

How do we learn models? Introducing the *supposed range vs. real range* hypothesis

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Abstract—We present a general learning model explaining in more depth how we learn (or fail to learn) models and theories. It specifically addresses the phenomenon of preconceptions introduced by the constructivist pedagogical approach. Nature of cognitive conflicts as well as existence, birth and resolution of preconceptions are clarified through a two-stage model based on the formalization of validity conditions of models and theories. Illustration of our hypotheses is provided by various examples, from limited-range models to wide-range scientific theories. Specific consequences on research are also discussed.

Index Terms—artificial learning, learning model, preconceptions, metamodel, research modelling

I. INTRODUCTION

This paper presents an original learning model characterized by the following elements:

- it focuses on models and theories as the *object* of learning and/or teaching (i.e. as a specific type of knowledge)
- it reproduces and explains the fundamental phenomenon of *preconceptions* (or misconceptions), i.e. the fact that the learning process sometimes fails because a cognitive conflict occurs between the new knowledge to be installed and the previous knowledge already owned by the learner.

In other words, we present a model explaining "how we learn (or fail to learn) models". The self-referring property of the model in this proposal will of course be addressed in the paper.

The paper is structured as follows.

Section II will precise our definitions of the main concepts used in this paper: *models* (their use, nature, scope, relation with learning and knowledge), *metamodels* and *preconceptions*.

The next two sections will detail the two levels of our original learning model, which respectively concern model validity (Section III) and model learning (Section IV).

At last, Section V will discuss the possible application of our conclusions in scientific research.

II. MAIN CONCEPTS AND DEFINITIONS

A. Models and reality

First of all, let us define what we call a "model". In the context of this paper, a model is firstly an object M representing another object R we call "reality" (Fig. 1). The reality and the model are two objects, of material or

cognitive nature, that can each be observed in some way and that are linked by the following relation: to some extent, the model can be used as a substitute (in terms of behaviour) to the reality [3] (see also §II.B).

As an example from the physics domain, a lumped-element schematic circuit of a power supply is a model (M) of this power supply (R). The schematic circuit M is a substitute to the power supply R since it can be used to predict the values of the electrical variables that can be measured on R.

As an example from the education theory domain, in the first half of the XXth century the behaviourist approach exposed the idea that a stimulus/response/reinforcement loop (M) applied by a teacher should result in a learning activity by the student [9]. (The validity of this theory is not of concern here, see also Section III.)

We illustrate this "substitution" capability of a model by an arrow as in Fig. 1.

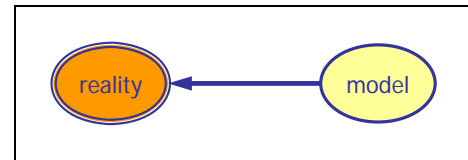


Figure 1. Any model is linked to a "reality".

Since the previous definition is very general, let us now try to precise what we include as models in this paper.

B. Models nature and use (scope of the paper)

Because they reproduce and/or explain (to some extent) reality, models allow us to derive *strategies* to control our environment. This "control" function seems us to be the main purpose why humans build models. (For a deeper discussion about models, control and learning, we refer the reader to [14].)

In this perspective, a first category of models are objects that (in addition to the definition of §II.A) are directly "executable" or "computable", i.e. that automatically produce output values in response to input values. Such a computation is nothing else than a *simulation* of the corresponding reality via the model, an operation that gives as result a prediction of the behaviour of the reality. A lumped-element electrical circuit is an example of such type of models.

We also include as "models" objects that express one or several relations between some sub-elements of a reality. The proposal "the Earth rotates around the Sun" (classically known as the "heliocentric model") is an example of it. Even if they are not directly computable,

such models often allow to derive some predictions about the represented reality's behaviour, so that the control purpose we associated to models remains.

As a last point, we'll consider here that there is no fundamental difference between a model and a theory, a theory simply being a specifically complex, coherent and widely applicable model [16]. (This last property makes already and implicitly appear the interesting concept of "applicability range" of a model, which will be the core of our discussion in Section III.)

Accordingly to the above definitions, models we address in this paper as objects to be learned vary from models with a rather limited range (e.g. electrical schematic circuit of a device, block diagrams of all kinds, equations and formulas, etc) to wide-range scientific theories (e.g. relativity theory, classical or quantum physics, etc).

C. Models multiplicity

From the above definitions, it is obvious that any model has some "properties" in common with the reality it represents. More than "properties", we prefer the term "behavioural aspects" (or more simply "aspects"), stating for an association of output values to input values of an object (a model or a reality) in a –at least supposed– *cause-to-effect* link.

A symmetrical proposal –far less obvious for many people– is that a model doesn't possess *all* the properties of the corresponding reality. This fact, explicitly confirmed for example in [6], is inherent to the definition of a model as a distinct object of the represented reality: if the model had all the properties of the reality, it would be the reality itself.

