Advanced Interactive Mathematical Tasks (DLO) for Moodle

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Abstract—In this paper, we will present the reasons for implementing DLO (Digital Learning Objects), DST (Digital Simulation Tools), and other learning models and programs. The focus will be on mathematics in Moodle (LMS-Learning Management System) and on the possibilities of its use. Especially in the construction of new advanced interactive tasks in derivatives and integrals. The main goal is to motivate students and encourage them to overcome their difficulties through an auto-study, giving them more confidence and making students keen on Mathematics. In particular, the idea of doing e-mathematics exercises within Moodle as friendly as possible for all users, lecturers and students – will be presented.

Keywords—eLearning systems, DLO, DST, ICT, e-mathematics, Moodle, LMS

1 Introduction

Information and communication technology (ICT) is increasingly becoming a more significant and more critical part of students' everyday life [20–22, 30–36]. Consequently, the role of e-learning has transformed completely as recent advances in Information Technology (IT) and the advent of Web 2.0 technologies enabled the creation of learning content that is no longer based on textbooks and learning guides [31–34]. The traditional idea of the "classroom" now incorporates the use of both physical and virtual space [48]. A significant trend in universities has been to implement so-called learning management systems (LMS) (Moodle and other web-based learning systems), which are used as a common platform where students and teachers can interact digitally [1, 47, 52].

For that reason, we started with implementing the interactive DLOs into LMS (Moodle) and motivating the students to study different fields of basic mathematics such as analysis and algebra. Namely, the classical way of pedagogical process is changing. We live in the modern world where technology is no longer something new, and this is evolving so fast that we have to follow it or even predict it if we want to keep up with the time. Logistics as a science is an interdisciplinary and multidisciplinary field and is one of the fastest-growing areas.

Therefore, we have to add a modern way of study in the faculty's pedagogical part and thus a more attractive way of studying mathematics. Consequently, we decided on such interactive DLOs with tutorials presented later in the article.

The paper will also present some existing and new ways of using e-Learning for teaching mathematics and some methods used worldwide. The primary purpose is to highlight the new exciting, and dynamic approach to teaching mathematics, mainly the new technique addressing derivatives and integrals supported by Flash software to improve interactivity and promote the automatic assessment of interactive activities developed using e-Learning Modules (e-LM).

We started building advanced interactive tasks mainly because we realized that basic content submission techniques were unfortunately not enough for great results. Even though students gain enough knowledge in lectures and tutorials, they have problems, especially when working independently and not having a teacher to assist them. As a result, students can only help themselves through the process of tasks already solved, with mutual assistance or with the help of an instructor. To this end, we decided to create interactive tasks that would guide the individual from step to step with the help of graphical and theoretical help. As a result, the student can test himself in solving various cases that he can create for himself.

The primary purpose of the assignment is that we have infinite possibilities and, consequently, infinite tasks that the system solves in the background and gives to the student in steps. So the main goal is to motivate students, encourage them to overcome their difficulties through an auto-study, giving them more confidence and making students keen on Mathematics. First, some basic definitions will be given for LMS, DLO, and DST, followed by an example and an algorithm for concrete use.

2   Theoretical framework

2.1   The LMS

The open-source LMS Moodle (http://moodle.org/) can be found in many education segments, and higher education is no exception. Its popularity, except for the fact it is free, is mainly based on its flexibility, adaptability, and the possibility of personalization while, on the other hand, the system contains many standard features which make the learning process easy to implement [1]. The University of Maribor has adopted Moodle as one of its official LMS.

Moodle can be used in multiple roles: as a repository of teaching materials, as a platform for making course announcements and as a discussion forum for student-student and student-instructor interaction. Also, the students could submit assignments, take quizzes, conduct group projects and receive feedback from the instructor.

It is worth mentioning that there is also a mobile version of Moodle, which according to the results of the survey presented in the article [37] is not very popular with users and has quite a few shortcomings. The study of the authors and the results are as follows.

The study investigated how students use Moodle via smart mobile devices to carry out different Moodle activities and the possible reasons behind such usage patterns.

One hundred twenty-two university students filled a survey on self-reported usage in a course offered by the faculty of Preschool Education at the University of Crete. Follow-up interviews were conducted to solicit students' perceptions on mobile access to Moodle and the underlying reasons.

The results show significant differences in students' usage of various Moodle activities via mobile phones. Students' responses also suggest that Moodle is used merely as an electronic document repository and not as a practical learning tool due to the limitations of mobile access on usability and reliability. Students had to use their preferred browsers to access Moodle on their mobile devices.

In general, students in this study did not prefer using their mobile phones to access Moodle due to the limitations of mobile access on usability and reliability. However, most of them used mobile phones to access Moodle when necessary. In terms of Moodle activities, it was found that students preferred carrying out easy and low-stake Moodle tasks on their mobile phones, such as accessing learning materials. The students expressed the need for a more user-friendly mobile access.

2.2 Digital learning objects (DLO)

From time to time, many definitions have been given to DLO. According to the research of Norman & Porter [29], referring to the DLO, they characterize them as:

- A piece of content structured to support learning through the possible inclusion of educational objectives, content, resource, activities, and assessments.
- A piece of content designed to ensure reuse in different educational settings.
- A piece of content can be stored in different digital learning management (LMS) systems or used in different delivery methods.

The DLO or Reusable Learning Objects (RLO) are digital learning activities that teachers can use to introduce Information and Communication Technologies (ICT) in the educational process. According to Billings [6], in his research, DLOs are software programs based on visual contact and can be reused to achieve the teaching goals set by the teacher. He says that they are generally small programs, which, among other things, indicate the teaching objectives that are to be achieved in their description. This definition is completed by the research of Redmond et a1. [41], which characteristically states that the DLO combines both visual and audio information. Through their interactive use by teachers and students, they facilitate the latter to create conceptual connections and achieve learning benefits.

As mentioned in Sotirova's [46] research, their main features are accessibility, reuse, interoperability, and adaptability to different software. Beyond that, some other features mentioned in Sinclair, Joy, Yau, & Hagan [45] are resilience, detectability, analytic ability, and manageability.

Although, as we mentioned earlier, the innovations offered by the Digital Learning Objects in education are treated with distrust by the teachers due to many shortcomings, they are inextricably linked to the evolution of technology. The use of ICT in the educational process [11, 39] research on the possibilities of DLO argues that it introduces not only innovative teaching methods but also promotes collaborative learning [15, 40].

2.3   Simulation programs (DST)

Thompson et al. [49] describe, in their research, digital simulation programs (DST) as programs – software simulating natural phenomena in safe conditions and as realistically as possible. They help learners understand, record, and analyze various phenomena related to the Natural Sciences, think critically about them, repeat the experiment and solve any problems they may encounter [2, 8, 28]. Alessi and Trollip characterize them as simulations that help shape the teaching, the environments in which they are formed, and the interpersonal interactions of the participants, making them a valuable tool for student training and the evaluation of the students and their teachers.

Many students have a negative attitude toward learning Physics. Therefore, effective ICT teaching encourages students to cultivate their positive attitudes towards the subject. Zacharia's [51] research highlights that an educational process occurs when the teaching of Natural Sciences is combined with interactive learning to facilitate understanding of the essence of each phenomenon.

Summarizing the findings of the research, we would say that simulation:

- Helps learners build on their existing knowledge [18].
- Allows students to handle the phenomena' variables and observe the resulting results [51].
- Improves the environment in which the experiment will take place to make it more understandable to learners [17].
- Provides constructive feedback for learners to process [42].
- Students can observe natural events that are not immediately visible because they can be either too big or too small, or too slow or too complicated and dangerous to watch live [4, 20, 50].

