Fault Detection for Dynamic Systems based on Multirate Sampling

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Aibing Qiu[1], Junjie Shi[2] and Shengfeng Wang[1]
[1]Nantong University, Nantong, China
[2]Nantong Shipping College, Nantong, China

***Abstract*—In many complex systems such as distributed multi-sensor systems, different sensors may work with multiple sampling periods. In this paper, the problem of fault detection for this kind of systems is considered. Firstly, by mapping the multirate measurements onto fast time instants, a new measurement equation with variable dimensions is obtained. Based on this, a multirate Kalman filter based residual generator is designed to deliver residual signal at a fast rate, which is further evaluated in a dynamic window. The proposed scheme has no need to process some troublesome issues such as noise correlation and causality constrains caused by traditional lifting technique. A numerical simulation is presented to illustrate our approach.**

***Index Terms*—Fault detection, multirate Kalman filtering, residual evaluation**

I. INTRODUCTION

As modern industrial systems tend to be more and more complex and large in scale, the demand for higher system safety and reliability is continuously growing. However, no matter how to enhance the quality and reliability for individual components such as sensors, actuators, controllers and plants, an overall fault-free system cannot be guaranteed. Once a fault occurs, it may result in a degradation of system performance or even a crash. So it is important to detect fault promptly so that appropriate remedies can be applied. In this context, fault detection and isolation (FDI) has received considerable attentions from academia and industry in past three decades [1-3]. On the other hand, in many large scale systems like distributed multi-sensor systems and chemical process, the sampling rates of different sensors may not be equal due to numerous reasons such as hardware constraints and physical property of measurable variables, introducing the so-called multirate sampling [4-5]. Multirate systems can achieve some objectives that can not be accomplished by single rate systems and can provide a better tradeoff between performance and implementation cost [6]. However, some difficulties on design complexity and real time FDI have also been brought by multirate sampling.

A traditional way to deal with multirate sampling is using lifting technique to convert multirate systems into a slow rate LTI system [4]. Based on this technique, disturbance decoupling fault diagnosis observer, parity space based residual generators, H_∞ and H_2 optimal residual generators have been developed in [7-9], respectively. However, all above fault detection methods deliver residual signals at the end of the repetition period, which may result in detection delay and belong to slow rate fault detection schemes. It is reasonable to hope that residual can be obtained one after another in the repetition period to detect fault in time, i.e., fault rate fault detection scheme. Iman and Zhong et al. focus on the problem of optimal fast rate fault detection for multirate sampled systems [10-11]. Their research ideas are similar. Firstly, a set of slow rate residual generators is designed based on the lifted model of multirate sampled system. Then the fact that the optimal solution is not unique makes it possible to find some residual generators satisfying causality constraints. Finally, the slow rate (vector) residual will be inversely lifted to detect fault at the fast sampling rate. In [10], the design is under the framework of parity space, the key of the method is using parity matrix to replace parity vector so that a vector residual can be obtained. Applying the inverse lifting operation can deliver a scalar residual at every instant in the repetition period. Different from the residual generator in [10], an optimal fast rate fault detection filter designed in [11] by using of factorization technique generates the residual only when measurements are available. It should be stressed that in [10-11] most efforts focus on how to deal with the so-called causality constraints. The proposed methods are involved and not systematic. Therefore, some fast rate fault detection methods which can avoid causality constraint are pursued. As for stochastic multirate sampled systems, a show rate Kalman filter based residual generator is designed with the aid of lifting technique [12]. In [13], by combing the lifting and elementary transformation technique, the multirate sampled systems is firstly converted into a LTI system satisfying causality constrain. A sequential Kalman filtering algorithm is then proposed to generate residual at the fast rate. Although the proposed scheme can avoid the troublesome causality constraint, the correlations between process noise and measurement noise and the auto-correlations between measurement noises caused by lifting technique make the design to be very complicated.

In this paper, a simple and intuitive fast rate fault detection scheme is developed for multirate sampled system. Instead of lifting technique, we use measurement mapping technique to build a new measurement equation. Based on this, a multirate dynamic Kalman filtering method is designed to generate residual at the fast rate. On account of the variation of residual dimension, the residual is evaluated in a dynamic window to guarantee a constant threshold. The proposed scheme not only has no need to cope with causality constraint but also avoid the problem of noise correlations.