A second argument can be given in reference with the control purpose of models (§II.B): simulating a model fundamentally consists in obtaining a prediction of the reality's behaviour *with a given benefit* (in terms of delay, complexity, real consequences, etc) over the direct use of the reality. Such a benefit is only possible, in the principle, if the model is different from the reality: using a model has only a meaning if this one is *discordant* with the reality on some aspects.

Our conclusion is that any model is partly identical and partly different (in terms of behaviour) of the reality it represents, which seems to us the very meaning of the relation (the arrow in Fig. 1) between a model and its reality.

As a direct consequence, many different models –each capturing different aspects– may always be associated to a same reality [16]. This principle is illustrated in Fig. 2.

As an example, many different electrical equivalent circuits can be associated to a same real electrical transformer [12]. Similarly, various educational approaches (behaviourist, cognitivist, socio-constructivist, etc), each with its own benefits, have been followed to analyze a teaching situation [9][11].

With the concurrent existence of various models for a same reality arises the question of the validity of each of these models: this is the object of Section III.

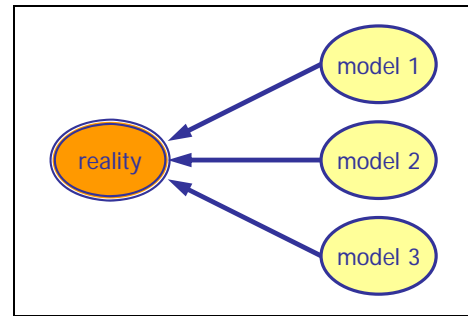


Figure 2. Several models may be linked to a same reality.

D. Models, knowledge and learning

It is important to understand that models, in this paper, will play a twofold role in reference to learning:

- firstly, we'll *expose* one specific and original model to represent (and hopefully to better control) the real actions of learning and teaching: this model will be the *result* of our analysis. It is composed of two subparts respectively presented in Sections III and IV.
- secondly, models in general will be considered as the *object* (more exactly as one possible object) of a learning activity. In other words, models addressed in this paper (including the learning model we expose) are considered as knowledge.

Various arguments support this last idea:

- many examples of models and theories cited in this paper are part of usual physics or educational theory courses
- uncomputable models described in §II.B (a relation between concepts) correspond to the definition of "principles" presented as a specific level of knowledge in some taxonomies of pedagogical contents [2][10]
- these taxonomies also cite "concepts" as another type of knowledge: we'll precisely mention, in §IV.G, that a tight parallel may be made between concept learning and model learning
- Cornuéjols explicitly mentions learning as "building models" [5].

In such a perspective, we'll limit ourselves from now on to models of cognitive nature, i.e. to models that exist (1) as ideas in someone's mind or (2) as an appropriate data structure and content in an artificial system. *Learning* can then be viewed as the fact, for an individual (let's call it a "student"), to gain access to a new model (a new knowledge) he didn't possess before; and *teaching* as the fact, for another individual (let's call it a "teacher"), to transmit (independently of the way this operation is obtained) a given model to a student.

Hence in the next paragraphs (and more specifically in Section IV), we'll particularize the Fig. 2 to a situation where different models of a same reality are distributed among various individuals involved in a teaching relation.

E. Preconceptions

In order to explain why a teaching/learning activity sometimes fails (which is the real situation R we try, as a teacher, to better control), educational theory has proposed the idea of "preconceptions".

In this paper, the term "preconception" states for a knowledge (more specifically here: a model) owned by the student and acting as an obstacle ("epistemological obstacle" [1]) for him to gain access to the teacher's model. Oppositely to other knowledge elements than can be directly articulated by the student to his previous knowledge, in case of preconception the teacher's model enters in conflict ("cognitive conflict") with previous knowledge of the student that however seemed previously efficient to him. To solve this conflict and access higher level knowledge, the student must go throughout a "rupture" ("epistemological rupture") by destroying in part his previous knowledge ("deconstruction"). This rupture often appears as a specifically difficult task.

A very classical example of preconception in physics consists in thinking that material objects, in the absence of applied forces, spontaneously stop their movement. However Newton's law, a very fundamental law in physics, oppositely tells that the spontaneous state of an object (if no force is applied) is a constant speed [7].

As a more local example, we observed repeatedly in an engineering school evaluation that several students modify the correct result of their calculation (turning it wrong) of a RC circuit –one of the simplest electrical circuit– because they implicitly think that "the voltage may not be negative when the source is always positive". This last proposal, which is not correct for the RC circuit, may be viewed as a preconception.

The concept of preconception applies to individual's knowledge as well as to scientific knowledge. On the scientific side, introduction of Einstein's relativity theory or of the heliocentric model (against the geocentric one stating that "the Sun rotates around the Earth") may, among many other examples, be considered as epistemological ruptures. More examples from biology or mechanics can still be found respectively in [1] and [7].

