

## Optimization of Velocity Mode in Buslaev Two-Contour Networks Via Competition Resolution Rules

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**Abstract**—In computer networks based on the principle of packet switching, the important transmitting function is to maintain packet queues and suppress congestion. Therefore, the problems of optimal control of the communication networks are relevant. For example, there are  $n$  users, and no more than a demand of one user can be served simultaneously. This paper considers a discrete dynamical system with two contours and two common points of the contours called the *nodes*. There are  $n$  cells and  $l < n$  particles, located in the cells. At any discrete moment the particles of each contour occupy neighboring cells and form a cluster. The nodes divide each contour into two parts of length  $d \leq n/2$  and  $n - d$  (non-symmetrical system). The particles move in accordance with rule of the elementary cellular automaton 240 in the Wolfram classification. Delays in the particle movement are due to that more than one particle cannot move through the node simultaneously. A competition (conflict) occurs when two clusters come to the same node simultaneously. We have proved that the spectrum of velocities contains no more than two values for any fixed  $n$ ,  $l$ , and  $d$ . We have found an optimal rule which minimizes the average velocity of clusters. One of the competition clusters passes through the node first in accordance with a given competition rule. Two competition resolutions rules are introduced. The rules are called input priority and output priority resolution rules. These rules are Markovian, i.e., they take into account only the present state of the system. For each set of parameters  $n$ ,  $d$  and  $l$ , one of these two rules are optimal, i.e., this rule maximizes the average velocity of clusters. These rules are compared with the left-priority resolution rule, which was considered earlier. We have proved that the spectrum of velocities contains no more two values for any fixed  $n$ ,  $l$ , and  $d$ . We have proved that the input priority rule is optimal if  $l \leq n/2$ , and the output priority rule is optimal if  $l > n/2$ .

**Keywords**—Dynamical systems, optimal control, self-organization, mathematical models, information and communication systems, Buslaev networks.

## 1 Introduction

A class of dynamical systems introduced by A.P. Buslaev can be used for modeling the work of information and communication systems. Currently, the main tool for transmitting information is computer networks based on the principle of packet switching. Computer networks are generally a set of communication terminals connected by data channels and switches (network nodes). Connection and data exchange in the network are implemented according to a particular data transfer Protocol, i.e. a set of rules for moving packets. Data packets are received, buffered, processed, and addressed in network nodes and routers.

One of the most important routing functions is to maintain packet queues and suppress congestion. If the rate at which packets are received is higher than the rate at which they are processed, then packets can form a queue. For threads that exceed a certain threshold, the number of delivered packets begins to drop as the number of sent packets increases, and as the load increases further, the number of delivered packets may become zero. This situation is called a network collapse.

Optimal network management under congestion conditions depends on the choice of queue maintenance rules and determines the efficiency of network usage.

Assume that there is a resource in information and communication network, for example, in a packet-switched data network. Suppose that there are  $n$  users, and no more than a demand of one user can be served simultaneously. Assume that the  $i$ -th user sends a demand periodically, and the duration of this demand service is equal to  $l_i$  time units. The duration of time interval from the end of the preceding demand service till the moment, when the user sends the next demand, is equal to  $c_i - l_i$ . Therefore, if there are no delays, then duration of time interval between moments in that the  $i$ th user sends demands is equal to  $c_i$ . However there can be delays due to waiting the resource release, [21, 22]. The problem is to estimate the number of  $i$ th demands such that these demands are served per a time unit. If  $c_i, l_i$  ( $i = 1, \dots, n$ ) are random values, then we have a problem of the queueing theory. If the values  $c_i, l_i$  are constant numbers, then the system is deterministic, and this system can be interpreted as follows. There are  $n$  contours. The length of the  $i$ th contour is equal to  $c_i$ . There is a moving segment, called a cluster, in the  $i$ th contour, and the length of the cluster equals  $l_i$ . There is a unique common point of the contours. This point is called the node. The velocity of any cluster is equal to 1 if there is no delay. A cluster stops if this cluster comes to the node when another cluster is passing the node. If there are several clusters at the node simultaneously, then these clusters cross the node subsequently in accordance with a given discipline.

