limit" but "the limit is never reached" or it is "unattainable."

Another *image schema* that, according to Lakoff and Núñez [19], has great relevance within the cognitive mechanisms that occur in the context of mathematics is the "container," which is formed from the combination of "outside" and "inside" schemas. In this case, it was observed that when the "container" was combined with the *image schema* "barrier," the *tacit model* known as *bounded-finite* [22] was obtained, which assumes that a bounded set ("container") cannot have ("barrier") an infinite number of elements. Similarly, it was shown that by combining the same two *image schemas* differently, the *tacit model* known as *infinite-unbounded* [22] appeared in students' discourse with phrases such as "this infinite process must necessarily produce unbounded sets," assuming an infinite collection ("container") must be non-limited (without "barrier").

Besides, the analysis revealed that the "container" schema, joined with the *image schemas* "part/whole" [18] and "barrier," originated the *tacit model* known as *bounded-unbounded* [22], which assumes that an unbounded set ("container" without "barrier") must have more elements than a bounded one ("part/whole"). It was observed when, for example, students stated that "the perimeter is infinite" because the process produces "an infinite number of triangles" and therefore, the perimeter of an infinite number of triangles must be greater than "the perimeter of a single triangle" or greater than "the perimeter of a finite number of triangles."

Similarly, when this *image schema* known as the "container" was merged with the "part/whole," the *tacit model* known as *inclusion* [12] was produced, which was observed when students argued that a set ("container") must have a greater number of elements than any of its proper subsets ("part/whole") expressed in their discourse when they declared that "in a line, there are more points than in one of its segments" because "it includes the other segments."

Furthermore, it was observed that when the *image schema* known as the "container" was combined with the "scale" and "link/correspondence" schemas [23], the *tacit model* known as *dependency* [12] was obtained, associating ("link/correspondence") the segment as a geometric space ("container") with a numerical distance ("scale"), which was observed in the students' arguments when they affirmed that "there are more points in a longer segment than in a shorter segment." In this case, students were considering the number of points in a segment proportional to the segment's length.

In the same way, it was shown that the mixture of the schema "link/correspondence" with the *image schemas* "part/whole" and "scale" gave rise to the *squeezing* model [12], assuming all infinite sets ("link/correspondence" between "part/whole") have the same size ("scale"). This was observed in the students' responses, which stated, "All infinite sets are equal" because "one cannot speak of infinities greater than others."

On the other hand, the union of the *image schema* called the "container" with the schemas "link/correspondence," "scale," and "surface" [23] triggered the appearance of the *tacit model* known as *point-mark* [24], relating ("link/correspondence") the infinite sets ("container" without "barriers") of numbers ("scale") with geometric places ("surfaces"), which was shown in the students' explanations when they considered that a "segment" is made up of "points."

It was also observed that the combination of the schema "container" with "link/correspondence" and with that of "barrier" formed a *tacit model* known as *squeezing*, that is, the difficulty ("barrier") of establishing relationships ("link/correspondence") between various representation systems ("container"). This was declared in students' arguments when they pointed out that in the outline of an ellipse, there are more points than in the outline of a square, giving different explanations that tried to justify why an infinite set would have a cardinal that is less than the cardinal of another infinite set.

The qualitative content analysis carried out allowed us to closely examine the unconscious cognitive mechanisms that were responsible for the formation of these *tacit models* in students' learning processes that were revealed by the written record of students' arguments and explanations. Furthermore, these models were also exposed by students' interactions recorded and generated by their exchange of arguments during the stimulated recall interviews.

In summary, the analysis showed the dynamics by which *image schemas* were combined and integrated in different ways to form each of these *tacit models*, as can be seen in Figure 1.

