# Nonlinear Measure about $l^2$-norm with Application in Synchronization Analysis of Complex Networks via the General Intermittent Control

http://dx.doi.org/10.3991/ijoe.v9iS4.2702

Zhang Qunli
Heze University, Heze, Shandong, P. R. China

***Abstract*—Based on the nonlinear measure about $l^2$-norm, some simple yet generic criteria are derived by using the general intermittent control, which ensure the exponential stability of dynamic systems. The numerical simulations whose theoretical results are applied to robust synchronization of complex networks demonstrate the effectiveness and feasibility of the proposed technique.**

***Index Terms*—nonlinear measure about $l^2$-norm, the general intermittent control, exponential stability, synchronization; complex networks.**

## I. INTRODUCTION

Since its origins in the work of Fujisaka and Yamada[1-3],Afraimovich,Verichev, and Rabinovich[4], and Pecora and Carrol[5], the study of synchronization of chaotic systems [6- 13] is of great practical significance and has received great interest in recent years.In the above literature, the approach applied to stability analysis is basically the Lyapunov's method. As we all known, the construction of a proper Lyapunov function usually becomes very skillful, and the Lyapunov's method does not specifically describe the convergence rate near the equilibrium point of the system. Hence, there is little compatibility among all of the stability criteria obtained so far.

The concept named the nonlinear measur about $l^2$-norm[14-19] has been applied to the investigation of the existence, uniqueness or stability analysis of the equilibrium. Intermittent control [20-24] has been used for a variety of purposes in engineering fields such as manufacturing, transportation, air-quality control and communication. A wide variety of synchronization or stabilization using the periodically intermittent control method has been studied (See [21-28]). Compared with continuous control methods[7-15], intermittent control is more efficient when the system output is measured intermittently rather than continuously. All of intermittent control and impulsive control are belong to switch control. But the intermittent control is different from the impulsive control, because impulsive control is activated only at some isolated moments, namely it is of zero duation, while intermittent control has a nonzero control width. Therefore, it is essential and important to investigate the exponential synchronization of networks with mixed delays by periodically intermittent control.

A special case of such a control law is of the form

$$U(t)=\begin{cases}-k(y(t)-x(t)), & (nT\le t<nT+\delta),\\ 0, & (nT+\delta\le t<(n+1)T),\end{cases}\qquad(1)$$

where $k$ denotes the control strength, $\delta>0$ denotes the switching width, and $T$ denotes the control period. The general intermittent controller

$$U(t)=\begin{cases}-k(y(t)-x(t)), & (h(n)T\le t<h(n)T+\delta),\\ 0, & (h(n)T+\delta\le t<h(n+1)T),\end{cases}\qquad(2)$$

where $h(n)$ is a strictly monotone increasing function on $n$ has been studied (See [29]). In this paper, based on nonlinear measur about $l^2$-norm and Gronwall inequality, the general intermittent controller

$$U(t)=\begin{cases}-k(y(t)-x(t)), & (h(n)T\le t<h(n)T+\delta),\\ 0, & (h(n)T+\delta\le t<h(n+1)T),\end{cases}\qquad(2)$$

or

$$U(t)=\begin{cases}-k(y(t)-x(t)), & (h(n+1)T\le t<h(n+1)T+\delta),\\ 0, & (h(n+1)T+\delta\le t<h(n)T),\end{cases}\qquad(2')$$

is designed, where $h(n)$ is a strictly monotone function on $n$ , then the sufficient yet generic criteria for synchronization of complex networks with and without delayed item are obtained.

## II. PRELIMINARIES

Let $X$ be a Banach space endowed with the $l^2$-norm $\|\ \|$, i.e. $\|x\|=\sqrt{x^T x}=\sqrt{\langle x,x\rangle}$, where $\langle\ ,\ \rangle$ is inner product, and $\Omega$ be a open subset of $X$ . We consider the following system:

$$\frac{dx}{dt}=F(x(t))+G(x(t-\tau)),\qquad(3)$$

where $F,G$ are nonlinear operators defined on $\Omega$ , and $x(t)$, $x(t-\tau)\in\Omega$ , and $\tau$ is a time-delayed positive constant, and $F(0)=G(0)=0$ .