3   Use of software (DSO, DST and other e-learning models)

Before presenting some existing and new ways to use e-learning in teaching mathematics and some methods used around the world, we will present an interesting study conducted by the following article (The management of digital learning objects of natural Sciences and digital experiment simulation tools by the teacher.)

In the study, Poultsakis et al. [38] examine how and to what extent teachers handle Digital Learning Objects (DLOs) for the Natural Sciences and whether they use – and to what extent – Digital Simulation Tools (DST) Experiments Natural Sciences in distance teaching to achieve the teaching objectives.

It will also investigate whether they are aware of both DLO and Digital Experiment Simulation Tools and how they use them in the learning process.

In order to obtain the results, they used various methods of analysis with the use of the SPSS program.

Below, the results are reflected according to the groups and sub-groups of questions that arise from their research questions.

For Physics and Science, 37.59% answered that they "sometimes" use the DLO in the learning process, while 35% and 15% of the teachers answered that they use them "often" or "always." Respectively 43.99% answered that they "often" use the DST in

the learning process, while 29.39% and 19.59% use them "always" and "sometimes," respectively.

By recording their views about why they would be interested in engaging with the DLO, they received responses such as: "Through their use, better learning outcomes are achieved us they contribute to learning. They mobilize the active participation of all students (through visualization), which they find attractive, exciting, and ultimately functional as it makes it easier [or them to understand the different concepts pleasantly."

Many more answers emphasized the importance of the DLOs in the educational process, and many also agreed that "it is the future of education." For the DST, the answers that aroused great interest were the following: "It is a simple way for the students to see the results of an experiment and its process without collecting any material, and it also allows the representation of experiments that are not feasible in the classroom."

Answers such as "They help the students understand the subject better" and "they are interesting" were often slightly changed, as the DST's necessity and ease are often stressed in education. As a general comment, we would say that the DLOs have fanatical supporters, fewer in population than those who choose the DST due to convenience and immediacy. In conclusion, what prevails is the intention of the teachers to deal with both the DLOs and the DSTs. Finally, in the results, we observe a significantly more significant difference – 30.6 percentage units between the positive and negative answers for the IBAs.

Let us introduce the factors that lead teachers to use the DLOs and DST by the research Poultsakis et al. [38].

According to the teachers' answers, 36.89% of the respondents agree that the DLOs facilitate the educational process. 47.29% believe that it is a pleasant way of teaching. 42.49% strongly agree that students show more interest in this way of teaching. 43.29% agreed that students are interested in being actively involved in this way of teaching. 40.89% agree that they have noticed an improvement in student performance. 35.29% strongly agree that the DLOs can be used in the context of Interdisciplinary. 51.29% agree that the DLOs meet the teacher's goals, while 45.69% agree that they are easy to use. 32.89% agree that they have various options, while 40.89% agree that it is an alternative way of teaching.

39.29% agree that the DLOs are based on modern teaching methods. Some answers that show the will of these teachers to use the DLO are "Some phenomena are not described only with words or with an image. MOVEMENT is very desirable. Also, the meanings are given much faster. "There is a better achievement of goals and responses by the students," and "They help children with learning difficulties." For the DST, 42.1% of the respondents agree that DST facilitates the educational process. 43.69% completely agree that it is a pleasant way of teaching. 39.89% completely agreed that students show more interest in this way of teaching. 39.89% agreed that students are interested in actively participating in this way of teaching, while 40.69% agreed that they had noticed an improvement in student performance. 32.39% agree that they are adapted to specific age groups, and 36.89% agree that the DLOs can be used in the context of Interdisciplinary. Finally, 44.49% agree that the DLOs meet the goals of the teacher and 35.39% agree that they are easy to use. 31.69% completely agree that

they have a variety of options. 41.49% agree that it is an alternative way of teaching. Finally, 42.99% agree that the DLOs are based on modern teaching methods. Also, answers such as "They work supportively for students with learning difficulties or disabilities," "They save enough teaching time," and "I consider them necessary" emphasize even more the "thirst" of the teachers to use the DST.

4 Use of software (DSO, DST and other e-learning models) in Mathematics

Our work is based on various papers about Moodle and mathematics; it reviews the novelties in the area of LMS (Learning Management System) such as Moodle, summarizes all known concepts of learning mathematics with the help of ICT (Information and communications technology) and tools used to upgrade traditional learning of mathematics. When preparing e-literature, different software is used. Various e-learning materials could be prepared according to e-learning and a particular subject. Because of the complexity of mathematical content, e-learning materials should be supported with animations, simulations, pictures, quizzes, tests, etc. Below we will present which software is generally used around the world.

In their paper, P. A. Rodrigues and others introduced i-Assign, an interactive package for Moodle that allows users to create interactive activities in any Moodle context. The interactivity offered by i-Assign is due to the simplicity and ease of integration of interactive content created using e-LMS. Several experiments have evaluated the impact of introducing interactivity [43].

In their paper, M. Blanco et al. focus mainly on the Moodle quiz modules. This module allows you to create quizzes with different types of questions, and questions are tailored to the specific goals that need to be achieved at different steps in the teaching and learning process. In order to explore how to use these new strategies to develop a large number of quiz questions, they implemented projects funded by the UPC Institute of Educational Sciences. In particular, the goals were to analyze the students' answers and perform an analysis to determine the relevance of the questions [7].

J. Azevedo and others have built and implemented a new and ambitious mathematical project called MatActiva, developed on the Moodle platform, whose main goal was to motivate students. They also want to improve student-teacher communication and create a learning environment that is in line with the real needs of all students [3].

In this article, S. Kotzer and others presented an overview of e-learning environments implemented through the Moodle platform. These environments include interactive activities that combine simulations, short videos, virtual experiments, games, etc. The fundamental mission was to improve interactive learning. The environment was developed in partnership with teachers as an improvement in face-to-face teaching [25].

In this paper, M. Kondratieva and others focus primarily on teaching Euclidean geometry at the university level through interactive mathematical learning environments. This paper mainly describes sequences of learning activities that combine geometric constructions with research, observations and explanations of facts related to triangle geometry. The author concluded that the role of constructions in an interactive environment was necessary for a proper understanding of the subject [24].

In the paper, G. N. Haddif and M. Yerushalmy focused primarily on developing sophisticated computer-based e-tasks designed to function as a dynamic, multi-link interactive (MLR) environment that provides feedback to students. Construction e-tasks require students to take advantage of a technological advantage by constructing examples that meet specific conditions. The e-task is automatically verified and reports on intermediate actions and responses [14].

H. Drier, North Carolina State University, focuses on the unique use of Excel spreadsheets and interactive modelling and simulation of mathematical situations. The flexibility and power of Microsoft Excel spreadsheet software enable teachers to engage in essential math activities and use spreadsheets as an interactive environment in their teaching [12].

In their theoretical study, D. Martinovič and others attempt to explore the potential of a dynamic and interactive mathematics learning environment (DIMLE) in conjunction with the Technology Teaching Content Framework (TPACK). Dynamic and Interactive Learning Mathematical Environments (DIMLE), such as Cabri, GeoGebra, Geometer Sketchpad, Fathom and the like, provide teachers with unique opportunities to broaden their mathematical knowledge and understanding of their students, especially in areas that are both dynamic and otherwise difficult to understand [27].

Kodippili et al.'s contribution were to determine whether online homework using MyMathLab (MML) would lead to more extraordinary academic performance than traditional homework produced by instructors. Twenty-seven students enrolled in the compulsory algebra course participated in this study. This study shows insufficient evidence that students in a group with interactive, computer-generated homework can perform better than students in traditional paper books [23].