The present paper studies closed chains of contours such that the rules can be chosen in these systems for resolution of particle competitions, which occur when two particles come to the node simultaneously. The efficiency criterion is the average velocity of the clusters. These systems belong to the class of Buslaev networks. This class was introduced in [1]. Each system of this class contains common points called nodes. A discrete and a continuous version of these systems are considered. In the discrete version, there are cells. Particles are located in the cells. Two types of particle movement are considered. In the first version, at any discrete moment, any particle

moves onto a cell forward if the cell ahead is vacant and does not move if the cell ahead is occupied.

In accordance with the same rule, particles move on a one-dimensional lattice in the simplest traffic model, which was considered in [2], [3], when analytical results were obtained for characteristics of this system. In [2], [3], analytical results are obtained for characteristics of this system. In particular it has been proved that, if the flow density (relation of the particles number to the number of cells) is not more than  $1/2$ , then there is the self-organization, i. e., from a moment, any particle moves without delays. It was noted in [3], that the rule of movement in these models is equivalent to rule of the cellular automaton 184 (ECA 184) in the Wolfram classification [4]. Some generalizations of the model, considered in [2], [3], were studied in [5]– [7]. In particular, stochastic versions of the model were considered.

A traffic model on a toroidal lattice was introduced in [8]. In this model, the particles move in accordance with the rule similar to the rule. In [9]– [11], conditions for the self-organization and collapse (from a moment, no particle moves).

Another type of movement in the contour network was introduced in [12]. This type of movement is called the cluster movement. In discrete version, the particles, located in neighboring cells, form clusters. The particles of the same cluster move simultaneously. In the continuous version of contour network, the clusters are segments moving with constant velocity if there is no delay.

The delays in the cluster (particle) movement are due to that more than one particle cannot cross the node simultaneously.

To obtain analytical results, networks with regular structures periodic structure were considered (in particular, closed and open chains of contours, [13] – [15]), two contour systems, [16]— [19], systems with one common node — the flower, [20].

In [19], it was shown how the spectrum of the system varies depending on the direction of motion. This article explores the effect of competition resolution rules on the spectrum of a system.

Section 2 of this paper considers a two-contour system with two nodes. This system is equivalent to a non-symmetrical closed chain of two contours. Two nodes divide each contour into two arcs. They are the smaller and the greater arc. We compare the efficiency of the left-priority competition resolution rule, the rule in accordance of that the cluster, moving from the greater arc to the smaller arc, (the output priority rule) is chosen, and the rule in accordance of that the cluster, moving from the smaller arc to the greater arc, (the output priority rule) is chosen. We have proved that the input priority rule is optimal in the sense of a given criterion if the length of cluster is not greater than the smaller distance between the nodes, and the input priority rule is optimal in the sense of a given criterion if the length of cluster is greater than the smaller distance between the nodes.

## 2 Description of the System

Let us consider a dynamical system, containing two contours 1 and 2, Fig. 1. There  $n$  cells in each contour, where  $n$  is an even number. At any contour, there is one clus-

ter containing  $l < n$  particles, located in neighboring cells at any discrete moments  $t = 0, 1, 2, \dots$ . The indexes of the cells are  $0, 1, \dots, n - 1$ . The cells  $i - 1, i + 1$  (subtraction and addition by modulo  $n - 1$ ) are neighboring for the cells  $i, i = 0, 1, \dots, n - 1$ . The cluster, located in the contour  $i$ , is called the cluster  $i, i = 1, 2, \dots$ . At any discrete moment, each particle moves onto one cell forward if there is no delay. There are two common points (nodes), common for this contour and two neighboring contours. The node 1 is located between the cells 0, 1 in the contour 1 and between the cells  $d, d + 1, d \leq n/2$ , in the contour 2. The node 2 is located between the cells  $d, d + 1$  in the contour 1, and between the cells 0, 1 in the contour 2.