**Definition 1**[6,22,25,30] System (1) is called to be exponentially stable on a neighborhood $\Omega$ of the equilibrium point, if there exist constants $\mu > 0, m > 0$, such that

$$\|x(t)\| \le m\exp(-\mu t)\|x_0\| \qquad (t > 0), \qquad (4)$$

where $x(t)$ is any solution of (1) initiated from $x(t_0) = x_0$.

**Definition 2**[15,16,18,19] Suppose that $\Omega$ is an open subset of $R^n$, and $G:\Omega \to R^n$ is an operator. The constant

$$m_\Omega(G) = \sup_{\substack{x\neq y\\ x,y\in\Omega}} \frac{\langle G(x) - G(y), x - y\rangle}{\|x-y\|^2} = \sup_{\substack{x\neq y\\ x,y\in\Omega}} \frac{(x-y)^T(G(x)-G(y))}{\|x-y\|^2}$$

is called the nonlinear measure of $G$ on $\Omega$ with the $l^2$-norm $\|\ \|$.

**Lemma 1**[29] Suppose that $\Omega$ is an open subset of $R^n$, and $F:\Omega\to R^n$ is a bounded operator. The function $f(r)=\|(F+rI)x-(F+rI)y\|-r\|x-y\|,\ (r\ge 0, x\in\Omega)$, is monotone decreasing function on $r$; thus the limit $\lim_{r\to\infty} f(r)$ exists, and

$$\lim_{r\to\infty} f(r) = \frac{\langle F(x) - F(y), x-y\rangle}{\|x-y\|},$$

here, the operator $F+rI$ mapping every point $x\in\Omega$ denotes $F(x)+rx$.

**Lemma 2**[29] If the operator $G$ in the system (3) satisfies

$$\|G(x)-G(y)\| \le l\|x-y\| \qquad (5)$$

for any $x, y\in\Omega$, wher $l$ is a positive constant. The solutions $x(t), y(t)$, initiated from $x(t_0)=x_0\in\Omega,\ y(t_0)=y_0\in\Omega$, of the system (3) satisfy $\|x-y\|\le\|x_0-y_0\|\exp\{\lambda(t-t_0)\},\ \forall t\ge 0$, where $\lambda = m_\Omega(F)+\exp\{-m_\Omega(F)\tau\}l$.

**Corollary 1** Let $G(x(t-\tau))=0, \lambda = m_\Omega(F)$ be defined as in Definition 2, then the result similar to Lemma 2 is obtained.

## III. SYNCHRONIZATION VIA GENERAL INTERMITTENT CONTROL AND EXAMPLES

Consider a delayed complex dynamical network consisting of $N$ linearly coupled nonidentical nodes described by

$$\dot{x}_i(t) = f(x_i(t)) + g(x_i(t-\tau)) + \sum_{j=1}^{N} a_{ij}x_j(t) + u_i(t), i = 1,2,\cdots,N \qquad (6)$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in R^n$ is the state vector of the $i$th node, $f, g: R^n \to R^n$ are nonlinear vector functions, $u_i(t)$ is the control input of the $i$th node, and $A=(a_{ij})_{N\times N}$ is the coupling figuration matrix representing the coupling strength and the topological structure of the complex networks, in which $a_{ij} > 0$ if there is connection from node $i$ to node $j(i\neq j)$, and is zero, otherwise, and the constraint $a_{ii} = -\sum_{j=1,j\neq i}^{N} a_{ij} = -\sum_{i=1,i\neq j}^{N} a_{ij}$, $(i,j=1,2,\ldots,N)$, is set.

A complex network is said to achieve asymptotical synchronization if

$$x_1(t) = x_2(t) = \ldots = x_N(t) = s(t) \text{ as } t\to\infty, \qquad (7)$$

where $s(t)\in R^n$ is a solution of a real target node, satisfying

$$\dot{s}(t) = f(s(t)) + g(s(t-\tau))$$

For our synchronization scheme, let us define error vector and control input $u_i(t)$ as follows, respectively:

$$e_i(t) = x_i(t) - s(t), i = 1,2,\ldots,N.$$

When $h(n)$ is a strictly monotone increasing function on $n$ with $h(0)=0, \lim_{n\to+\infty} h(n) = +\infty$,

$$u_i(t)=\begin{cases} -ke_i(t), & (h(n)T\le t< h(n)T+\delta),\\ 0, & (h(n)T+\delta\le t<h(n+1)T),\end{cases} \quad (k>0, i=1,2,\ldots,N)$$