In their article, A. Cohen and others introduce the MathDox system developed by themselves. MathDox is a language for interactive XML-based mathematical documents that can be converted into web pages by MathDox Player. These sites are dynamic and interactive, support mathematical rendering, provide easy access to computer algebra systems, and are a convenient mathematical input system. MathDox demonstrates its potential in demonstrating algorithms, testing skills through exercises, or explaining new concepts with dynamic on-screen calculations [10].

A. Shawar et al. say that on the Arab peninsula, software Flash and Java software are mainly used for teaching mathematics. The emphasis is on simulations and other flexible activities, such as quizzes [44].

Iglesias, Carbajo and Sastre state that not only Java but also Javascript is the most helpful software for teaching mathematics in Spain. They also stress the importance of e-learning. In Spain, one of the critical issues of e-learning is also virtual laboratories based on Flash, Java, and Javascript. Using virtual laboratories, knowledge of mathematics can be upgraded with simulation, which makes a mathematical problem more realistic and dynamic. The simulation is intended for numerical and graphical animation. Further review is described in [13].

Like in Spain and Italy, Moodle is the most used e-learning programme, which is also connected with the already mentioned software. Bardelle's opinion about the use of Moodle is further described in [5].

S. Isotani and L. Brandao believe that problems may emerge due to the complexity of mathematics since specific simulation cannot check the results, which we can find

in [16]. For this reason, iGeam was created in Brazil, which can check the simulation results, even in geometry. iGeam compares the written result of the simulation with the right one. Kaufmann and Schmalstieg conducted one survey. In their survey, described in [19], the authors present a fully functional educational AR application for mathematics and geometry education. They implemented flexible methods to support various teacher-student interaction scenarios. They used hybrid hardware setups which allow the use of Construct 3D in today's classrooms and provides a testbed for these scenarios.

Loret et al. also believe that Flash is the most helpful software for creating animation, simulation and testing, while Flash Player and Adobe Flash are used for multimedia like, for example, games. Their opinion is further described in [26].

5 New interactive mathematical tasks (DLO)

According to the literature, it seems very useful to teach mathematics in a Moodle friendly programming language. We decided to develop our mathematical approaches based on the software Flash based on the reviewed literature mainly because we have not found similar interactive tasks in this field anywhere.

As already mentioned, the focus will be on derivatives and integrals. First, we would like to show how an engaging animation could lead the user by drawing graphs of functions to determine the definition area, zeros, critical points, falling and raising intervals, convexity and concavity. Second, we would like to show how animation can lead the user through the process of calculating the area using the definite integral.

The software will verify the results. The user will also be presented with the so-called ahelpa, which will lead him to the correct result. We will also give some examples with the option of choice. In particular, the user will have the possibility to choose the coefficients of polynomials. Thus, the user will be able to see different problems and the difference between them.

First, we will focus on derivatives and integrals.

5.1 Derivative

This section will focus on the derivative of a function of one variable. The aim for the student will be to draw graphs of the concrete function. First, the basic definitions will be given, followed by a few examples with the procedures to solve tasks in the form of animations. The students will be guided by drawing the graphs of functions: the determination of the modelling domain, zeros, stationary points, local extremes, intersection points, intervals of falling and rising, convexity and concavity.

Use of the derivative. This section will present the concrete example of solving derivatives as an animation. As mentioned in the introduction, the aim is to lead the students through the process of drawing graphs by using software tools. The students will have assistance/help solving concrete tasks during the tasks. The procedure is presented below.

Example 1: Use of the derivative

For a given function $f(x) = \dfrac{x^3}{x^2 - 4}$ define:

- Definition area
- Zero points
- Stationary points and local extremes
- Falling and rising intervals
- Inflexion points
- Intervals of convexity and concavity.

Next, the procedure for solving the given an example will be presented.

1. Define the definition area (*Df*) (see Figure 1).

First, the student should define the definition area.

Definition area: $Df = $ ☐ $\mathbb{R} \backslash \{-2, 2\}$

There are many possibilities to write down the result; only one record type will be specified in the computer program. In order to avoid any mistakes, the assistance will be given as follows:

Help: The definition area of a function is a set of all real numbers for which a rule specifies an actual number.

Definition area, record as $Df = \mathbb{R} \backslash \{A, B\}$, whereby A and B are real numbers.

Fig. 1. Inserting the values for the definition area

2. Define the zeros of the given function (see Figure 2).

In the second step students will have to define zeros of the given function, which are:
$x_1 = \boxed{0}$, $x_2 = \boxed{0}$, $x_3 = \boxed{0}$

Help: In order to find zero points, you have to solve the equation $f(x) = 0$. You have to write the solutions as $x_i = $, $i = 1, 2, 3$.
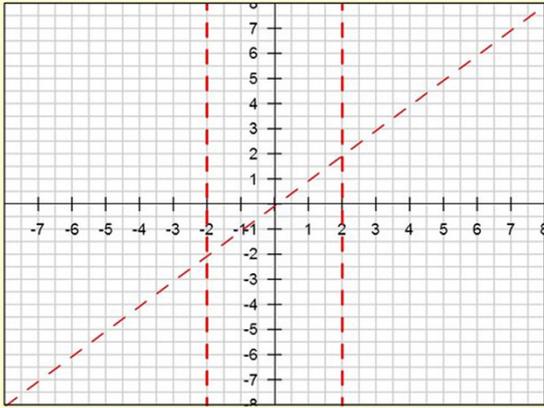
Fig. 2. Plotting the modelling domain (the definition area describes all real numbers, except at the poles) and inserting the values for the zeros

3. Define the stationary points of the function. Specify the *x* and *y* coordinates of the points. Define the maximum, the minimum and/or inflexion point of the function (see Figure 3).

In the next step, the user has to determine the stationary points, which in this case are as follows:

$Sx_1 = -3.46$, $Sx_2 = 0$, $Sx_3 = 3.46$. Hence $Sy_1 = -5.19$, $Sy_2 = 0$ and $Sy_3 = 5.19$. The point $S2 = 0$ is the inflection point.

Help: Stationary point is determined as $f'(x) = 0$. The solution should be written down as a decimal and rounded to 2 decimal numbers. Use a dot-decimal notation. Enter it from left to right depending on the position in the graph. After finding the stationary points, the student will have to define the local extremes. The assistance will be as follows.

Help: If $f''(S) < 0$, then the point is maximum; if $f''(S) > 0$, then the point is minimum, whereby S is the concrete stationary point. If $f''(S) = 0$, the point is an inflexion point.

Fig. 3. Plotting zeros (a green dot represents zero) and specifying the *x* and *y* coordinates of the stationary points

4. Determine the falling or rising intervals of the given function (see Figure 4).

The next task for the student is to determine the falling or rising interval of the given function. The intervals are plotted on the graph using an animation. The results are as follows: the rising interval (–*inf*, –3.46) ∪ (3.46, *inf*), the falling interval (–3.46, –2) ∪ (–2, 2) ∪ (2, 3.46).

Help: If $f'(x) > 0$ for all $x \in (a, b)$, the function is increasing (a, b), whereas if $f'(x) < 0$ for all $x \in (a, b)$, the function is decreasing (a, b). Instead of infinity, you should write inf.