F. Metamodels

Let us finally introduce the notion of metamodel we'll use in further sections. In this paper, we call "metamodel" a model representing... models in what they possess as generic properties. Since this definition is not easy to manage, it can be said that a metamodel is a set of proposals about models in general. The elements we gave in §II.A to §II.D (example: "a model can be used as a substitute to the corresponding reality") are part of our own metamodel.

Figure 3 (to be compared to Fig. 1) illustrates the fact that a metamodel (on the right) is the specific case of a model having as represented object (or reality, on the left): models themselves.

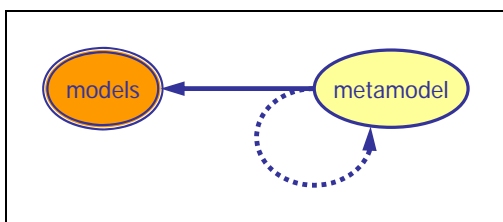


Figure 3. A metamodel is a model representing models (note the dotted reflexive arrow and the "models" appearing as reality on the left).

Compared to other models, a metamodel has the additional property of being reflexive: it refers (or applies) to itself since it belongs to the type of realities it represents (models). This specific property is represented by the dotted arrow in Fig. 3.

G. Structure of the proposed learning model

The learning model we propose throughout this paper is composed of two related subparts or "levels" (Fig. 4):

- the lowest level (noted $M_{MetaLim}$) is a metamodel: it details what is a model (in general) according to our hypotheses. It will be opposed in Section III to another metamodel ($M_{MetaBin}$, §III.A), both of them focusing on model validity.
- the highest level of our model (noted M_{Precon} and presented in Section IV) details the *process* of learning models. Exploiting the metamodel of Section III, it presents a coherent set of hypotheses about how preconceptions exist, appear, generate cognitive conflicts and can be resolved.

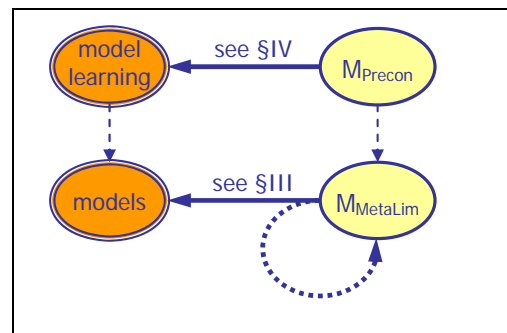


Figure 4. The two-stage structure of the learning model presented in this paper

III. LOW-LEVEL SUBPART: THE LIMITABLE METAMODEL

This section focuses on models validity, which is the keypoint of the low-level subpart of our learning model.

A. The binary metamodel

From our experience, models and theories (knowledge) are often thought to be either "valid" or "invalid", "right" or "wrong". This is specifically true in a teaching context, in which assessments are often based on binary ("right"/"wrong") evaluations. This is also the case in a scientific context where refuted theories are often considered wrong.

Comments we directly collected from colleagues showed us that the behaviourist pedagogical approach, for example, has for many teachers a very clearly "wrong" status [9][11]. Another evidence is the term "misconceptions" used by Espinoza to mention models underlying errors made by students in mechanics [7]: the prefix used ("mis-") clearly indicates that these models are considered wrong. (This last example is specifically interesting since knowledge identified as misconceptions denotes at the same time historical obstacles of the scientific discipline as well as obstacles encountered by today students.)

In this context, learning and teaching appear as the fact to replace a "false" knowledge (owned by the student) by

a "true" knowledge (initially owned by the teacher). Such a vision seems to us very common and refers to a validity status of a model that is (1) intrinsic to the model itself and (2) of binary nature (the model is either valid or invalid). A third property, based on the idea that if two models are different they can not be simultaneously right, is that (3) for a same reality, only one valid model may exist at a time. These three properties about model validity define what we call the "binary metamodel" ($M_{MetaBin}$).

Figure 5 illustrates this way of thinking: even if different models of a same reality may be formulated (e.g.: "the Earth is flat" and "the Earth is spherical"), only one is valid.

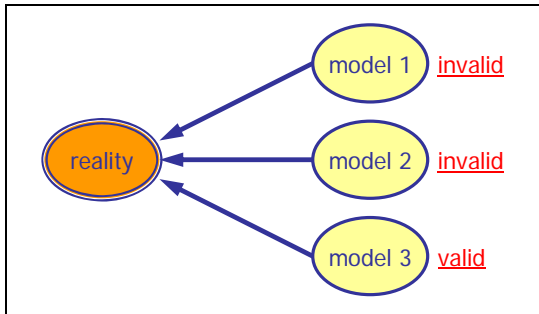


Figure 5. In the binary metamodel, a model is valid or invalid

B. Limitations of the binary metamodel

It is easy to illustrate that the binary metamodel is rather poor to describe validity aspects of models and theories we use in reality.