When $h(n)$ is a strictly monotone decreasing function on $n$ with $h(0)=+\infty, \lim_{n\to+\infty} h(n) = 0$,

$$u_i(t)=\begin{cases} -ke_i(t), & (h(n+1)T\le t< h(n+1)T+\delta),\\ 0, & (h(n+1)T+\delta\le t<h(n)T).\end{cases} \quad (k>0, i=1,2,\ldots,N)$$

In this work, the goal is to design suitable function $h(n)$ and parameters $\delta$, $T$ and $k$ satisfying the condition (7). The error system follows from the expression

$$\begin{cases} \dot{e}_1(t) = f(x_1(t)) - f(s(t)) + g(x_1(t-\tau)) - g(s(t-\tau)) + \sum_{j=1}^{N} a_{1j}e_j(t) + u_1(t),\\ \dot{e}_2(t) = f(x_2(t)) - f(s(t)) + g(x_2(t-\tau)) - g(s(t-\tau)) + \sum_{j=1}^{N} a_{2j}e_j(t) + u_2(t),\\ \quad\vdots \qquad\qquad \vdots\\ \dot{e}_N(t) = f(x_N(t)) - f(s(t)) + g(x_N(t-\tau)) - g(s(t-\tau)) + \sum_{j=1}^{N} a_{Nj}e_j(t) + u_N(t).\end{cases} \qquad (8)$$

When $h(n)$ is a strictly monotone increasing function on $n$ with $h(0)=0, \lim_{n\to+\infty}h(n)=+\infty$, we obtain the following result:

**Theorem 1** Suppose that the operator $g$ in the network (6) satisfies condition (5), and $m_\Omega$ is defined as Definition 2, $\lambda = m_\Omega(F)+\exp\{-m_\Omega(F)\tau\}l$, where

$$m_\Omega(F) = \sup_{\substack{x_i(t)\neq s(t),\\ i=1,2,\ldots,N}} \frac{< F(t), e(t) >}{\|e(t)\|^2},$$

$$F(t) = ((f(x_1(t)) - f(s(t)) + \sum_{j=1}^{N} a_{1j} e_j(t))^T, \ldots$$
$$(f(x_N(t)) - f(s(t)) + \sum_{j=1}^{N} a_{Nj} e_j(t))^T)^T,$$
$e(t) = (e_1^T(t), e_2^T(t), \ldots, e_N^T(t))^T$, $l = \max\{l_1, l_2, \ldots, l_N\}$,

$l_i$ satisfies $\|g(x_i(t-\tau)) - g(s(t-\tau))\| \le l_i \|e_i(t-\tau)\|$.

Then the synchronization of networks (6) is achieved if the parameters $\delta$, $T$ and $k, \eta$ satisfy

$$\inf((\rho+\lambda)\delta \frac{h^{-1}(t\text{-}\delta/T)}{t} - \lambda) \ge \eta > 0, \tag{9}$$

where $\rho = k - \lambda > 0$, $h^{-1}(g)$ is the inverse function of the function $h(g)$.

**Proof** From Lemma 2, the conclusion is valid:

$$\|e(t)\| \le \|e(h(n)T)\| \exp\{-\lambda(t - h(n)T)\} \tag{10}$$

for any $h(n)T \le t < h(n)T + \delta$;

$$\|e(t)\| \le \|e(h(n)T + \delta)\| \exp\{\lambda(t - h(n)T - \delta)\} \tag{11}$$

for any $h(n)T + \delta \le t < h(n+1)T$ .

$$\|e(t)\| \le \begin{cases} \|e(0)\| \exp\{-\rho t + (\rho+\lambda)h(n)T - n(\rho+\lambda)\delta\}, \\ \qquad (h(n)T \le t < h(n)T + \delta), \\ \|e(0)\| \exp\{\lambda t - (n+1)(\rho+\lambda)\delta\}, \\ \qquad (h(n)T + \delta \le t < h(n+1)T), \end{cases}$$

$$\le \begin{cases} \|e(0)\| \exp\{-((\rho+\lambda)\delta \frac{h^{-1}(t-\delta/T)}{t} - \lambda)t\}, \\ \qquad (h(n)T \le t < h(n)T + \delta), \\ \|e(0)\| \exp\{-((\rho+\lambda)\delta \frac{h^{-1}(t)}{t} - \lambda)t\}, \\ \qquad (h(n)T + \delta \le t < h(n+1)T), \end{cases}$$

$$\le \|e(0)\| \exp\{-\eta t\},$$

when $t \to +\infty$, $\|e(t)\| \to 0$ is obtained under the condition (9). So the synchronization of the network (6) is achieved.