Interval of rising: (–*inf*, –3.46) ∪ (3.46, *inf*),

Interval of falling: (–3.46, –2) ∪ (–2, 2) ∪ (2, 3.46).
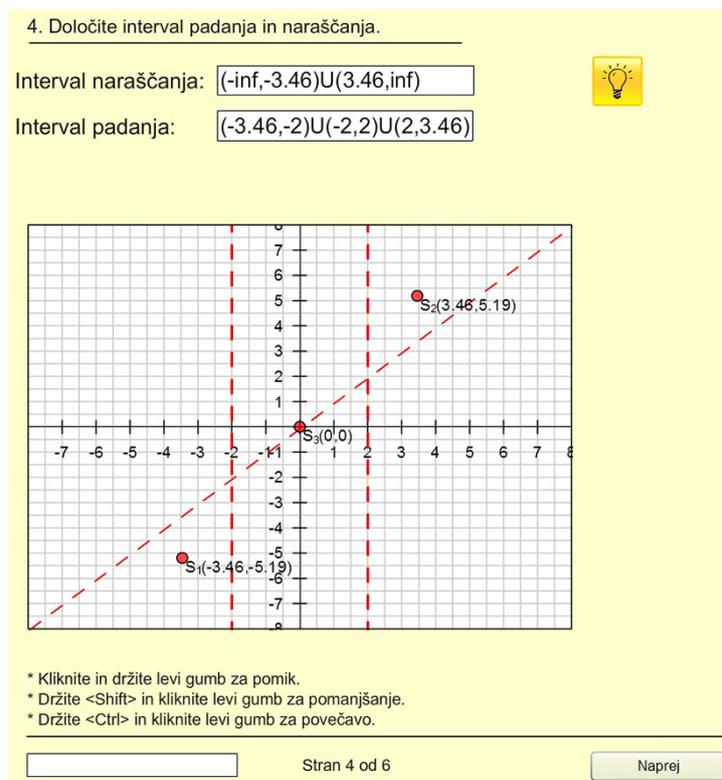
Fig. 4. Plotting the stationary points (the stationary points are presented in red) and determining the falling and rising interval of the given function

5. Determine the intervals of concavity and convexity (see Figure 5).

The next task is to determine the intervals of concavity and convexity of the given function. Upon the entry of the solutions, the graph intervals of convexity and concavity are plotted using the animation. In that case, the solution is as follows: the interval of convexity $(-2, 0) \cup (2, inf)$ and the interval of concavity $(-inf, -2) \cup (0, 2)$. Using the animation, the student will be notified that instead of the symbol for infinity, the word inf is written down.

Help: $f''(x) > 0$ for all $x \in (a, b)$ if and only if the function is convex, $f''(x) < 0$ for all $x \in (a, b)$ if and only if the function is concave. Interval of convexity: $(-2, 0) \cup (2, inf)$, Interval of concavity: $(-inf, -2) \cup (0, 2)$
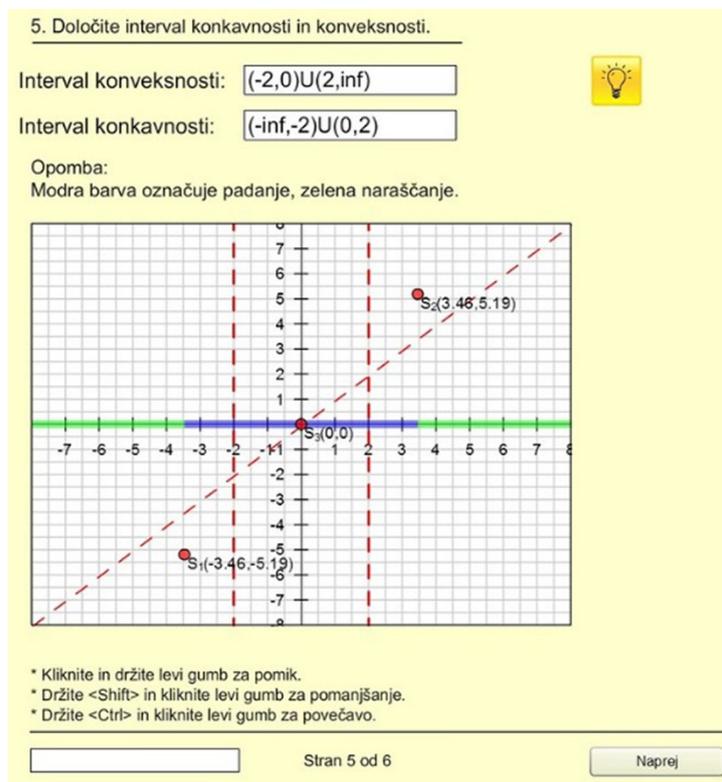
Fig. 5. Plotting the rising and falling intervals (the green colour denotes a rising interval, whereas the violet colour denotes the falling interval) and determining the intervals of concavity and convexity of the given function
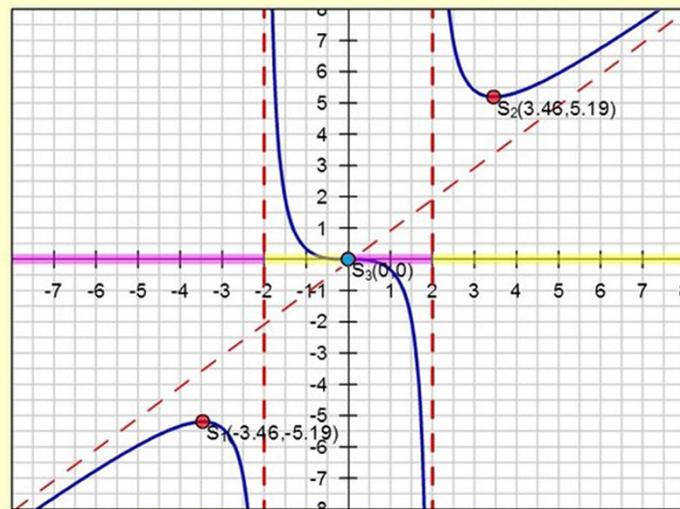
Fig. 6. Plotting the intervals of convexity and concavity (the yellow colour denotes the interval of convexity, whereas the purple colour denotes the interval of concavity) and the entire graph of the given function

The algorithm will not allow the following tasks to be performed until the previous task is solved correctly.

As soon as the user writes down the correct solution for the concrete task, the algorithm will plot the graph and enable the user to do the next task. The user will be able to see the right solution for each task in the graph. In the end, the entire graph of the given function will be seen.

Example of an algorithm for the specific use. In this part, another algorithm for the use of derivatives is shown. In order to make the study more dynamic and attractive, the students are offered a choice of function. For example, given a third-degree polynomial $f(x) = Ax^3 + Bx^2 + Cx + D$, the students can choose the integers for the coefficients A, B, C, D. Accordingly, the users could see various graphs of the functions. Bellow, an example is shown.

Example 2: Example of an algorithm

$$f(x) = \square\, x^3 + \square\, x^2 + \square\, x + \square$$

In the empty fields, the student should write down concrete integers. The sign and the number must be entered in the same field.

For example: $f(x) = +1x^3 - 4x^2 - 2x + 8$

The program should define the zero points, the stationary points, the points of inflexion and classify stationary points. In the end, all tasks of the selected function will be graphically presented. The proposed algorithm aims to understand the importance of changing the function's graph by choosing different coefficients. After the user response, the algorithm will plot all the data on the graph. The zeros in the concrete case are the following: $-1.41, 1.41, 4$ (*i.e.*, $\sqrt{2}, \sqrt{2}, 4$).

The algorithm below asks the user to insert the required field after the selected function's stationary point coordinates S_x and S_y. The graph draws these stationary points according to the value entered in the concrete place. In the selected case, the stationary points are the following: $Sx_1 = -0.23$, $Sy_1 = 8.24$ and the $Sx_2 = 2.90$ and $Sy_2 = -7.05$. The user should also determine the infection point. In this case, the points are as follows: $P_x = 1.33$, $P_y = 0.62$. A hint will be given in the solution box that the solution should be written in decimals rounded to 2 decimal numbers and separated by a dot. The algorithm requires the falling and rising interval to be determined as convexity and concavity. The solutions are entered and plotted on the graph. In this case, the upward interval is located in the area of $(-inf, -0.25) \cup (3, inf)$, whereas the falling interval is in the area $(-0.25, 3)$. In the selected case, the interval of convexity is located in the $(1.33, inf)$, whereas the interval of concavity is in the following area: $(-inf, 1.33)$. There will be a hint that instead of the word infinity, the word inf. is required. Another hint is that the solution should be written down in decimals, rounded to 2 decimal numbers and separated by a dot. The algorithm, therefore, enables a choice for the users. The animation offers the student an accurate description of a particular graph requirement for each drawing in the graph. The process is described below. Determine the stationary points and the intervals of convexity and concavity for any function.