As a very simple example, anyone can experiment that most of our daily actions can be very efficiently performed considering that "Earth is flat" (which is a model according to our definition of §II.B). Hence in many circumstances, this model can be thought to be *valid*, at least in its function of reality control.

Similarly, the idea that the spontaneous state of a material object is constant speed (Newton's law, see §II.E) doesn't often match our daily experience, unless we live in space.

As a third example from a previous work [12], we compared for a same real electrical transformer very complex models (finite element method, about 10000 equations) with far more simple ones (lumped element circuits, 10 equations). Predicted electrical values for both types of models were identical (within 1%) on main electrical aspects, so that both of these models can be thought to be equally valid on these aspects.

These simple examples show that a strictly binary metamodel is not able to capture in its whole complexity the reality of model validity. Many people will agree with this conclusion and will argue that they possess more complex metamodels (i.e. proposals about models validity) than the strictly binary one. However the references we give in §III.A and throughout the paper also force to admit that many teaching, learning and research activities are implicitly based on this binary metamodel: who would admit to say that "Earth is flat" is right? (see end of §IV.F for an answer).

C. The limitable metamodel

To clarify this, we formalize a second validity metamodel. We call this one the "limitable metamodel" (for a reason that will appear later) and note it $M_{MetaLim}$.

To any model M (and referring to a reality R), $M_{MetaLim}$ associates a *validity range* describing, by definition, which behavioural aspects of the reality R are identically reproduced by the model M.

For our flat earth model for example, this validity range is the surface in which you do not measure a difference (to a given precision) between the real Earth and a flat Earth model. Hence the validity range is the domain in which the model may be used to predict the reality's behaviour.

The validity range hypothesis, which is our key hypothesis for the lower level of our learning model, may seem obvious. The examples and citations we gave previously in this section show it is not the case. Hence let us now analyze what differs between the two metamodels.

The limitable metamodel firstly opposes to the binary metamodel in the fact that the notion of validity appears to be related, via the model range, to specific aspects we want to reproduce at a given moment, to a given precision and for a given reality. Validity of a model relates then to the *use* of this model as a substitute to the reality *in specific circumstances* (instead of being related to the model itself, see property (1) in §III.A).

Secondly, the binary validity (property (2) in §III.A) is replaced by a multi-valued variable (range): an infinity of positions exist between the "pure validity" and the "pure invalidity" of the binary metamodel. In the limitable metamodel, saying that "Earth is flat" is "valid" or "invalid" has in fact no meaning anymore: since any model share some properties with the represented reality, it is efficient and can be said to be valid; but oppositely since any model do not share all aspects of the represented reality (§II.C), it could also be said to be invalid. In this last vision, however, finding a valid model is impossible unless the model is the reality itself.

This last property allows the association of several "valid" (efficient) models to a same reality (which is the opposite of the property (3) in §III.A). It is coherent with the idea that "Earth is flat" and "Earth is spherical" are two fundamentally efficient models, the choice to be made depending on the specific circumstances of the targeted substitution operation. This situation is the one illustrated in Fig. 6 (to be compared to Fig. 5).

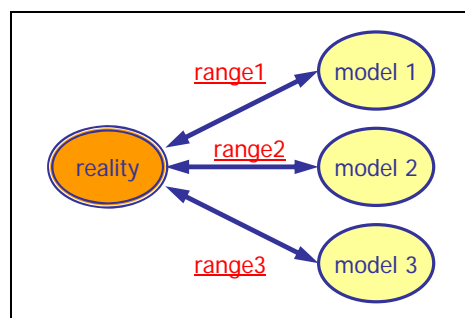


Figure 6. In the limitable metamodel, each model has a validity range (validity or invalidity do not exist anymore)

In that vision, learning consists in gathering a library of models as vast as possible and to associate to each of them

the most appropriate validity range. This is very different of the learning vision (§III.A) associated to the binary metamodel.

D. Is the binary metamodel "right"?

Since it opens the way to multiple valid representations, adopting the limitable metamodel has very interesting implications in learning and research, as we'll see in next sections.

One of the funniest and very coherent one is the fact that this adoption does not invalidate the previous, binary metamodel (since invalidity does simply not exist anymore in $M_{MetaLim}$): according to the limitable metamodel, the binary metamodel may coexist with the limitable metamodel but as a poorer representation of the real world (or in our vocabulary as a metamodel with a smaller validity range). This is specifically true if we try to model preconceptions, as we'll see in Section IV.

The limitable metamodel is moreover reinforced by a paradox arising in the binary metamodel itself: if a model may only be either valid or invalid and since the binary metamodel doesn't capture all the aspects of real models validity (§III.B), the binary metamodel, according to itself, concludes to its own invalidity. Hence the fact that a model is either valid or invalid is... invalid (in the binary metamodel). This paradox may be solved by using the limitable metamodel, as explained above.