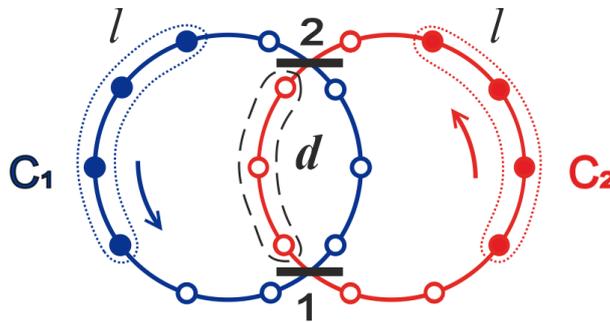


Fig. 1. Two contour system with two nodes,  $n=10, d=3, l=4$

The state of the system at time  $t$  is the vector  $A(t) = (x_1(t), x_2(t))$ ,  $t = 0, 1, 2, \dots$ , where  $x_i(t)$  is the number of the cell, containing the leading particle of the cluster  $i$  at time  $t, t = 0, 1, 2, \dots$ . The state of the system is admissible if no more than one cluster passes through the node (covers the node). The cells of each contour are numbered in the direction of movement.

A cluster stops if this cluster comes to the node when another cluster covers the node. This cluster does not move until the release of the node. If two cluster come two the node simultaneously, then a competition occurs. In this case only the cluster, winning the competition in accordance with the competition resolution rule.

The initial state  $A(0) = (x_1(t), x_2(t))$  is given. This state must be admissible.

### 3 Competition Resolution Rule and Formulation of Problem

This system was considered in [20] under the assumption that the cluster 1 always wins competition (the left-priority rule).

This paper considers the following competition resolution rules:

- The left-priority — Cluster 1 wins the competition (the left-priority rule)
- The priority input rule — The cluster 1 wins at the node 1, the cluster 2 wins at the node 2 (the winning cluster comes from the greater arc to the smaller arc)
- The priority output rule — The cluster 2 wins at the node 1, the cluster 1 wins at the node 2 (the winning cluster comes from the smaller arc to the greater arc).

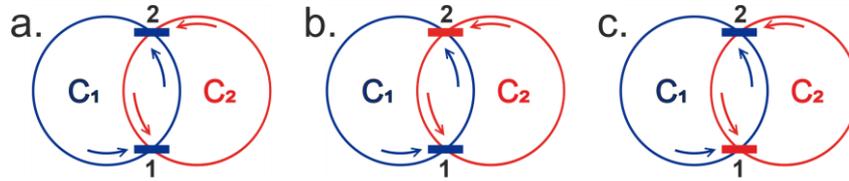


Fig. 2. Competition resolution rules: a). the left-priority; b). the input priority; c). the output priority.

#### 4 Average Velocity of Clusters. Self-Organization. Collapse. Spectrum

Since the set of the system is finite, the moments  $t_0$  and  $t_0 + T$  exist such that  $A(t_0 + T) = A(t_0)$ . Suppose  $t_0 + T$  is the first moment when the system state is repeated. Since the system is deterministic, the state will be repeated over  $T$  states. The sequence of the  $T$  states is called a spectral cycle with the period  $T$ .

Let  $A_i$  be the number of the transitions of the cluster  $i$  during the spectral cycle,  $i = 1, 2$ . The value

$$v_i = \frac{A_i(T)}{T}$$

is called the average velocity of the cluster  $i$ ,  $i = 1, 2$ . The value of the average velocity depends on  $l$ ,  $d$  and the initial state of the system.

Suppose that  $S_i(t)$  is the number of the cluster  $i$  transitions in the time interval  $(0, t)$ . It is evident that

$$v_i = \lim_{T \rightarrow \infty} \frac{S_i(T)}{T}, \quad i = 1, 2.$$

We say that, at time  $t$ , the system is in the state of free movement if the clusters move at any time  $t \geq t_1$ . If the system results in the state of free movement, then  $v_1 = v_2 = 1$ .

We say that, at time  $t$ , the system is in the state of collapse if the clusters do not move at any time  $t \geq t_1$ . If the system results in the state of collapse, then  $v_1 = v_2 = 0$ .

Suppose that the values  $l$ ,  $d$  are given. Then the set of spectral cycles with the values of average velocity is called the spectrum of the system.