There will be "help" for each task written in the previous section.

$$f(x) = \boxed{+1}\,x^3 + \boxed{-4}\,x^2 + \boxed{-2}\,x + \boxed{+8}$$

1. Insert an arbitrary number for a polynomial coefficient (see Figure 7).

Fig. 7. Defining the polynomial
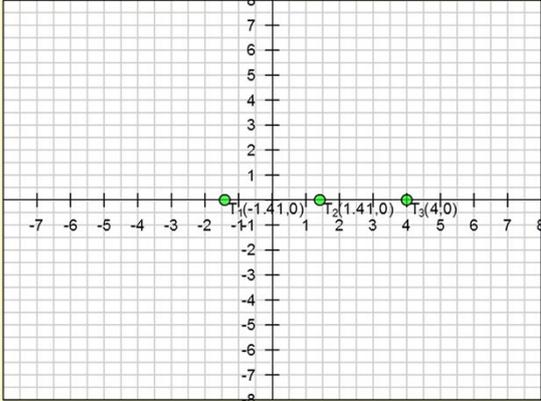
2. Plotting the zeros on the graph (see Figure 8).
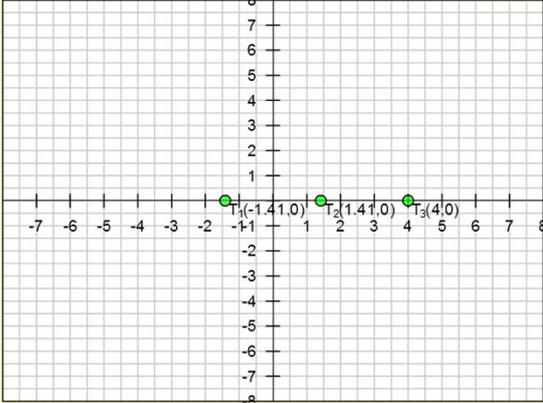
Fig. 8. Plotting the zeros on the graph

3. Define the stationary points of the function. Specify the *x* and *y* coordinates of the points. Define the maximum, the minimum and/or inflexion point of the function (see Figure 9).

Fig. 9. Specifying the *x* and *y* coordinates of the stationary points

4. Specify the interval falling or rising *f*(*x*) (see Figure 10).
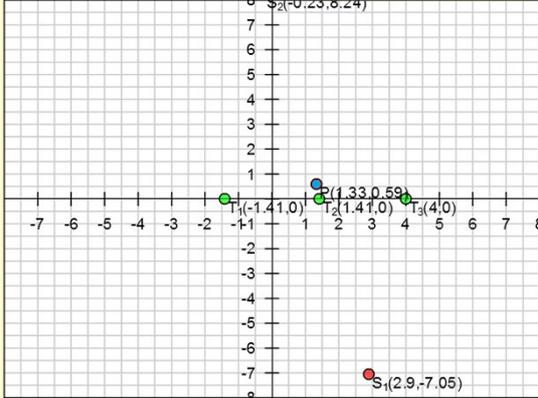
Fig. 10. Plotting the stationary points and determining the falling and rising interval of the given function

5. Define the intervals of concavity or convexity (see Figure 11).

Fig. 11. Plotting the rising and falling intervals and determining the intervals of concavity and convexity of the given function
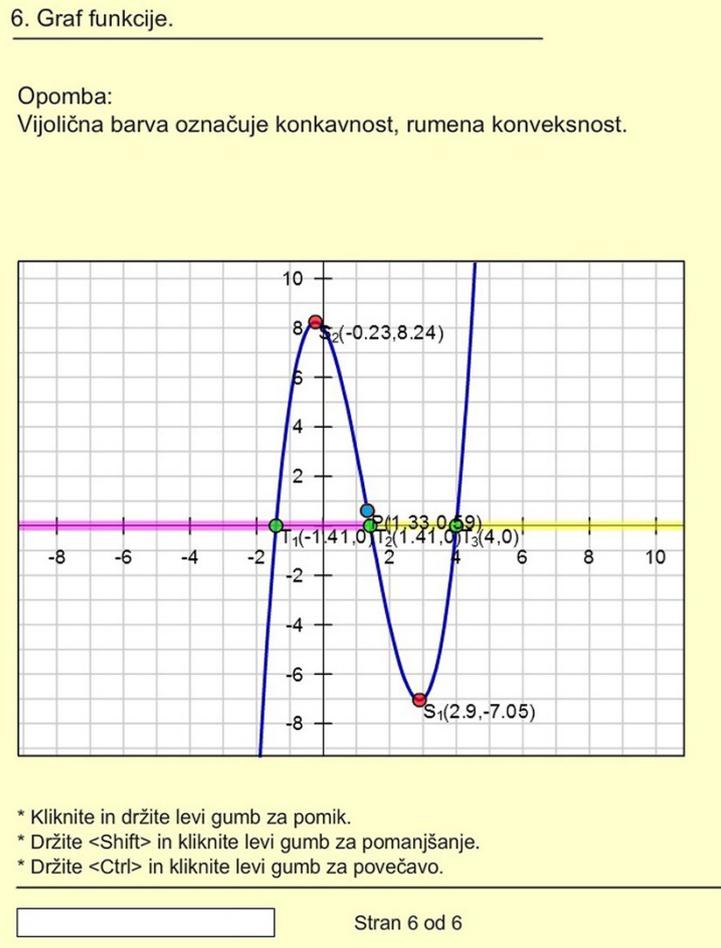
6. Plotting the graph (see Figure 12).

Fig. 12. Plotting the intervals of convexity and concavity and the entire graph of the given function

5.2 Integral

In integrals, the focus will be on creating software tools that would help the users understand the integrals better. For this purpose, we designed three different dynamic tasks. We will present the algorithm for calculating the area, which will be limited by any function depending on the choices of coefficients. Finally, we will give examples of real case integrals, in which the user has to determine the surface area and the volume of the geometric body.

Use of the Integral. In this section, the concrete example will be shown, in which, based on two given functions, the area limited by two given polynomials is to

be calculated. An animation will show the mapped area or the result. Like the tasks for the derivatives, we create an integrals task, whereby, in the case of the correct solution, the plotting of the graph is seen.

Example 1: Use of the integral

First, a summary of the program's implementation to calculate the area between the functions $f(x)$ and $g(x)$ will be given.

The procedure for solving the given an example is presented below.

1. Define the zeros of the function $f(x) = x^2 + 2x - 3$ (see Figure 13).

In the first step, the function's zeros should be defined. In this case, the zeros are as follows: $x_1 = -3$, $x_2 = 1$. The help sign provided guides with the user to the solutions for better understanding. In this case, the formulas for calculating zeros of functions are given. After entering the correct values for the zeros, the animation moves directly to the next task, and the obtained result is plotted on the graph.