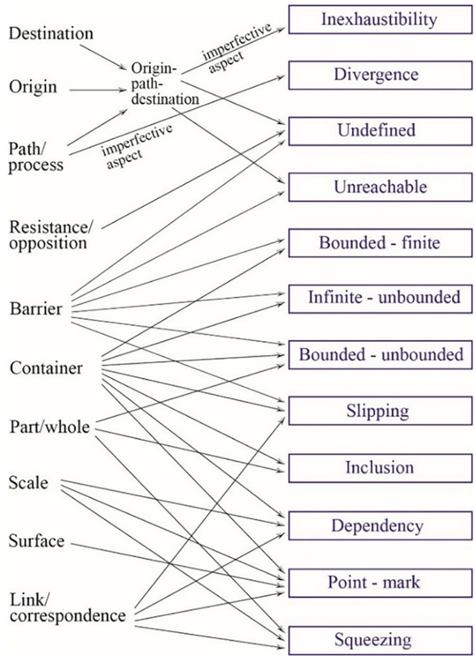

Fig. 1. *Image schemas* in the formation of *tacit models*

Although the combination of *image schemas* formed unconsciously does not always lead to erroneous or misleading patterns of reasoning, in this case, the origin of these erroneous structures, called *tacit models,* is found in these combinations. More precisely, the analysis showed that the origin of a *tacit model* can be an *image schema* that is formed through a direct experience of the physical world or an *image schema* that has been formed through a combination of two or more of these schemas. Table 1 lists the erroneous or inadequate patterns of reasoning or misconceptions represented by each of these *tacit models* found in students' discourse.

In the second phase of our study, during the stimulated recall interviews, it was noted that all the teams formed by pairs of students made use of several of the models that we have already discussed at length. The *tacit models* known as *divergence*, *point-mark*, and *dependency* appeared with special persistence in the discourse of students, and the rich interactions developed between them did not manage to change these erroneous models of reasoning, even though these interactions managed to draw some attention to these models by making them conscious, at least to a certain extent.

Table 1. Tacit models found in students' discourse

Tacit Model	Misconception
Undefined	An infinite sum cannot be calculated due to the undefined number of terms
Divergence	The result of the infinite sum of finite quantities cannot be finite
Unreachable	The limit is a value that cannot be reached
Inexhaustibility	An infinite sum cannot be calculated because it is always possible to continue the process of adding terms
Point-mark	Points can be identified with marks in the geometric line
Dependency	Numerical distances can be identified with segments viewed as geometric spaces

(Continued)

Table 1. Tacit models found in students’ discourse *(Continued)*

Tacit Model	Misconception
Squeezing	All infinite sets are of the same size
Inclusion	A part of an infinite set must be smaller than the whole of the set
Slipping	There is no one-to-one correspondence between an infinite set and its proper subset
Infinite-unbounded	An infinite sequence must be unbounded
Bounded-finite	A bounded set must have a finite number of elements
Bounded-unbounded	An unbounded set must have more elements than a bounded set

An example of this phenomenon occurred when a pair of students discussed the bijective correspondence between elements of two sets concerning the *tacit model* known as *squeezing*, arguing that maybe such correspondence could be made. Although they didn’t find a way to do it, they were clearly on the right path to overcome the obstacle posed by this model, and we could assume a higher level of awareness regarding this model was achieved by them.

On the other hand, we noticed that some pairs of students managed to overcome some of these *tacit models* in some way paradoxically by combining or contrasting them with another erroneous *tacit model* based on the common *image schemas* by which they were formed. For example, some students managed to overcome the *squeezing* model and concluded that there are infinities larger than others by intertwining this *tacit model* with that of *inclusion* through their debate and discussions about the common *image schemas* that make them up (see Figure 1), that is, the “part/whole” and “link/correspondence” *image schemas*.

It is important to highlight that this intertwining of *tacit models* occurred throughout the entire study, through the *image schemas* that were associated with each other, thus weaving the fabric of *tacit models* appearing in students’ discourse and its evolution over time during their interactions in the online learning environment selected for the study.

Sometimes students were able to get the correct answer prompted by their peers, but through the incorrect reasoning hidden in these erroneous assumptions given by these *tacit models*. For example, one student stated, “All these curves have an infinite quantity of points that cannot be counted, regardless of how they are distributed. That is, I would say that they all have the same number of infinite points.” His last sentence is the right answer, but in this case, he arrived at this conclusion by implicitly implying that these curves had an infinite number of points that cannot be counted; therefore, he simplified his reasoning by assuming that if he cannot find a difference between these infinities, they must be equal, which is a mistaken deduction.

We also noticed that the same *image schemas* that had been present in the formation processes of these *tacit models* at the beginning of these interviews were, on many occasions, combined in a new way through student interactions, giving rise to adequate cognitive structures, which led to a proper understanding of the concepts under study.

To summarize all the above, Table 2 shows the *tacit models* present in students’ written discourses, the ones that were overcome through cognitive and metacognitive interactions during the stimulated recall interviews, and the *image schemas* that were involved in those cognitive processes for each of the 16 teams formed by pairs of students that participated in the study.