Suppose that the values  $l$ ,  $d$  and the initial state of the system are given. We say that the rule 1 is not less efficient than the rule 2, if, for  $i = 1, 2$ , the value  $v_i$  for the rule 1 is not less than the value of  $v_i$  for the rule 2.

Let us consider the class of rules such that choosing the winning cluster does not depend on time and the rule is deterministic. These rules are Markovian, i.e. they take into account only the system state at the present moment. This class contains just four rules. These rules can be represented by the vectors  $(i_1, i_2)$ ,  $i_j = 1$  or  $i_j = 2$  depending on that the cluster 1 or the cluster 2 wins the competition at the node  $j$ ,  $j = 1, 2$ . If,

for given values  $n, l, d$ , there exists a rule in the considered class such that, for any initial state, this rule is not less efficient than any other rule of this class, then this rule is called optimal. Since the process of the chain work is a Markov chain, then the following is true. Suppose, for given values  $n, l, d$ , a rule is optimal in the class, containing the rules (1,1), (1,2), (2,1), (2,2). Then this rule is optimal in the class of rules such that the choosing winning cluster can depend on time or/and be random. Thus is sufficient to consider only the rules (1,1), (1,2), (2,1), (2,2). Since the rules (1,1) and (2,2) are symmetrical, we can consider only the rules (1,1), (1,2), (2,1), i. e. the left-priority rule, the input priority rule, and the output priority rule.

However, due to symmetry, under optimal rule, at any node, whether always the cluster passing from the smaller arc to the greater arc wins competition or the cluster passing from the greater arc to the smaller arc wins competition. Therefore, the rule (1,1) the left priority rule also can be not considered, and we can consider only the prior input rule and the priority output rule.

Only these two rules are symmetrical Markovian rules.

## 5 Behavior of the System and Optimal Control

Considering the system behavior for different relations between the system parameters and different initial states, we have proved the following statements.

1. Suppose

$$(l \leq d) \& \left( (l \leq \frac{n}{2} - d) \vee (l \geq n - 2d) \right).$$

Then the system results in a state of free movement from any initial state for all three competition resolution rules. **Thus, the efficiency of these rules is the same.**

2. Suppose

$$(l > d) \& (l \geq n - 2d).$$

Then the system results in a state of collapse from any initial state for all three competition resolution rules, i.e. **the efficiency of these rules is the same.**

3. Suppose

$$(l \leq d) \& \left( \frac{n}{2} - d < l < n - 2d \right) \tag{1}$$

Then, for the left-priority rule, the input priority rule, and the output priority rule, the system, depending on the initial state, the system results of a state of free movement or a spectral cycle with the average velocity

$$v_1 = v_2 = \frac{n}{2(l + d)}$$

is realized.

If a competition at the node 1 occurs, then the system occurs in a state of free movement in the case of the left-priority rule or the input rule, and a spectral cycle

$$v_1 = v_2 = \frac{n}{2(l+d)}$$

in the case of the output priority.

If a competition at the node 2 occurs, then the system occurs in a state of free movement in the case of output rule, and the system results in a spectral cycle

$$v_1 = v_2 = \frac{n}{2(l+d)}$$

in the case of the left-priority rule or the output priority rule.

**Thus, if (1) holds, then the input priority rule is optimal.**

4. Suppose

$$(l > d) \& \left( l \leq \frac{n}{2} - d \right) \tag{2}$$

Then, for the left-priority rule, the input priority rule, and the output priority rule, the system, depending on the initial state, the system results of a state of free movement or in the state of collapse.

If a competition at the node 1 occurs, then the system occurs in a state of collapse in the case of the left-priority rule or the input priority rule, and the system results in the state of free movement in the case of the output priority rule.

If a competition at the node 2 occurs, then the system occurs in a state of collapse in the case of input priority rule, and the system results in a state of free movement in the case of the left-priority rule or the output priority rule.

**Thus, if (2) holds, then the output priority rule is optimal.**

5. Assume that

$$(l > d) \& \left( \frac{n}{2} - d < l < n - 2d \right) \tag{3}$$

Then, for the left-priority rule, the input priority rule, and the output priority rule, depending on the initial state, the system results of a state of collapse or a spectral cycle with the average velocity

$$v_1 = v_2 = \frac{n}{2(l+d)}$$

is realized.