Help: To find zero points, you have to solve the equation $f(x) = 0$. The zeros of quadratic functions could be calculated with the following equation $x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
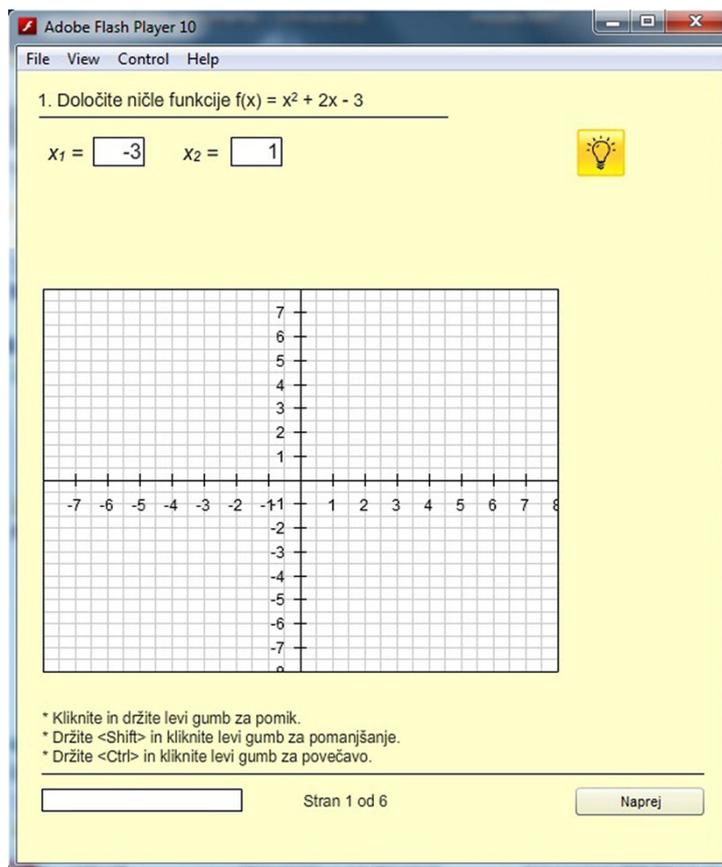

Fig. 13. Entering the zeros

2. Define the critical function point (see Figure 14).

The next task for the user is to find and specify the coordinates of the critical point of the function. The formulas for calculating the coordinates of the critical point are given as help. Thus, the zeros, the critical point of the given function and the function itself are plotted on the graph.

Help: Critical point could be calculated with the following equation or derivation:
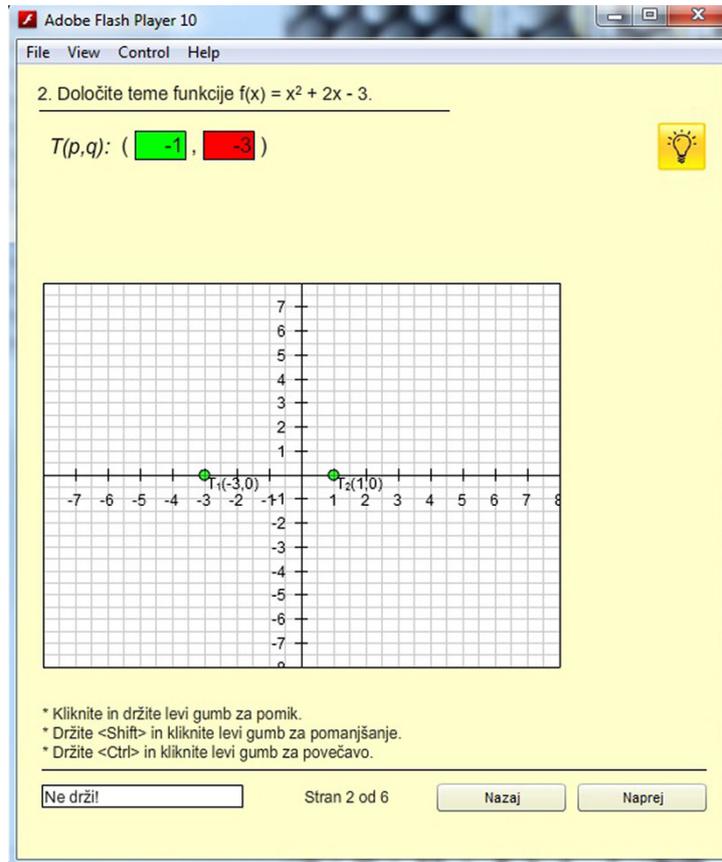
$$T(p,q): q = \frac{(4ac - b^2)}{4a}, p = \frac{-b}{2a}.$$

Fig. 14. Entering the coordinates of the critical points and plotting the zeros

3. Define the zeros of the function $g(x) = x – 1$ (see Figure 15).

In the next step, the zero of the function $g(x) = x – 1$ must be defined. In our case, this is $x_1 = 1$.

Help: To find the zero points, you have to solve the equation $f(x) = 0$. You have to write the solutions as $x_i = i = 1$. The zero point of the linear function
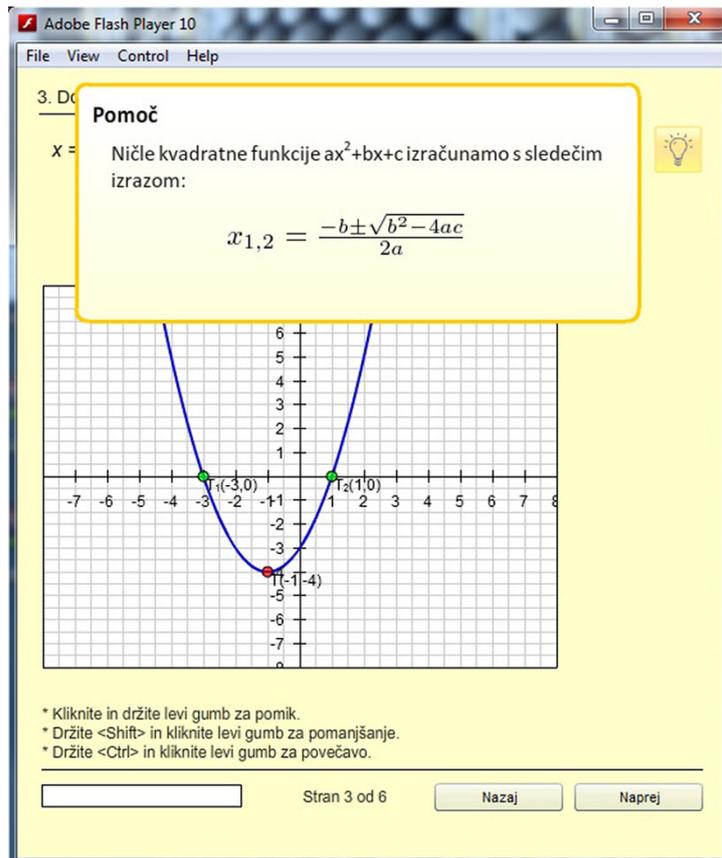
Fig. 15. Enter the second function's zeros and plot the function $f(x)$

4. Calculate the intersections between the functions $f(x)$ and $g(x)$ (see Figure 16).

The next task for the user is to find and specify the coordinates of the intersections between the functions.

The formulas for calculating the intersections are given as help.

Help: Intersections between the functions could be calculate as $f(x) = g(x)$.