IV. HIGH-LEVEL SUBPART: THE PRECONCEPTION MODEL

A. Considered situation

In this section, we expose how the limitable metamodel $M_{MetaLim}$ may help to explain preconceptions in learning.

In that purpose, we'll consider a typical teaching situation with two individuals: a "teacher" and a "student" (these names are mostly given to facilitate the explanation). As stated in §II.C and §II.D, in the more general situation, each of them may have one or several models about a same reality R. For the sake of simplicity (situations with more models may be explored), we'll consider that the teacher owns (i.e. knows) two models noted M1 and M2 (M2 being more powerful than M1 to control reality). The student owns the same model M1 as the teacher, but doesn't know M2.

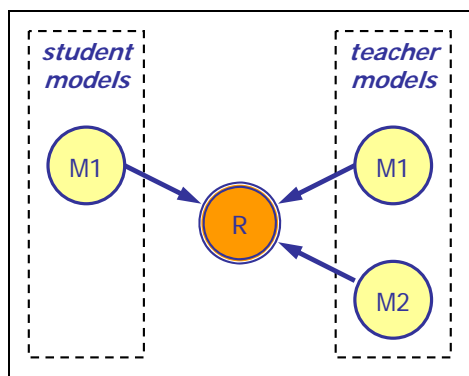


Figure 7. A typical teaching situation

B. Binary metamodel vs limitable metamodel analysis

Let us now discuss the validity of each of these models. As we'll see, the result of the analysis fully depends on the metamodel we choose to perform it.

Following the binary metamodel, only one model (independently of the individuals) is "right": this is the case of M2 in our example, M1 being "wrong" (see Fig. 8). If we suppose that the teacher acts according to the binary metamodel, he should then systematically reinforce M1 as wrong when used by the student. In consequence the student should be very rapidly discouraged of using M1 in response to answers about R from the teacher.

This representation, in addition to the weaknesses already pointed out in §III.B, doesn't seem to be able to reproduce the various typical elements of preconceptions (cognitive conflict, epistemological rupture, etc).

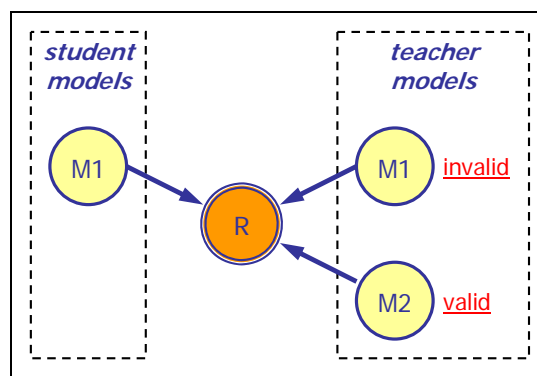


Figure 8. The considered situation analyzed through the binary metamodel

Following the limitable metamodel, the analysis significantly differs. In this case, the binary validity status disappears and is replaced by the notion of validity range. Hence we have to associate such a range to any of the above models.

One more element must be introduced here: a distinction has to be made between the "real range" (the validity range as defined in §III.C, i.e. a set of behaviours that the model and the reality share in common, independently of human or artificial individuals owning these models) and the "supposed range", i.e. the validity range that an individual owning a model associates to this model. In the general case, the supposed range (individual's representation about the model validity range) may differ from the real range. Moreover, it can be partly of fully unconscious for his owner.

In Fig. 9, the validity range associated by the student to M1 is noted S1 ("supposed range"). To explain preconceptions, we may take the validity ranges owned by the teacher as references ("real ranges"), hence we'll respectively note R1 and R2 the ranges associated by the teacher to M1 and M2 (R2 being wider than R1).

(In a more general situation, we should also consider supposed ranges by the teachers, real ranges existing independently of both individuals. This should model, for example, that a teacher may be "wrong" –according to a binary metamodel–, a situation which does certainly exist in the real life. However this more general case is not mandatory to explain the principle of preconceptions by the student.)

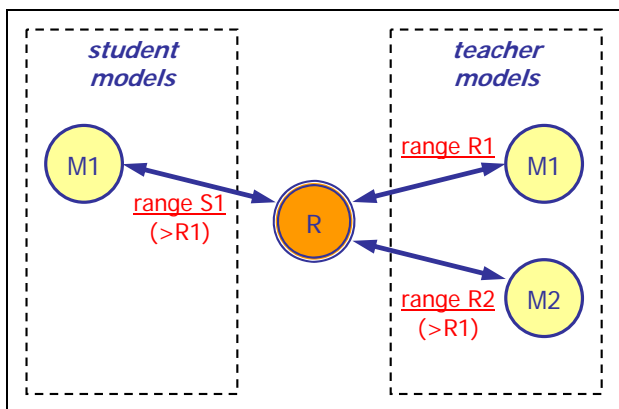


Figure 9. The considered situation analyzed through the limitable metamodel

C. Nature of preconceptions

Here comes our main hypothesis about preconceptions: the nature of a preconception consists in the fact that, for a same model M (M1 here) referring to a same reality R, the *student validity range* ("supposed range") is wider, on some behavioural aspects, than the *teacher validity range* ("real range"). In other words, a preconception simply consists in the fact that the student has a too wide idea of the applicability conditions of the model he already owns (compared to the teacher's vision taken as reference, see Fig. 9).

