

REMLABNET – Solar System, New Virtual Laboratory with Mathematical Model Using Vector Functions for Primary, Secondary and High Schools and Universities

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Abstract—The main role of this article is to create a mathematical model using virtual laboratories for primary, secondary and high schools, and universities. The aim of the article is such a mathematical model to be applicable at all levels of education, with a substantial idea to teach students to think more scientifically. Analysis showed us two possible methods of creating a given model. Possible implementation is with differential equations or vector functions. After a thorough consideration and analysis of the subject we chose a vector function that interfere with a larger spectrum of students compared to differential equations. The main contribution of the work is already mentioned and that is a virtual laboratory, which in our case describes the Solar system and its movement using vector functions. This laboratory is useful, from a primary school, where students are getting to know the Solar system to secondary and high schools, and colleges, where educators can use this model on physics and mathematics lessons when taking over teaching vectors.

Keywords—remote laboratories, virtual laboratories, solar system, vector functions, education, REMLABNET, science

1 Introduction

In an antiquity, the world had to look quite random. It must be seemed that disasters, for example, floods, epidemics, earthquakes, or explosions of volcanoes, occurs without warning or apparent causes. But gradually people had to notice certain regularity in nature behaviour. These regularities were most obvious in the movement of heavenly bodies on the sky. [1] It was built on a solid mathematical foundation more than three hundred years ago by Isaac Newton, whose theory of gravitation we still use today. [2] The movement of the planet around the Sun describes Kepler's laws [3] where planets describe an elliptical pathway. However, the ellipse is just a description for simplifying the situation when it is considered only about the Sun itself. The planets actually act by gravitational forces with each other. At first glance, it seems to be no problem (science) and is sufficient if we placed in motion equations all planets and we get a solution. In

fact, it's a bit more complicated and we call it n problems of bodies or points. In our case we have tried the problem of three points. This we show in the following chapters.

On a theoretical level, we need to put right what we are aiming for in this paper. Our aim is to achieve the most efficient and correct results, and that is why we also turned to the officially introduced data. For example, NASA [4], Swinburne University [5] and more. In this work, we analyse the solar system and offer it to students so that they can imagine the extent of our solar system. They compare the data measured in our virtual laboratory with the actual results, as shown on the NASA website. [4](Table 1):

Table 1. Sidereal periods in solar system

	MERCURY	VENUS	EARTH	MOON	MARS	JUPITER	SATURN	URANUS	NEPTUNE	PLUTO
Mass (10 ²⁴ kg)	0.330	4.87	5.97	0.073	0.642	1898	568	86.8	102	0.0146
Gravity (m/s ²)	3.7	8.9	9.8	1.6	3.7	23.1	9.0	8.7	11.0	0.7
Distance from Sun (10 ⁶ km)	57.9	108.2	149.6	0.384	227.9	778.6	1433.5	2872.5	4495.1	5906.4
Orbital Period (days)	88.0	224.7	365.2	27.3	687.0	4331	10,747	30,589	59,800	90,560
	MERCURY	VENUS	EARTH	MOON	MARS	JUPITER	SATURN	URANUS	NEPTUNE	PLUTO

Let's now state what we do next with the given data. Our consortium of three universities deals mainly with the topic of remote laboratories [6]. The first documented e-experiment was built by PhD students in USA [7] and the whole history of e-experimentation is described in [8]. Over time, of course, virtual laboratories began to be created, which are not directly connected to physical HW, but the main theme of remote and virtual laboratories is developing in the last two decades and has become a world-sought-after theme, which also was helped by the pandemic of Covid-19. Remote laboratories have become an integral part of the teaching process, especially in the field of technical and natural sciences, but also the science and research itself. [9] Today's internet time has requested the need to modernize the learning process and then transfer to the online world. [10] Wherever we look we see the use of modern technologies that have become an integral part of normal life. For this reason, it is necessary to adapt to the primary, secondary and high schools and universities, thus gaining not only attractions for pupils and students themselves but especially becomes modern and progressive from the perspective of their further use in the practice itself.