Table 2. Tacit models overcome through interactions during the stimulated recall interviews

Team No.	Tacit Models found in Students' Discourse	Tacit Models Overcome through Interactions	Image Schemas Involved
1	Inexhaustibility, dependency, divergence, point-mark, unreachable, slipping	Unreachable, slipping	Origin-path-destination, path-process, container, scale, surface, barrier, link-correspondence
2	Inexhaustibility, dependency, point-mark, unreachable, inclusion	Inexhaustibility, unreachable, inclusion	Origin-path-destination, part-whole, container, scale, surface, barrier, link-correspondence
3	Inexhaustibility, dependency, divergence, point-mark	Inexhaustibility	Origin-path-destination, container, scale, surface, link-correspondence, path-process
4	Inexhaustibility, dependency, point-mark, unreachable, squeezing	Squeezing	Origin-path-destination, path-process, container, scale, surface, barrier, link-correspondence
5	Inexhaustibility, dependency, divergence, inclusion, undefined, infinite-unbounded, bounded-unbounded	Inclusion, undefined, infinite-unbounded, bounded-unbounded	Origin-path-destination, path-process, container, scale, link-correspondence, barrier, part-whole, resistance-opposition
6	Inexhaustibility, dependency, divergence, point-mark, bounded-finite	Inexhaustibility, divergence, bounded-finite	Origin-path-destination, path-process, container, scale, link-correspondence, barrier, surface
7	Inexhaustibility, dependency, divergence, point-mark, unreachable, inclusion	Inexhaustibility, unreachable, inclusion	Origin-path-destination, path-process, container, scale, link-correspondence, barrier, surface, part-whole
8	Inexhaustibility, dependency, point-mark, unreachable, squeezing	Inexhaustibility, squeezing	Origin-path-destination, container, scale, link-correspondence, barrier, surface, part-whole
9	Inexhaustibility, dependency, divergence, point-mark, unreachable, undefined	Point-mark, unreachable, undefined	Origin-path-destination, path-process, container, scale, link-correspondence, barrier, surface, resistance-opposition
10	Inexhaustibility, dependency, divergence, point-mark, unreachable, squeezing	Unreachable, squeezing	Origin-path-destination, path-process, container, scale, link-correspondence, barrier, surface, part-whole
11	Inexhaustibility, dependency, divergence, point-mark, undefined, bounded-finite	Inexhaustibility, dependency, undefined	Origin-path-destination, path-process, container, scale, link-correspondence, surface, barrier, resistance-opposition
12	Inexhaustibility, dependency, divergence, unreachable, undefined, infinite-unbounded	Unreachable, undefined, infinite-unbounded	Origin-path-destination, path-process, container, scale, link-correspondence, barrier, resistance-opposition
13	Inexhaustibility, dependency, divergence, point-mark, unreachable, slipping	Inexhaustibility, unreachable, slipping	Origin-path-destination, path-process, container, scale, link-correspondence, surface, barrier
14	Inexhaustibility, dependency, divergence, point-mark, unreachable, bounded-finite, bounded-unbounded	Inexhaustibility, unreachable, bounded-finite, bounded-unbounded	Origin-path-destination, path-process, container, scale, link-correspondence, surface, barrier, part-whole
15	Inexhaustibility, dependency, divergence, point-mark, unreachable	Inexhaustibility, unreachable	Origin-path-destination, path-process, container, scale, link-correspondence, surface, barrier
16	Inexhaustibility, dependency, divergence, inclusion, unreachable	Inexhaustibility, inclusion, unreachable	Origin-path-destination, path-process, container, scale, link-correspondence, barrier, part-whole

It was observed that the stimulated recall interviews taking place in an online collaborative environment and involving the verbalization of cognitive processes in a retrospective way by the students provided excellent tools for detecting misconceptions and for reflecting on their reasoning processes and erroneous ways of reasoning, showing, at the same time, how metacognition and cognition complexly interact with one another and how both are interrelated with self-regulated learning [25].

The metacognitive strategies [26] observed in this case, used by each team of students and induced by their cognitive engagement and interactions through the online environment, are presented in Table 3. It can be noted that, in general, all the students recognized contradictory information, acted on feedback from each other, often chunked the task or scaffolded it from easiest to hardest to better understand it, and compared and reflected on their answers.