We will now show that this hypothesis, although very simple, appears to be powerful to explain preconceptions associated to the process of *learning* models and theories.

D. Cognitive conflict and epistemological obstacle

Let us first explain how this hypothesis well reproduces the idea of cognitive conflict by the student. As we'll see, we must consider a teacher acting accordingly to a binary metamodel.

Suppose the teacher questions the student about R. Two situations may be distinguished:

- the question only concerns behavioural aspects of R that are inside the R1 range
- the question concerns behavioural aspects of R that are outside R1 (but inside S1 and, for the sake of simplicity, inside R2)

In the first case, the M1 model appears to be fully operational: by definition of the real validity range, M1 correctly predicts the behaviour of R inside R1. Moreover, M1 and M2 may not be distinguished, as long as they are used inside the R1 range to predict the behaviour of R. Hence, although the teacher uses M2 (his "right" model about R) and the student uses M1, the answers given by the student according to M1 will meet the teacher's expectations.

As a consequence, the teacher won't detect that the student uses M1, hence will (unwillingly) reinforce, by its positive evaluation, the student in his idea of using M1 in relation to R.

Oppositely, suppose now the teacher questions the student concerning aspects of R that are outside R1: the student will continue to use M1 (since the aspects are inside S1) and the teacher will again use M2. This time,

the answers of the student and of the teacher will differ, hence the student's answer will be evaluated as wrong by the teacher.

In this situation, the student will see the validity of his unique model, previously "confirmed" by the teacher, suddenly turn to "right" from "wrong". This moment will certainly be very disturbing for the student because of the uncoherent message delivered by the teacher about M1. We interpret this moment as a *cognitive conflict* in student's mind, which gives to this concept a very formalized explanation.

Moreover we interpret the fact that the M1 model always was previously evaluated as right, as an obstacle, for the student, to access another model about the same reality. This obstacle well matches, in our vision, the idea of *epistemological obstacle* often cited about preconceptions. However the valid status of M1 acts as an obstacle only in the binary metamodel.

As a conclusion, we may note that the cognitive conflict and the epistemological obstacle only exist if the student and the teacher both think to model validity in terms of the binary metamodel.

E. Origin of the preconception: implicit generalization process

A complementary hypothesis about the origin of preconceptions well matches the previous ones: the supposed validity range is too wide because it is the result, by the student, of an "inappropriate implicit generalization".

This hypothesis consists in saying that when a teacher presents or uses a model, he focuses in many cases on the model itself, without discussing clearly its validity range. (From our experience, the model and the reality are even often not distinguished at all, the model being presented as the reality itself). Since the limits of the validity range are defined by discrepancies which are not shown or discussed, it is logical to think that the student implicitly associates a too wide (supposed) validity range to the presented model. Since this range is inappropriate and its association implicit, we call this process "inappropriate implicit generalization".

This process also corresponds to the situation where any individual, performing a set of practical experiments within an unconsciously restricted range, always found a good agreement between the model and the represented reality. The experiments only showed that the model is efficient within the tested range but since all experiments were positive and the range was unconscious, the individual may think that the model is always valid. Hence he associates a too wide supposed validity range compared to the model's real range.

As an example, we think that Einstein's relativity theory or quantum physics (M2) are difficult to access because classical physics (M1) is "valid" in all the daily life experiments we perform. Since discrepancies between classical physics and these theories concern domains far beyond our daily experience, we implicitly think (and this seems confirmed everyday by our interaction with our environment) that classical physics is "always" valid. This results in a cognitive conflict (see §IV.D) when Einstein's relativity or quantum physics are presented to us for the first time.

The example of Newton's law unmatching our daily life experience (§II.E) may be explained on the same basis.

F. Resolving the preconception: model range limitation

Based on the previous elements, resolving the preconception in itself should simply consist, for the student, in rewriting (and more specifically limiting or reducing) its supposed range.

When the discordance between the supposed range and the real range is discussed explicitly, there is in general no difficulty for the student to perform this step (as we have tested it). This in turn allows the student to access the higher level of knowledge of the teacher (M2), which often then simply appears as a superset of the previous knowledge (M1).

The main difficulty related to preconceptions appears to be the fact that this discordance has firstly to be identified between the teacher and the student representations. This identification may be made difficult by the fact that the teacher representations (models, metamodels and ranges he owns) may be partly unconscious for the teacher itself.