The basic aspect of education in the current context is the flexibility of education, understood as the possibility of re-organizing education depending on various interests or needs. [11] Excellent opportunity for making such modernization bring us just a distant and virtual laboratory. Both creates a student environment in which they can easily orient and educate and educate, using basic knowledge they can usually dispose of today. In teaching, we often encounter a problem of weak timeframe booked for the

volume of knowledge that students should cover. [12] Remote and virtual laboratories from-the teacher from worrying with aids for hours and facilitate it preparation for an hour, thereby gaining time. Pupil, with the help of remote and virtual laboratories, can discover and verify the authenticity or mathematical, physical or chemical phenomena and the laws that would never be examined, respectively. It could be more challenging. Using remote and virtual laboratories and teachers acquire the necessary time flexibility.

For a better understanding, let's give an example and clarify what remote and virtual laboratories are. One of such laboratories is, for example, a laboratory exploring radiation. [13] This experiment serves to develop foundation and manipulation with radioactive materials in education and practice (Figure 1). Without these remote laboratories, the student would only acquire theoretical knowledge of radiation, but with this experiment can verify their knowledge in practice and also to try and absorption of gamma radiation different materials.



Fig. 1. View of the remote laboratory “Radiation”

Laboratories like this is oriented in our REMLABNET consortium and is located on the web domain www.remlabnet.eu. New form (developmental version of REMALBNET) is oriented on the test page <https://www.remlabnet.truni.sk>, where are inputting our remote and virtual laboratories.

2 Purpose

Our assigned work was to create a laboratory that could be used at different levels of education and has been publicly accessible to teachers to include as a tool in the

teaching process. The quantity of the possibilities offered the choice to solve solar system. Primary school teachers can use this laboratory at the first stage in the fourth year of the science at the topic of the universe. Here, the teacher can show the pupils as our solar system works and show them as it looks in the universe. Furthermore, this model is used for pupils of the 9th grade of primary schools, where they learn about the solar system and planets in her physics lessons. From our point of view, the most interesting application is used in secondary schools where this model is also useful for mathematics. Here the teacher can explain and show the pupils using vectors in practice. This model can be extended and specially to supplement theoretical knowledge. Information in the Moodle (our learning management system) environment becomes one such addition. Here, in addition to ordinary information about planets, we also mention the curvature of the universe, so our student can gain more comprehensive knowledge about the origin and evolution of the universe. Figure 2 shows typical information for the planet Uranus.



Fig. 2. Typical information published in theoretical part of laboratory

The main area of our interest in this work is the movement of planets and their interaction. For this reason, it is necessary to express not only the elliptical movement of the planet, but also gravitational forces acting among them. We need a simple, fast and

comfortable solution for the variables. In order to continue whether in programming or minimally in terms of relationships, we must create a mathematical model of behaviour of the system that is in our case solar system containing planets.

The concept of a mathematical model is meant as an abstract model that uses a math language to describe system behaviour [14]. The mathematical model has the following advantages [15]:

- standardized registration formalization,
- precisely defined rules for manipulating mathematical symbols and structures,
- the possibility of using ICT for processing,
- and solving the model created.

In our mathematical model [16] we will deal with these variables that have the largest impact on a virtual laboratory created by us for our simulation:

- F – Force
- G - Gravitational Constant
- m – Mass
- r - Speed
- a – Acceleration

As we mentioned and how of these variables follows, we must create a mathematical model not only for movement, but also their interaction. It is clear from the new findings that the planets move in a straight trajectory and their elliptical orbit is caused by the curvature of space-time. Such a pathway is then called the ecliptic. In our case, however, we will describe mainly their interaction and we will stick to Newton's laws. That's why we have two methods in particular to choose from:

1. Differential equations
2. Vector functions

Theoretical information about vector functions were derived from [17] [18] [19] and about differential equations from [20] [21] [22] [16]. For this reason, we concluded that it will most suit the theoretical potential that we have been able to use. As another reason, we can also choose the use in practice. Differential equations belong to mathematical analysis. Their teaching is only implemented at universities with a technical or mathematical focus. On the other hand, vectors are part of the teaching plan at primary, secondary and universities with different degrees of difficulty. Therefore, we know that a mathematical model containing a vector function will be more useable in our area and our applications as a model using differential equations, despite the fact that differential equations could be a solar system to describe a relatively easier. They also use direct vectors, so that the theoretical incorporation and study of vectors is also required to handle and resolve the mathematical model using differential equations. When we take a mathematical model that uses a vector function, the entire calculation for the student uses our model much more clearly from the point of view of the amount of information necessary for its understanding.