The first function is drawn in the graph, followed by the second function, a linear function. The animation then requires defining and calculating the bounded area between the functions.
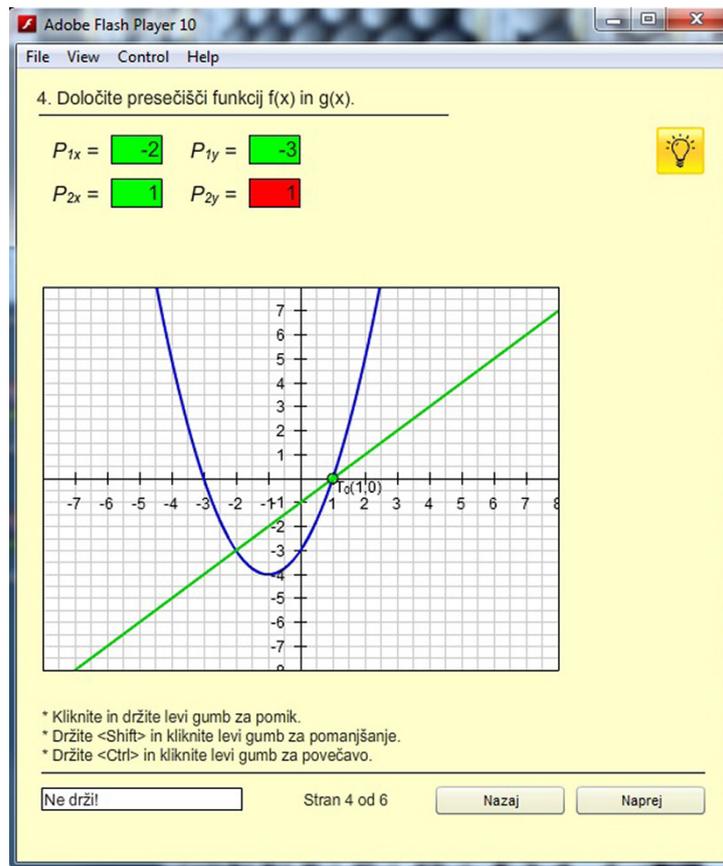

Fig. 16. Entering the coordinates of the intersections and plotting both graphs

5. Write down the definite integral and calculate the bounded area (see Figure 17).

The user has to set the limits of definite integrals and calculate the area bounded by the functions.

Help: In the first case, the area between $f(x)$ and $g(x)$ on the interval $[a, b]$ should be determined. We also assume that $f(x) > g(x)$. The formula for the area, in this case, is $S = \int_{a}^{b} (f(x) - g(x))dx$.
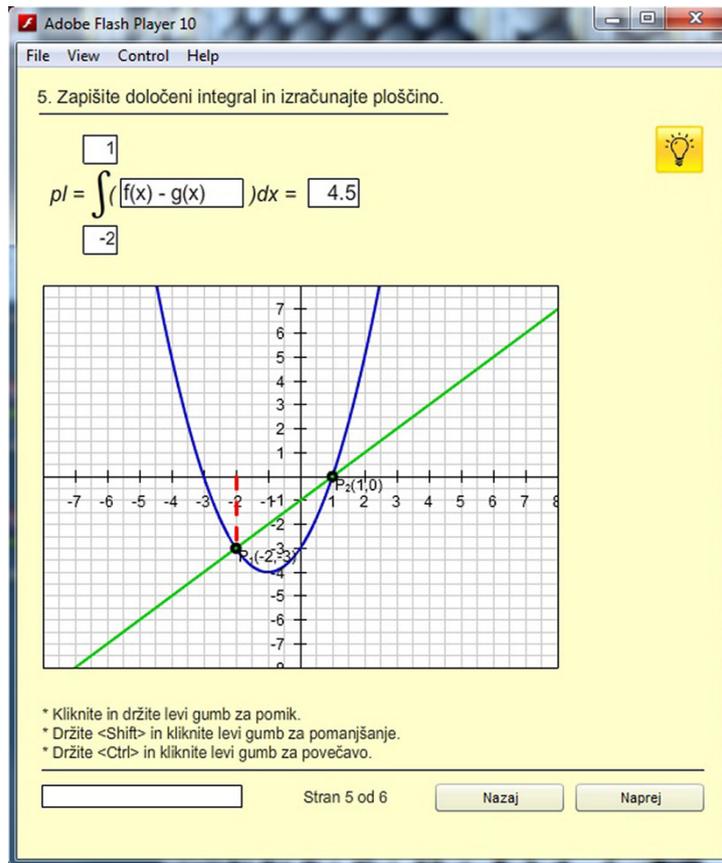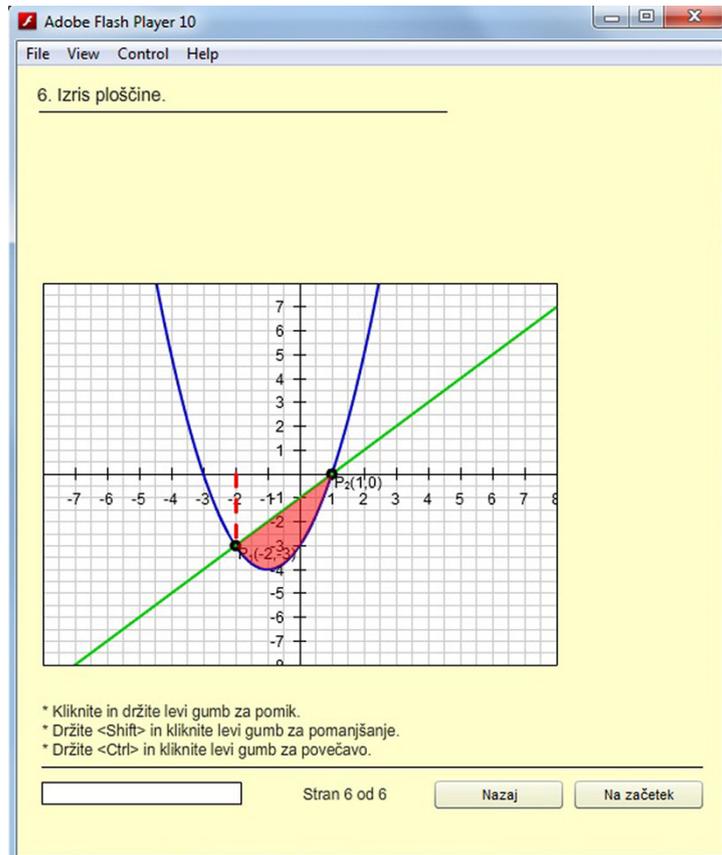

Fig. 17. Setting the limits of the definite integral

6. Plotting the bounded area (see Figure 18).

Upon providing the correct calculation and the necessary data, the calculated area of the graph is coloured.

Fig. 18. Plotting the bounded area

Algorithm for a concrete application of the integral. In this part of the paper, another algorithm for using the integral is presented. In order to make the study more dynamic and attractive, the students are given a choice of the functions. For example, given the second-polynomials $f(x) = Ax^2 + Bx + C$, the students could choose integers for the coefficients A, B, and C. With different inputs, the user can change the complexity of the task and practice the use of integrals. Accordingly, the students could see various graphs of the functions. Bellow, an example is shown.

Calculate the area bounded by the functions $f(x)$ and $g(x)$. In the box, insert the arbitrary integer, for example, +1, –2. By entering a value, which must be from the set of integers, the user specifies the functional notation in the empty boxes. Thus, the task could be set to establish the area between the parabolas, the parabola and a linear function, two linear functions and the abscise axis. The conditions for all the above

scenarios are predetermined, and consequently, there is a planned programming environment that guides the user to the solution. In this case the example with the functions $f(x) = x^2 - 2x + 1$ and $g(x) = -x^2 + 4x$ will be presented. The algorithm below first requires determining the zeros of the function. The zeros are plotted on the graph by entering the appropriate values in the fields. The definition and formulas are provided to help calculate the function's zeros. In the next step, the program requires the intersections of the functions to be determined if they exist. If there are no function interfaces, the program provides the relevant information and requires a new value to be entered. In the case of intersections, the program draws them on the graph. The program will also notify the user to write down the solution rounded to two decimals and separated by a dot. In the last part of the task, the algorithm must define the limits of the definite integrals and write down the formula for calculating the bounded area of the selected functions.

For each task, "help" will be offered as written in the previous section.