If a competition at the node 1 occurs, then the system occurs in a state of collapse in the case of the left-priority rule or the input rule, and a spectral cycle

$$v_1 = v_2 = \frac{n}{2(l+d)}$$

in the case of the output priority.

If a competition at the node 2 occurs, then the system occurs in a state of collapse in the case of input priority rule, and the system results in a spectral cycle with the average velocity

$$v_1 = v_2 = \frac{n}{2(l+d)}$$

in the case of the left-priority rule or the output priority rule.

**Thus, if (3) holds, then the output priority rule is optimal.**

Let us prove, for example, the statement 3. Suppose that the condition (1) is true

$$(l \leq d) \& \left( \frac{n}{2} - d < l < n - 2d \right)$$

In the case of the left-priority rule, the statement 3 is proved in [19]. If no competition occurs, a spectral cycle is realized such that this spectral cycle does not depend on the competition resolution rule.

Let a competition occur at the node 1 at time  $t_0$ . If a competition occurs at the node 2, then the behavior of the system is analogous due to symmetry.

For the left-priority, we have the following sequence of system states

$$\begin{aligned} A(t_0) &= (0, d), & A(t_0 + l) &= (l, d), & A(t_0 + d) &= (d, n - d + l), \\ & & A(t_0 + n - d + l) &= (n - d + l, d), \\ & & A(t_0 + n + l - d) &= (n + l - d, 0), \\ & & A(t_0 + n + l) &= (l, d). \end{aligned}$$

Therefore,  $A(t_0 + n + l) = A(t_0 + l)$ . Hence the system is in a state of free movement.

For the output priority rule, we have

$$\begin{aligned} A(t_0) &= (0, d), & A(t_0 + l) &= (0, d + l), & A(t_0 + d + l) &= (d, 2d + l), \\ & & A(t_0 + n - dl) &= (n - d - l, 0), \\ & & A(t_0 + d + 2l) &= (d + l, 0), \\ & & A(t_0 + 2d + 3l) &= (0, d + l). \end{aligned}$$

Hence,  $A(t_0 + 2d + 3l) = A(t_0 + l)$ . We have a spectral cycle with period  $2(d+l)$ . During the spectral cycle,  $n$  transitions of each cluster occur. Then the average velocity equals to

$$v_1 = v_2 = \frac{n}{2(l+d)}.$$

The statement 3 has been proved. The proofs of the statements 1, 2, 4, 5 are analogous.

The domains listed in statements (1) — (5) are shown in figure 3. The domains are highlighted in the color that corresponds to the optimal competition resolution rule. The points *A* and *B* belong to the domain 1.