When $h(n)$ is a strictly monotone decreasing function on $n$ with $\lim_{n\to+\infty} h(n) = 0$, $h(0) = +\infty$, we obtain the following result:

**Theorem 2** Suppose that the operator $g$ in the systems (6) satisfies condition (5),and $m_\Omega$ is defined as Definition 2, $l$ ,$e(t)$ are the same as Theorem 3. Then the synchronization of networks (6) is achieved if the parameters $\delta$ , $T$ and $k, \eta$ satisfy

$$\inf((\rho+\lambda)\delta \frac{h^{-1}(t)}{t} - \lambda) \ge \eta > 0, \tag{12}$$

where $\rho = k - \lambda > 0$, $h^{-1}(g)$ is the inverse function of the function $h(g)$.

The proof of Theorem 2 is similar to that of Theorem 1. It is omitted, here.

**Corollary 2** Let $g(x(t-\tau)) = 0, \lambda = m_\Omega(F)$ be defined as in Definition 2, and the condition (9) or (12) , respectively, is satisfied. Then the result similar to Theorem 1 or Theorem 2 is obtained.

**Corollary 3** Supposing that $h(n) = p_1 n$, $\delta = p_2 T$, $p_1 > 0$ , and the rest of restricted conditions are invariable. Then the synchronization of the network (6) is achieved if the parameters $\delta$, $T$ and $k$, $\eta$ satisfy

$$(\rho+\lambda)\delta \frac{p_2}{p_1} - \lambda \ge \eta > 0, \tag{13}$$

**Corollary 4** when $a_{ij} = 0, i, j = 1, 2, \ldots, N$ , the result similar to Theorem 1 or Theorem 2 is obtained if the condition (9) or (12) , respectively, is satisfied.

In the simulations of following examples, we always choose $N = 40, T = 5, d = 4, k = 16$, the matrix

$$P=\begin{pmatrix}
-14 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 \\
2 & -8 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
2 & 0 & -6 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
3 & 1 & 0 & -9 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & -5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -6 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -3 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 2 & 0 & 1 & 1 & 0 & 0 & -7 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & -8 & 1 & 0 & 2 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 2 & -9 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & -7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & -10 & 0 & 1 & 1 & 0 & 0 & 2 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & -5 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 1 & -5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & -7 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & -5 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & -6 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 & -7 & 3 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 2 & -7
\end{pmatrix},$$

$$Q = diag(-14, -8, -6, -9, -5, -6, -3, -7, -8, -9, -7, -6, -10, -5, -5, -7, -5, -6, -7, -7).$$

**Example 1** Consider a delayed Hopfield neural network[31-32] with two neurons:

$$\dot{x}(t) = -Cx(t) + Df(x(t)) + Bf(x(t-\tau)), \tag{14}$$

where $x(t) = (x_1(t), x_2(t))^T$, $f(x(t)) = (\tanh(x_1(t)),$ $\tanh(x_2(t)))^T$, $\tau = (1)$, and

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, D = \begin{pmatrix} 2.0 & -0.1 \\ -5.0 & 3.0 \end{pmatrix}, B = \begin{pmatrix} -1.5 & -0.1 \\ -0.2 & -2.5 \end{pmatrix}.$$

It should be noted that the network is actually a chaotic delayed Hopfield neural network.

We reach the value $l < 9.15, m_\Omega(F) \le 0.7993$, here $F(x(t)) = -Cx(t) + Df(x(t)), g(x(t-t)) = Bf(x(t-t))$. the function $h(n) = n^2/(n+1)$, , $h(n) = 0.3/n$ , which are the strictly monotone increasing or decreasing function on $n$ , respectively, then they can be clearly seen that the synchronization of network (6) is realized in Fig.1、Fig.2 ( A≠0), where $A = diag(0.2P, 0.2P)$, and Fig.3、Fig.4 (A=0), respectively.