3 Methods

After a thorough analysis and thought of a suitable method for solving our issue, we have chosen the use of vector functions. In this particular case (call it Problem of the 3Point), we solve a virtual laboratory with three planets. We reflect from the 3-planet output system, where each planet has its vector. In Figure 3, we have shown assumption as this system can be shown. Since we have already mentioned above, we are dealing with two coordinate vectors and are therefore in the case of 3Point program we are in a two-dimensional space where the third coordinate is neglected because it is not needed for this first simulation.

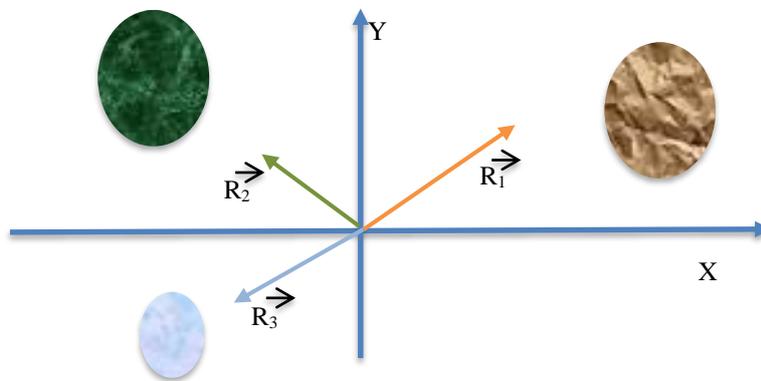


Fig. 3. The system of three planets

The whole of this simulation is the action of planets and their forces to each other. The ideal equation for our mathematical model is Newton's gravity law. From this definition we are dedicated to the equation about the strength that two bodies operate on each other. Strength F can be expressed in this shape:

$$F = G \frac{m_1 m_2}{r^2} \quad (1)$$

G tells us a gravitational constant, m_1 and m_2 indicate the weight of two different planets and r denoting their distance. For expressing force F we have chosen the use of the second Newton's law because it says that the force is the cause of movement change. The bodies, in our case of planet, will move either evenly or accelerate, slowed or change the direction. From definition, we can derive equation of force F :

$$F = ma \quad (2)$$

This expression of force F after being reached to the equation (1) we will receive the expression of each planet interaction with other planets:

$$ma = G \frac{m_1 m_2}{r^2} \quad (3)$$

After further steps of forming a mathematical model, we used the theory of vector functions. These include vector derivative, vector reading, and many more. At the end we received a set of six equations, where each one has featured a coordinate for one of the planets.

$$\ddot{x}_1 = \frac{Gm_2}{r_{12}^3}(x_2 - x_1) + \frac{Gm_3}{r_{13}^3}(x_3 - x_1) \quad (4)$$

$$\ddot{y}_1 = \frac{Gm_2}{r_{12}^3}(y_2 - y_1) + \frac{Gm_3}{r_{13}^3}(y_3 - y_1) \quad (5)$$

$$\ddot{x}_2 = -\frac{Gm_1}{r_{12}^3}(x_2 - x_1) + \frac{Gm_3}{r_{23}^3}(x_3 - x_2) \quad (6)$$

$$\ddot{y}_2 = -\frac{Gm_1}{r_{12}^3}(y_2 - y_1) + \frac{Gm_3}{r_{23}^3}(y_3 - y_2) \quad (7)$$

$$\ddot{x}_3 = -\frac{Gm_1}{r_{13}^3}(x_3 - x_1) - \frac{Gm_2}{r_{23}^3}(x_3 - x_2) \quad (8)$$

$$\ddot{y}_3 = -\frac{Gm_1}{r_{13}^3}(y_3 - y_1) - \frac{Gm_2}{r_{23}^3}(y_3 - y_2) \quad (9)$$

Since these coordinates are expressed by functions, we can write them to vector functions:

$$\vec{f}_1 = \left(\frac{Gm_2}{r_{12}^3}(x_2 - x_1) + \frac{Gm_3}{r_{13}^3}(x_3 - x_1), \frac{Gm_2}{r_{12}^3}(y_2 - y_1) + \frac{Gm_3}{r_{13}^3}(y_3 - y_1) \right)$$

$$\vec{f}_2 = \left(-\frac{Gm_1}{r_{12}^3}(x_2 - x_1) + \frac{Gm_3}{r_{23}^3}(x_3 - x_2), -\frac{Gm_1}{r_{12}^3}(y_2 - y_1) + \frac{Gm_3}{r_{23}^3}(y_3 - y_2) \right)$$

$$\vec{f}_3 = \left(-\frac{Gm_1}{r_{13}^3}(x_3 - x_1) - \frac{Gm_2}{r_{23}^3}(x_3 - x_2), -\frac{Gm_1}{r_{13}^3}(y_3 - y_1) - \frac{Gm_2}{r_{23}^3}(y_3 - y_2) \right)$$