Table 3. Tacit models found in students' discourse

Team No.	Observed Metacognitive Strategies
1	Recognizing contradictory information, reflective comparisons, acting on feedback from the other, scaffolding from easiest to hardest, brainstorming, thinking of the process
2	Recognizing contradictory information, reflective comparisons, thinking of the process, acting on feedback from the other, scaffolding from easiest to hardest, brainstorming, categorizing
3	Recognizing contradictory information, reflective comparisons, thinking of the process, acting on feedback from the other, chunking the task
4	Recognizing contradictory information, reflective comparisons, thinking of the process, acting on feedback from the other
5	Reflective self-questioning, recognizing contradictory information, reflective comparisons, thinking of the process, acting on feedback from the other, brainstorming, chunking the task
6	Recognizing contradictory information, reflective comparisons, thinking of the process, acting on feedback from the other, chunking the task, scaffolding from easiest to hardest, mental scripting
7	Reflective self-questioning, recognizing contradictory information, reflective comparisons, thinking of the process, acting on feedback from the other, chunking the task, scaffolding from easiest to hardest
8	Reflective self-questioning, recognizing contradictory information, thinking of the process, acting on feedback from the other, reflective comparisons
9	Reflective self-questioning, recognizing contradictory information, thinking of the process, acting on feedback from the other, brainstorming, reflective comparisons, mental scripting
10	Reflective self-questioning, recognizing contradictory information, thinking of the process, acting on feedback from the other, brainstorming
11	Recognizing contradictory information, thinking of the process, acting on feedback from the other, reflective comparisons, scaffolding from easiest to hardest
12	Recognizing contradictory information, reflective comparisons, thinking of the process, acting on feedback from the other, chunking the task, scaffolding from easiest to hardest
13	Recognizing contradictory information, reflective comparisons, thinking of the process, acting on feedback from the other, mental scripting, scaffolding from easiest to hardest
14	Reflective self-questioning, recognizing contradictory information, reflective comparisons, thinking of the process, acting on feedback from the other, brainstorming, chunking the task, scaffolding from easiest to hardest
15	Recognizing contradictory information, thinking of the process, acting on feedback from the other, brainstorming, scaffolding from easiest to hardest
16	Recognizing contradictory information, reflective comparisons, thinking of the process, acting on feedback from the other, brainstorming, categorizing, scaffolding from easiest to hardest

5 CONCLUSION

This study allowed us to examine the formation processes of unconscious cognitive mechanisms appearing during the learning process of a given mathematical concept at the university level and how students' interactions and the metacognitive resources as self-regulated learning processes derived from them could help them overcome errors caused by these models in their reasoning in an online collaborative learning environment.

Through the study of students' cognitive interactions during stimulated recall interviews, some valuable insights were gained into the cognitive processes examined. In particular, the contraposition of contradictory and unconscious ideas was very useful from a cognitive perspective, helping students to understand the abstraction processes that developed from these conflicting cognitive structures. The *image schemas* that were transformed and combined adequately through these interactive and self-regulated cognitive mechanisms in the online collaborative environment led, often, to the overcoming of the cognitive obstacles posed by these *tacit models*. Thus, conversely, the conflicts underlying these unconscious *tacit models* were essential for the emergence of new *image schemas* conducive to a proper understanding of the concepts under study.

This work is also aligned with previous studies that considered the role of conscious versus automatic metacognitive processes. Some have questioned, for example, if metacognition does, by definition, require conscious processing or if metacognitive activities can also appear on a less conscious level. Other researchers are skeptical about the methodology to capture automatic or unconscious metacognitive processes [25]. This issue needs to be addressed by further study and by advancing new measurement methods and techniques where multiple approaches may be combined to capture metacognitive processes during learning in different contexts.

In our particular case, the online learning collaborative environment supported by digital technology proved to be very useful in the development of self-regulated cognitive mechanisms that allowed students to overcome these unconscious patterns and models. It showed that through the active exchange of arguments and similar, opposite, or contradictory ideas and statements, they were able, progressively, to refute and resolve contradictions, often reaching, an adequate understanding of the concepts under study. Thus, within this progression, students developed their mathematical discourse.

This collaborative environment also revealed students' cognitive interactions, through which some of these *tacit models* became conscious. More precisely, the analysis shed light on the *image schemas* involved and the metacognitive strategies through which students examined their unconscious misconceptions, reflecting on previous mistakes and inadequate schemas. These findings confirm results from similar studies that state-stimulated recall data gathered in online collaborative environments is useful in revealing relatively higher-level cognitive and metacognitive processes [12], [27], [28], and [29].

This work also allowed us to get information about the unconscious mathematical structures students are confronted with and the conscious patterns of reasoning they must develop to overcome difficulties and obstacles in the learning processes of mathematical concepts, in a similar way proposed by similar studies [6], [22], and [30].

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