In case of a detected preconception, two levels of intervention may be considered for the teacher. A first, local level consists in making the student unconsciously limit his supposed range of a *given* model M1, for example by confronting him with a discrepancy of his model with the reality within his supposed range (but outside the real range of the concerned model). This should make the student access more easily to M2, without requesting that he integrates the limitable metamodel.

A second, global level of intervention consists in giving him conscious access to the limitable metamodel in itself, in which cognitive conflicts do not exist anymore (since they are explained and can be explicitly solved by limiting the supposed range). This can be initiated, for example, by exposing the limits of the binary metamodel, as we did in §III.B.

In any of these cases, the process to follow for the student is twofold: limiting the previous model (M1) supposed range and develop a new model (M2), with its own supposed range. The limitation operation, explicitly appearing in the M_{MetaLim} metamodel, is precisely the way the student may integrate both levels of knowledge without contradiction: Earth may not be flat and spherical at the same time, up to the moment we realize that a very limited area of a sphere may be viewed as being flat.

Note that by limiting the supposed range of his previous model, the student must "deconstruct" in part his previous knowledge (as identified previously about preconceptions). It appears however that this part is the validity range of the model, not the model itself.

G. A parallel with concept learning

Besides models, other types of knowledge do exist, among which *concepts* [2][10]. It can be shown that concepts, to some extent, may be structured as an *association* between a label and a set of properties, exactly as a model is *associated* to a validity range [13].

This formalization makes appear a very tight link between those two types of knowledge, by the fact that the high level subpart of our model (M_{Precon}) also applies to preconceptions associated to *concepts*. This introduces an original and fundamental relation between concepts and

models (as knowledge and learning objects) that we think worth being mentioned and explored further.

V. APPLICATION TO SCIENTIFIC INVENTION

This section discusses consequences of the M_{MetaLim} and M_{Precon} models when specifically applied to scientific theories.

A. A parallel between learning and invention

The previous sections detailed a two-level model that facilitates individual learning: knowing that a cognitive conflict may (sometimes) be solved by limiting the validity range of his own previous knowledge appears to us as a very explicit and powerful learning strategy (which can be classified in the domain of metacognitive knowledge [2]).

Several authors consider that a parallel exists between individual learning and scientific invention. Among various examples, we already cited Espinoza [7], or the fact that "epistemological ruptures" may be identified in learning as well as in scientific research [1]. Clement explicitly cites: "if research can be viewed as creating *public knowledge*, learning can be viewed as creating *private knowledge* [4]. Moreover, these two processes may be very explicitly shown as being two variants of a unique "knowledge creation" process [14].

One question then arises: are our previous conclusions, developed in a context of learning or teaching, applicable in research? Would it be possible to consciously "limit" scientific theories (as we previously "limited" student models) to access higher level knowledge?

B. Supposed range limitation as an explicit research strategy

We think that many scientists act according to a binary metamodel, or more often to an unformalized mix between the binary and the limitable metamodels.

Very often indeed, science is presented as based on the fact that a model (or theory) is considered valid up to the moment an experimental discordance occurs with this model, which then has to be refuted. As already pointed out, the refutation operation is presented as a rejection of the model itself, which then appears to be "wrong". Examples of this vision may be found, at least in part, for example in [7][9][11][15][16].

Rejecting the model itself in case of experimental discordance reveals an underlying binary metamodel and leads to the loss of this model efficiency (existing inside its real range). This often results to creating various "schools", presented as being not compatible, inside a discipline. The story of educational science, in which introspection, behaviourism, cognitivism, socio-constructivism have succeeded to each other (alternating from "inside" analysis to "outside" analysis) is a typical example of it [9][11].

Alternatively to rejecting a model or theory in case of discordance, we may instead consider, as in a teaching situation, that an experimental discordance should lead us:

- to restrict the (supposed) validity range we associate to the concerned theory
- to try to develop a new theory, with a wider validity range, in order to integrate (i.e. reproduce) the new experimental observation.

The main differences with the binary metamodel approach are that the previous model remains fully "valid" (but its applicability is now better known) and that it is not thought incompatible with the new model to develop.

This vision of the scientific invention process is illustrated in Fig. 10, which is issued from Fig. 9 (learning and teaching) with only minor adaptations:

- the "student" is replaced by the research community, owning models and theories that are the state-of-the-art knowledge. This community interacts with the reality (R in the figure).
- the "teacher" does not exist as a human but represents the future models and theories (with appropriate validity ranges) to be discovered.

In Fig. 10, limitation of the supposed range S1 (relative to state-of-the-art knowledge M1) is seen as an enabling step to access future knowledge (R1, M2 and R2), which is not thought to be incompatible with state-of-the-art theories.