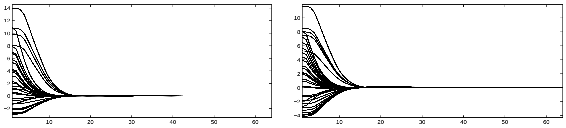

Figure 1. Synchronization error $x_{i1}-x_{11},\ x_{i2}-x_{12},(i=2,3,\ldots,40)$ when $h(n)=n^2/(n+1), A\neq 0.$

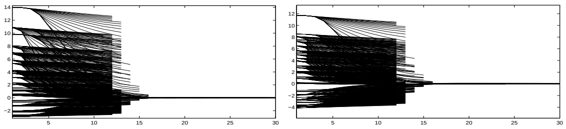

Figure 2. Synchronization error $x_{i1}-x_{11},\ x_{i2}-x_{12},(i=2,3,\ldots,40)$ when $h(n)=0.3/n,\ A\neq 0.$

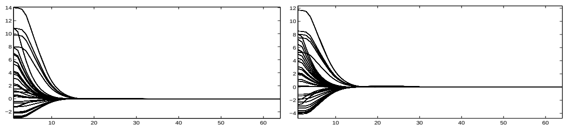

Figure 3. Synchronization error $x_{i1}-x_{11},\ x_{i2}-x_{12},(i=2,3,\ldots,40)$ when $h(n)=n^2/(n+1), A=0.$

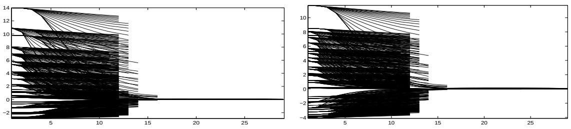

Figure 4. Synchronization error $x_{i1}-x_{11},\ x_{i2}-x_{12},(i=2,3,\ldots,40)$ when $h(n)=0.3/n,\ A=0.$

**Example 2** Consider hyper-chaotic Chen system[33] :

$$\begin{cases} \dot{x}=35(y-x)+w, \\ \dot{y}=7x-xz+12y, \\ \dot{z}=xy-3z, \\ \dot{w}=yz+0.5w. \end{cases}$$

We reach the value $m_\Omega(F)\le 5.0304,$ here $F(t)=(35(y-x)+w,7x-xz+12y,xy-3z,yz+0.5w)^T.$ We choose the function $h(n)=\ln(n+1),$ $h(n)=0.3/n,$ which are strictly monotone increasing or decreasing function on $n$, respectively, then they can be clearly seen that the synchronization of network (6) is realized in Fig.5、Fig.6 ( A≠0), where

$$A=\begin{pmatrix} 0.2(P+Q) & 0.2(P-Q) \\ 0.2(P-Q) & 0.2Q \end{pmatrix},$$

and Fig.7、Fig.8 (A=0), respectively.

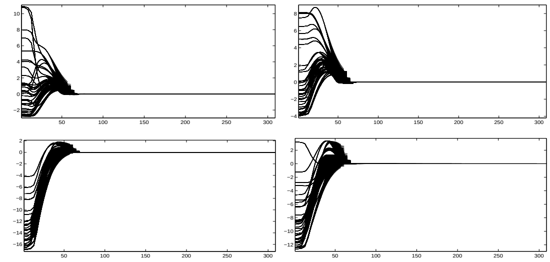

Figure 5. Synchronization error $x_{ij}-x_{1j},(i=2,3,\ldots,40,j=1,2,3,4)$, when $h(n)=\ln(n+1),\ A\neq 0.$

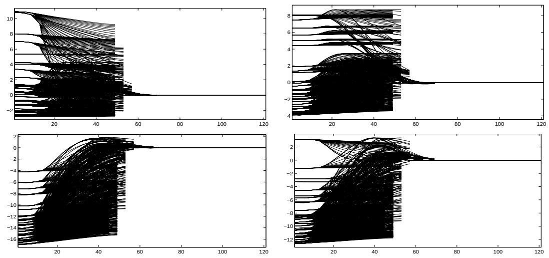

Figure 6. Synchronization error $x_{ij}-x_{1j},(i=2,3,\ldots,40,j=1,2,3,4)$ when $h(n)=0.3/n,\ A\neq 0.$

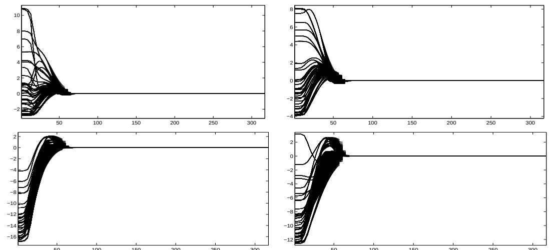

Figure 7. Synchronization error $x_{ij}-x_{1j},(i=2,3,\ldots,40,j=1,2,3,4)$ when $h(n)=\ln(n+1),\ A=0.$