1. Insert the arbitrary number for the coefficients of polynomials (see Figure 19).
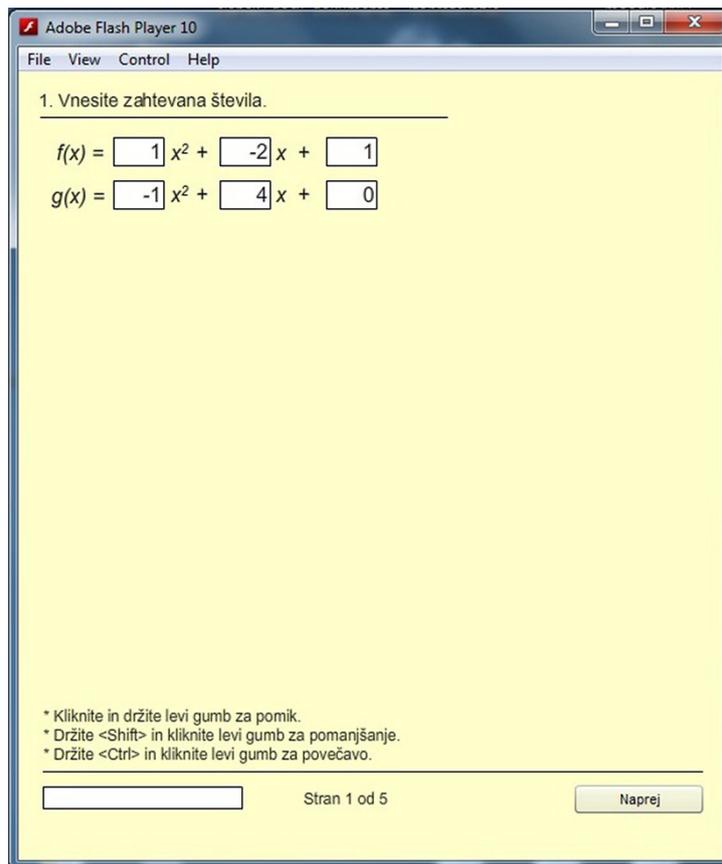
Fig. 19. Entering coefficients

2. Determine the zeros on the functions *f*(*x*) and *g*(*x*) (see Figure 20).

Fig. 20. Entering the zeros of the functions

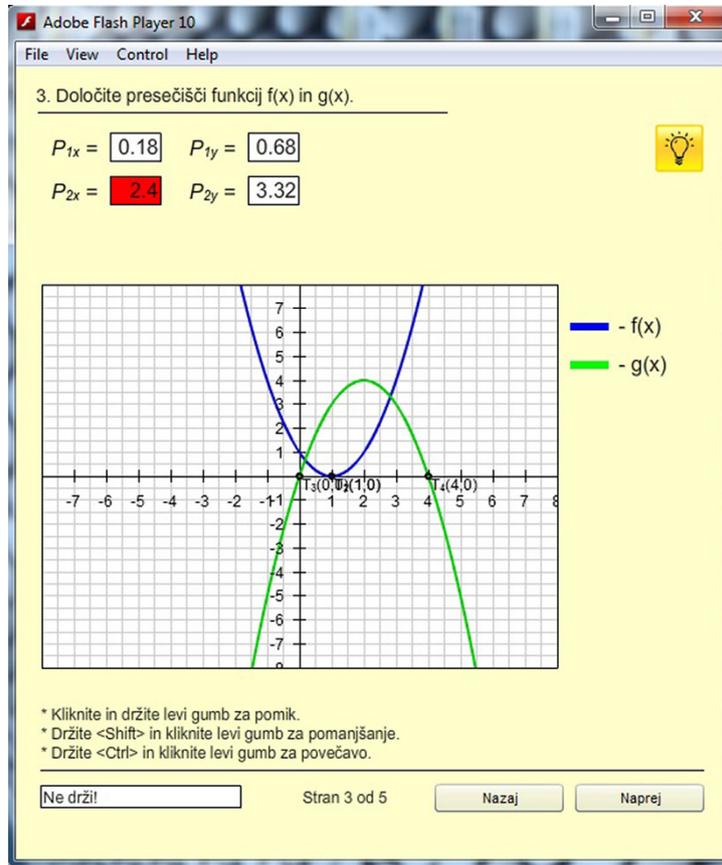**3. Determine the intersections of the functions if they exist (see Figure 21).**

Fig. 21. Entering the intersections and plotting

4. Define the limits of definite integrals and calculate the bounded area (see Figure 22).
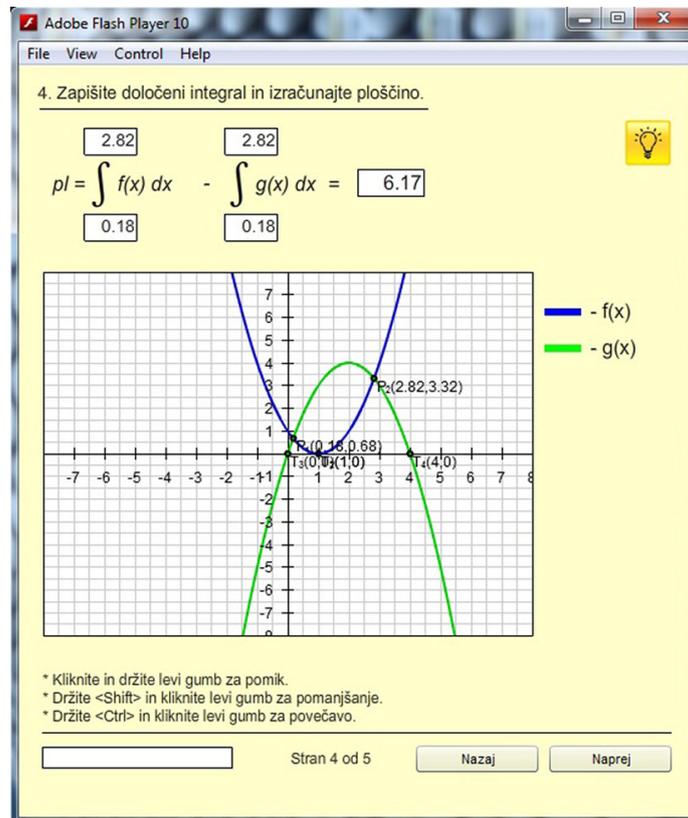

Fig. 22. Defining definite integrals of the functions

6   Conclusion

We live in a modern world where technology is no longer something new. Therefore, we implemented interactive virtual classroom exercises for a more attractive and dynamic mathematics study.

One of this "project" 's great objectives was to motivate students and encourage them to overcome their difficulties through a self-study giving them more confidence. We presented just one part of the upgraded teaching approach, and it guides the user through the example and the possibility of choosing the coefficient and sign of the polynomial function of the second and third degrees.

We had to say that student feedback on such interactive exercises is excellent. We have been using interactive tasks actively for at least three years now. As a result, this year, we also wanted to get feedback on how satisfied students were with the assignments, how long they used them before the exam, what they would like to change and whether they would like such assignments for other areas of mathematics.

In the survey, we included 150 students who passed the exam. The analysis showed a strong positive correlation ($r = 0.843$) between the hours spent by students solving interactive exercises and the final grade in the course itself.

Moreover, they expressed a wish for even more examples prepared in the described way. The exam results are also better than when we had only a classical pedagogical way of teaching mathematics.

The analysis also shows that students are more motivated than a classical pedagogical approach, and studying different parts of mathematics is even more attractive and dynamic.

In the future, we want to develop even more exciting exercises. Namely, we want students to have a broader insight into what they are learning. So, we will also upgrade the other areas of mathematics, whereby the limitation will not be to just one of the polynomials, but other functions of one variable will be included.

We firmly believe that with the use of those examples, the study of mathematics will be even more dynamic and attractive and improve the teaching-learning process.

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