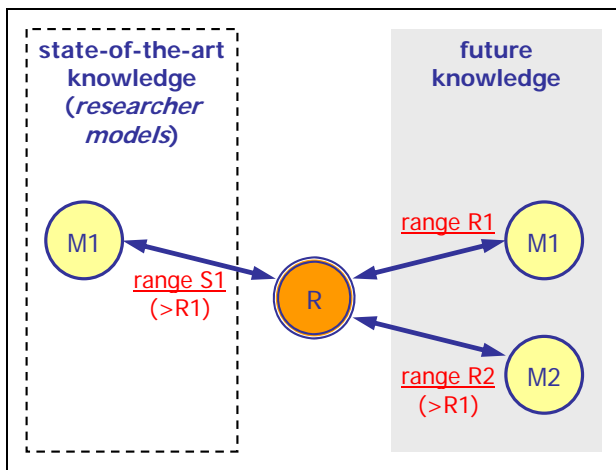


Figure 10. Transposition of the learning/teaching process (according to the limitable metamodel) to research

In some cases, we may even think that the new M2 model could be found simply by combining appropriately limited "concurrent" models which previously existed in a discipline. This idea has been retrieved in Lerot [8] for the field of linguistics. This is also the way we followed to integrate the behaviourist and cognitivist approaches in a unique and powerful pedagogical model [14].

Reviewing past and present models of a discipline in order to explicit their validity ranges (and limit these ranges when requested) then appears to be a powerful way to overcome epistemological obstacles and create new knowledge.

It can be thought as being exactly the same cognitive operation than the one performed by a student to solve a cognitive conflict.

CONCLUSION

We presented a general learning model explaining preconceptions (relative to models and theories) in many of their various aspects. This model relies on a discussion about model themselves that led us to distinguish between

two visions about model validity: the binary metamodel and the limitable metamodel.

The limitable metamodel appears to be more powerful to describe reality, and more specifically allows to derive an explicit strategy (i.e. supposed range limitation) to overcome epistemological obstacles, in individual learning as in scientific invention.

We think that the elements we developed should help students, teachers and researchers: (1) to better understand the validity conditions of their models and theories, (2) to learn more efficiently by providing a clearer view about what preconceptions are, and (3) to better create new and more powerful models and theories by bringing explicit and conscious strategies to resolve preconceptions.

REFERENCES

- [1] J. Ammar, J.P. Astolfi and M. Jeddi, "Les capacités métaboliques du monde microbien à travers l'histoire et les conceptions des étudiants de l'ISBM", AIPU Conference, Monastir, 2006
- [2] L.W. Anderson and D.R. Krathwohl, *A taxonomy for learning, teaching and assessing: a revision of Bloom's taxonomy of educational objectives*, abridged edition, New York: Addison Wesley Longman, 2001
- [3] A. Cleeremans and A. Content, *Introduction à la modélisation*, Université Libre de Bruxelles, 2003
- [4] M. Clement, "La situation de formation des enseignants du supérieur en Flandre", AIPU Belgium conference, 2006
- [5] A. Cornuéjols and L. Miclet, *Apprentissage artificiel: concepts et algorithmes*, Paris: Eyrolles, 2002
- [6] R. Davis, H. Shrobe and P. Szolovits, "What is a knowledge representation?", *AI Magazine*, American Association for Artificial Intelligence, Spring 2003: 17-33
- [7] F. Espinoza, "An analysis of the historical development of ideas about motion and its implications for teaching", *Physics Education*, 2005, 40(2):139-146
- [8] J. Lerot, *Abrégé de linguistique générale*, Louvain-la-Neuve: Cabay, 1983
- [9] F. Raynal and A. Rieunier, *Pédagogie: dictionnaire des concepts clés - apprentissage, formation, psychologie cognitive*, Paris: ESF Editeur, 1997
- [10] A. Rieunier, *Préparer un cours – tome 1: Applications pratiques*, Paris: ESF Editeur, 2000
- [11] A. Rieunier, *Préparer un cours – tome 2: Les stratégies pédagogiques efficaces*, Paris: ESF Editeur, 2001
- [12] F. Robert, *Modélisation et simulation de transformateurs pour alimentations à découpage*, Ph.D. thesis, Université Libre de Bruxelles, 1999
- [13] F. Robert, "Comment se forment et se dépassent les préconceptions: l'hypothèse de la généralisation abusive implicite appliquée aux concepts", AIPU Conference, Monastir, 2006
- [14] F. Robert, "Un modèle d'individu apprenant associant rétroaction double et représentations", AIPU Conference, Monastir, 2006
- [15] J. Tardif, M. Désilets, F. Paradis and G. Lachiver, "Le développement des compétences: cadre conceptuel pour l'enseignement" in J.P. Goulet, *Enseigner au collégial*, AGPC, 1995, pp. 157-168
- [16] J.C. Verhaeghe, J.L. Wolfs, X. Simon and D. Compère, *Pratiquer l'épistémologie*, Brussels: De Boeck, 2004

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