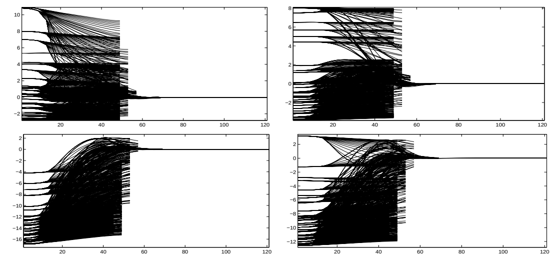

Figure 8. Synchronization error $x_{ij}-x_{1j},(i=2,3,\ldots,40,j=1,2,3,4)$ when $h(n)=0.3/n,\ A=0.$

## IV. CONCLUSION

Approaches for synchronization of complex networks via general intermittent which use the nonlinear operator named the measure about $l^2$-norm have been presented in this paper. Strong properties of global and exponential synchronization have been achieved in a finite number of steps. The techniques have been successfully applied to Chaotic delayed Hopfield neural networks and hyper-chaotic Chen system. Numerical simulations have verified the effectiveness of the method.

## REFERENCES

[1] Fujisaka H and Yamada T, Stability theory of synchronized motion in a coupled-oscillator system, Prog.Theor.Phys.69, 983, p:32-47.

[2] Fujisaka H and Yamada T, A new intermittency in coupled dynamical systems, Prog.Theor.Phys.74, 985, pp:918-921.

[3] Fujisaka H and Yamada T, Stability theory of synchronized motion in coupled-oscillator systems IV, Prog. Theor.Phys. 75, 986, pp: 1087-1104.

[4] V.S. Afraimovich, N.N. Verichev, and M.I. Rabinovich, Stochastic synchronization of oscillations in dissipative systems, Radio Phys. And Quantum Electron, 9, 986, pp:747-751.

[5] L.M. Pecora and T.L. Carroll, Synchronization in chaotic systems, Phys. Rev. Lett. 64, 900, p:821-824.

[6] Qian Ye, Huibin Zhu, Baotong Cui. Synchronization analysis of delayed hybrid dynamical networks with quantized impulsive effects. Control Theory & Applications, 30(1), 013, p:61-68.

[7] Qunli Zhang, Guanjun Jia, Chaos synchronization of Morse oscillator via backstepping design, Ann. of Diff. Eqs. Vol.22, o.3, 006, pp:456-460,

[8] Qunli Zhang. Synchronization of multi-chaotic systems via ring impulsive control, ontrol Theory and Applications, Vol.27, No.2, 010, pp:226-232.

[9] Ming Han, Yang Liu, Jiangquan Lu. Impulsive control for the synchronization of chaotic systems with time delay. Abstract and Applied Analysis, Volume 2013, rticle ID 647561, 013, p:1-13.

[10] Jinde Cao, Lulu Li, Cluster synchronization in an array of hybrid coupled neural networks with delay, Neural Networks, 22:4, 009, pp:335-342.

[11] Lulu Li, Jinde Cao, luster synchronization in an array of coupled stochastic delayed neural networks via pinning control, Neurocomputing, 74, 011, p:846-856.

[12] Hamid Reza, Peter Mass, elay-range-dependent exponential H synchronization of a class of delayed neural networks, Chaos, Solitons and Fractals, 41, 009, p:1125-1135.

[13] Jinde Cao, aniel W.C.Ho, ongqing Yang, rojective synchronization of a class of delayed chaotic systems via impulsive control, Physics Letters A, 73, 009, p:3128-3133.

[14] Wang Kai-ming, u Xin-li, ia Shuang-ying, onlinear measure about l 2-norm, Basic Sciences Journal of Texttile Universitties, ol.17, o.3, 004, pp:198-200.

[15] Ping Li, Jinde Cao, stability in static delayed neural networks: a nonlinear measure approach, Neurocomputing 69, 006, p: 1776-1781.

[16] Ping Li, Jinde Cao, Global stability in switched recurrent neural networks with time-varying delay via nonlinear measure, Nonlinear Dyn 49, 007, p:295-305.