Whereas we consider the mathematical model as an abstract model using a mathematical entry for describing the behaviour of the system, creating our mathematical model we have acquired a tool to create many systems where it is possible to show the use of vectors and vector functions in practice, but also teaching vectors themselves. Based on the created mathematical model, several didactic, virtual aids for teaching natural sciences are planned. One of the applications that use our mathematical model is a virtual laboratory containing three for each other with each other (planets or objects). This laboratory is called “3 point problem” and is shown in Figure 4.

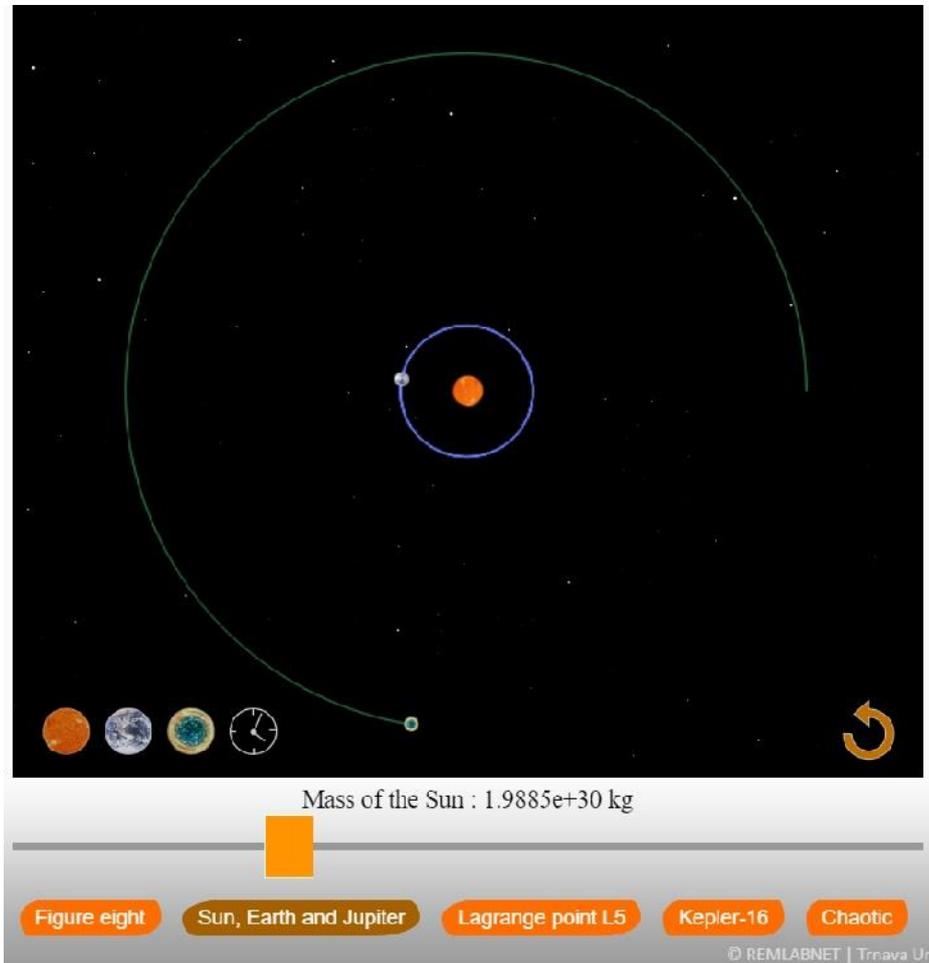


Fig. 4. Virtual laboratory - 3 point problem

After starting the 3 point problem application, which is available either through a REMLABNET system or by entering its external DNS record <https://3point.truni.sk>, will open the virtual laboratory itself. At this point, we would like to thank the creator of the core of the program, Mr. Evgenii Neumerzhitcki. We just modified his free JAVA code for our needs and completed the parts of the data collection. On Figure 4, there can be seen already ongoing simulation that is controlled by our equations.