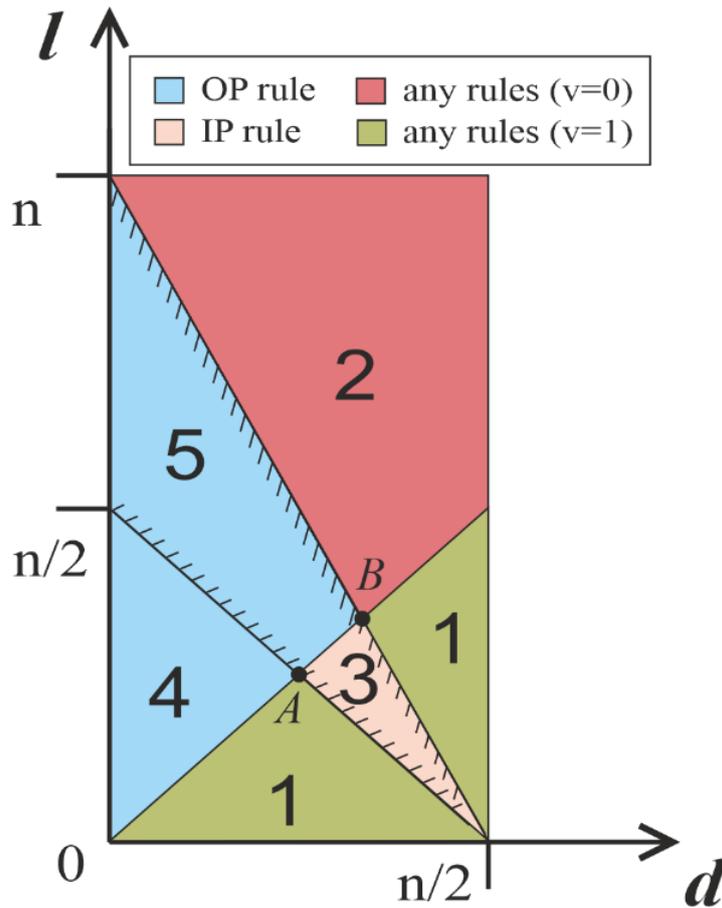


Fig. 3. Behavior of the system: 1)  $v = 1$ ; 2)  $v = 0$ ; 3)  $v = 1 \vee n/(2(d+l))$ ; 4)  $v = 1 \vee 0$ ; 5)  $v = 0 \vee n/(2(d+l))$ .

## 6 Conclusion

A non-symmetrical two contour system with two nodes is considered. There is a cluster of length  $l$  on each contour. The nodes divide every contour into two parts of length  $d \leq n/2$  and  $n - d$ . Main problem is to find the optimal competition resolution rule, which maximizes the average velocities of the clusters. We have proved that, if  $l \leq d$ , then the rule is optimal such that, in accordance with rule, the cluster,

passing from the greater arc to the smaller arc, and, if  $l > d$ , then the cluster, passing from the smaller arc to the greater arc, wins the competition.

It has been proved that **the free movement** occurs **for all initial states** if and only if

$$(l \leq d) \& \left( \left( l \leq \frac{n}{2} - d \right) \vee (l \geq n - 2d) \right)$$

Thus, the condition that  $l \leq n/2$  is necessary but not sufficient condition for the system to result in a state of free movement from any initial state. It follows from results obtained in [19] that, if one of the clusters moves counter-clockwise and the other cluster moves clockwise (co-directional movement), then the condition  $l \leq n/2$  is a necessary and sufficient condition for the system to result in a state of free movement from any initial state.

If  $(l > d) \& (l \geq n - 2d)$ , then the system results in **state of collapse from any initial state** for any competition resolution rule. It follows from the results obtained in [19], then, in the case of the co-directional movement, the necessary and sufficient condition of collapse is the inequality  $l > n - d$ .

If  $(l \leq d) \& \left( \frac{n}{2} - d < l < n - 2d \right)$  and the initial state is  $(x_1(0), x_2(0))$ , **then behavior of the system depends on the initial state** if and only if

$$(x_1(0) - x_2(t) = d) \vee (x_2(0) - x_1(t) = d) \quad (\text{subtraction by modulo } n).$$

If  $(x_1(0) - x_2(0)) = d$ , then  $v = 1$  for input priority rule, and

$$v = n/(2(d + 1))$$

for output priority and left-priority rule. If  $(x_2(0) - x_1(0)) = d$ , then  $v = 1$  for input priority and left-priority rule, and

$$v = n/(2(d + 1))$$

for output priority rule.

Assume that  $(l > d) \& (l \leq n/2 - d)$ . Then, if  $(x_1(0) - x_2(t) = d)$  then  $v = 0$  for input priority rule, and  $v = 1$  for output priority and left-priority rule.

If  $(x_2(0) - x_1(0)) = d$ , then  $v = 0$  for input priority and left-priority rule, and  $v = 1$  for output priority rule. For the other initial states, the average velocity of clusters does not depend on the competition resolution rule.

Suppose  $(l > d) \& (n/2 - d < l \leq n - 2d)$ , then,

$$\text{if } (x_1(0) - x_2(t) = d) \text{ then } v = 0 \text{ for input priority rule, and } v = n/(2(d + 1))$$

for output priority and left-priority rule. If  $(x_2(0) - x_1(0)) = d$  then  $v = 0$  for input priority and left-priority rule, and  $v = n/(2(d + 1))$  for output priority rule. For

the other initial states, the average velocity of clusters does not depend on the competition resolution rule.

## 7 Acknowledgement

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