[17] Hongtao Lu, Zhizhou Zhang, Lin He, Convergence Analysis of Genetic Regulatory Networks Based on Nonlinear Measures, Advances in Neural Networks-ISNN, 005, p:247-252.

[18] Yuanming Li, Haijun Jiang, Exponential stability of high-order Cohen-Grossberg neural networks with time delay, Journal of Xinjiang University(Natural Science Edition), Vol. 25, o. 1, 008, p:28-35.

[19] Anhua Wan, Miansen Wang, Jigen Peng, Exponential stability of Cohen-Grossberg neural networks, Journal of Xi'an Jiaotong University, Vol. 40, o. 2, 006, p:215-218.

[20] Junjian Huang, Chuandong Li, i Han. Stabilization of Delayed Chaotic Neural Networks by Periodically Intermittent Control, Circuits Syst Signal Process, ol.28, 009, p:567-579.

[21] Juan Yu, aijun Jiang, hidong Teng. Synchronization of nonlinear systems with delay via periodically intermittent control, Journal of Xinjiang University(Natural Science Edition), Vol.28, No.3, 010, p:310-315.

[22] Zuoyong Dong, Yajuan Wang, Minghui Bai, hiqiang Zuo. Exponential synchronization of uncertain master-slave Lur'e systems via intermittent control, ournal of Dynamics and Control, ol.7, o.4, 009, p:328-333.

[23] Weiguo Xia, Jinde Cao, Pinning synchronization of delayed dynamical networks via periodically intermittent control, Chaos, 19, 13120, 009.

[24] Junjian Huang, Chuandong Li, Qi Han, Stabilization of delayed chaotic neural networks via periodically intermittent control, ircuit Syst Signal Process, 8, 009, p:567-579.

[25] Junjian Huang, Chuandong Li, Qi Han, Stabilization of chaotic neural networks by periodically intermittent control, Circuits Syst Signal Process 28, 2009, p:567-579. http://dx.doi.org/10.1007/s00034-009-9098-3

[26] Smolen P, Baxter D, Byrne J, Mathematical modeling of gene networks review, Neuron 26, 2000, p:567-580. http://dx.doi.org/10.1016/S0896-6273(00)81194-0

[27] Wang Y, Hao J, Zuo Z, A new method for exponential synchronization of chaotic delayed systems via intermittent control, Phys Lett A 374, 2010, p:2024-2029. http://dx.doi.org/10.1016/j.physleta.2010.02.069

[28] Huang T, Li C, Yu W, Chen G, Synchronization of delayed chaotic systems with parameter mismatches by using intermittent linear state feedback, Nonlinearity 22, 2009, p:569-584. http://dx.doi.org/10.1088/0951-7715/22/3/004

[29] Qunli Zhang, The Generalized Dahlquist Constant with Applications in Synchronization Analysis of Typical Neural Networks via General Intermittent Control, Advances in Artificial Neural Systems Vol.2011, Article ID 249136, 7 pages http://dx.doi.org/10.1155/2011/249136

[30] Xiaoxin Liao, Theory and application of stability for dynamical systems, Beijing: National Defence Industry Press, 2000, p:15-40.

[31] Gopalsamy K., H. Z. He, Stability in asymmetric Hopfield nets with transmission delays, Physica D, Vol.76, 994, pp:344-358.

[32] H. Lu, Chaotic attractors in delayed neural networks, Phys Lett A, Vol. 298, 002, pp:109-116. http://dx.doi.org/10.1016/S0375-9601(02)00538-8

[33] Guo-liang Cai, Juan-juan Huang, Synchronization for hyperchaotic Chen system and hyperchaotic Rossler system with different structure, Acta Physica Sinica, Vol. 55, No.8, 2006, pp:3997-4003.

## AUTHOR

**Zhang Qunli** is with Department of Mathematics, Heze University, Heze, TX 274015, P.R.China (e-mail: qunli-zhang@126.com).

This work was supported in part by the National Natural Science Foundation of China (11071141), the Natural Science Foundation of Shandong Province of China (ZR2011AL018, ZR2011AQ008) and a Project of Shandong Province Higher Educational Science and Technology Program (J11LA06), the Research Rund Project of Heze University under Grant: XY10KZ01. This article is an extended and modified version of a paper presented at the International Conference on Mechanical Engineering, Automation and Material Science (MEAMS2012), held 22-23 December 2012, Wuhan, China. Received 21 January 2013. Published as resubmitted by the author 01 May 2013..