Another use where we applied our equations is a 3D space of the second application that shows us all the planets of the solar system. We are presenting it in Figure 5. For higher years we have switched to a 3D model where a student can approach and delay, rotate into different directions and then track the circulation of the planet around the sun from different angles. This 3D model uses the x, y and z axis, but since we do not take into account that our solar system moves in space, this is negligible to us. Therefore, even with this virtual laboratory, a mathematical model is applied to us.

It is clear that this system is more perfect in terms of processing. It allows us to compare the time cycles of individual planets (in years), which can also be seen in Table 1. While in the table we present the exact data obtained from the Nasa website, the system slightly rounds our orbital paths and mentions them only in years. Let's use the example of Mars. While Earth travels its trajectory in 1 year, Mars needs 1.8809 years to traverse its trajectory. Our system will round it to a decimal place, so it will state the value of 1.9 years. This is an absolutely sufficient value for teaching purposes. For scientific use, of course, we can use the exact values and adjust the program, but so far there has been no interest in similar interventions.

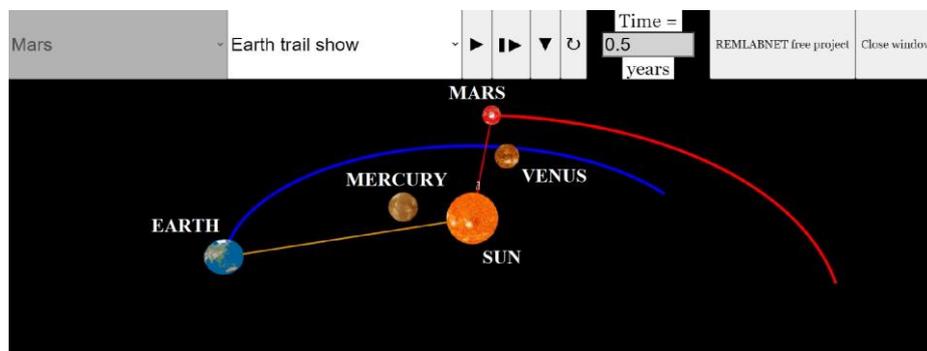


Fig. 5. 3D model of solar system, where we selected an interest in the trajectory of the planet Mars

4 Conclusion

The primary aim of the work was to create a mathematical model for a virtual laboratory. The objective set was fulfilled and used to use a vector function that appeared most easier for these reasons. We have applied the vector functions to the necessary form, where we have also had to use the necessary physical law and thus create a model useful in a wide sphere of teaching process. For primary schools, the model is used mainly on the subject of nature and physics, where students learn how the universe and the solar system works. For secondary schools this model is used for mathematics lessons when explaining vectors and their use in practice or even on physics lessons as a laboratory for solar system planets. High schools this mathematical model and the laboratory can be discussed with students to understand how they work vector features in practice and see their real use. Subsequently, they can enter further and deeper application of vector function methods.

Remote and virtual laboratories alone bring us a number of benefits. On the one hand, the pedagogue will be relieved from the suffocation with the carriage of various devices for hours, thus saving time and are also very convenient for capacitive needs, one experiment per pupil. For schools it is financially easier to implement a virtual laboratory in teaching to which access every student is to buy technique on the so-

called. Hands-on Laboratory. Another advantage is that students can look at the laboratory at home and thus helping him for further education. Such a laboratory is accessible via the Internet and access can be practically every 24 hours a day and 7 days a week. We live in a time when everything is quickly moved, and we need to customize very quickly. Thus, Virtual Laboratories bring us the opportunity to hold trends and approach students and in this way. What we can simulate in a virtual laboratory cannot be in the real world and it is possible one of the largest benefits of virtual laboratories. Future generations of scientists can explore and discover new things, quickly and easily, only with the help of electronic equipment and internet. We try to keep this trend and build it in the highest possible extent. The biggest contribution of this work is that it shows the linking of natural sciences such as mathematics, physics, but also chemistry and with the help of new technologies that provide informatics to build laboratories and paid research. Every research stimulus, which we give pupils and students can bring us new scientists and technicians in the future that this country needs. It approaches the students of the beauty of science and research, so I believe that we will work together to build new laboratories and give them freely available to students.

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