II. DESCRIPTION OF MULTIRATE SAMPLED SYSTEMS

Consider the following discrete time LTI system:

$$x(k+1) = Ax(k) + Bu(k) + Ef(k) + w(k) \tag{1}$$

$$y(k) = Cx(k) + Du(k) + Ff(k) + v(k) \tag{2}$$

where $x(k) \in \mathbb{R}^{n}$, $u(k) \in \mathbb{R}^{n_u}$, $y(k) \in \mathbb{R}^{m}$ denote the system state, the known control input and the measurable outputs, respectively. $f(k) \in \mathbb{R}^{n_f}$ is the unknown constant or time-varying fault to be detected. The process noise $w(k)$ and measurement noise $v(k)$ are zero mean white Gaussian random sequences with variances W and R. Further assume that the two noise sequences and measurement noises sequences are uncorrelated, i.e., $E[w(k)v^{\mathrm{T}}(k)] = 0$, $E[v(k)v^{\mathrm{T}}(k)] = R = diag\{R_1, \cdots R_m\}$. The initial state $x(0)$ is also assumed to be a Gaussian distributed random variable with the known mean $\bar{x}$ and variance P_0. The two noise sequences and the initial state are mutually independent. A, B, C, D, E, F are some known matrices with approximate dimensions

In multirate sampled systems, the control input $u(k)$ is generally updated at the fast sampling rate T [10-11]. Each component $y_i(k), i = (1, \cdots, m)$ in the system outputs is sampled at the different rate $n_i T$. Define $N = \mathrm{l.c.m.}(n_1, n_2, \cdots, n_m)$, where l.c.m. stands for least common multiple. It is easy to prove that the multirate sampled system described above is a periodic time varying system with the repetition period $T_s = NT$ [11]. The we can obtain that $Q_i = N/n_i$ represents the number of i th output data in one repetition period. $Q = \sum Q_i$ is the total number of all output data in one repetition period. As for this class of periodic system, discrete lifting technique is generally used to convert it into a slow rate LTI system with basic period T_s. However, discrete lifting technique extends the dimension of system output and makes the process and measurement noises correlated, which add difficulties to the analysis and synthesis of system.

To overcome the above problem and be capable of fast rate real time fault detection, the dynamic features of multirate sampled system will be considered at every fast rate time instant. Therefore, mapping all multirate sampled measurements onto fast rate time instants, measurements equation (2) can be rewritten as

$$\bar{y}(k) = \bar{C}(k)x(k) + \bar{D}(k)u(k) + \bar{F}(k)f(k) + \bar{v}(k) \tag{3}$$

where $\bar{y}(k)$ is the mapping of multirate sampled measurement at time instant k. $\bar{C}(k)$ is the row-combination of the corresponding measurement matrices C_i, satisfying $\mathrm{mod}(k, n_i) = 0$ for $i = (1, \cdots, m)$. It is obvious that $\bar{C}(k)$ is a time varying function with period T_s. Similarly for $\bar{D}(k)$, $\bar{F}(k)$, $\bar{v}(k)$ and $\bar{R}(k) = E[\bar{v}^{\mathrm{T}}(k)\bar{v}(k)]$.

Remark 1. By measurement mapping, multirate sampled system has the new form of (1), (3). Although the dimensions of system matrices in (3) are varying at different time instants, the process and measurement noises are still independent, i.e., $E[w^{\mathrm{T}}(k)\bar{v}(k)] = 0$ and $\bar{R}(k)$ is a diagonal matrix.

Remark 2. At some time instant k, if it does not hold that $\mathrm{mod}(k, n_i) = 0$ for all i $(i = 1, \cdots, m)$, no measurement can be obtained at the time. It could be considered that the variance of $\bar{v}(k)$ at time instant k is infinity, i.e., $E[\bar{v}^{\mathrm{T}}(k)\bar{v}(k)] = \bar{R}(k) = \infty$

III. FAULT DETECTION DESIGN

Based on the multirate sampled system description (1), (3), the fault detection scheme is given in this section. Firstly, a multirate Kalman filtering algorithm is developed to generate residual, and then a corresponding residual evaluation is designed.

A. Residual Generation

The Kalman filter based residual generator is one of the first residual generation schemes [1]. However, since the dimensions of system matrices in [3] is time varying and there is no measurement at some time instant, the corresponding multirate Kalman filtering algorithm has some differences with the conventional Kalman filtering, as shown in the following.

Suppose at time instant $k-1$, one has the state estimation $\hat{x}(k-1 \mid k-1)$ and the covariance matrix $P(k-1 \mid k-1)$. Then the multirate Kalman filtering algorithm is recursively implemented by the two following steps.

One-Step Prediction:

$$\hat{x}(k \mid k-1) = A\hat{x}(k-1 \mid k-1) + Bu(k-1) \tag{4}$$

$$P(k \mid k-1) = AP(k-1 \mid k-1)A^{\mathrm{T}} + W \tag{5}$$

where $\hat{x}(k \mid k-1)$ and $P(k \mid k-1)$ denote one-step prediction state estimation and one-step estimation error covariance matrix, respectively.

Update:

Since there may be no measurement at some time instant k, the update step should be given under different situations. One can divide the time instant k into two categories: $\mathrm{k}_1 = \{k \mid \mathrm{mod}(k, n_i) \neq 0, i = 1, \cdots, m\}$ and $\mathrm{k}_2 = \{k \mid \mathrm{mod}(k, n_i) = 0, i = 1, \cdots, m\}$.

Case 1: For time instant $k \in \mathrm{k}_1$

$$\hat{x}(k \mid k) = \hat{x}(k \mid k-1), \quad P(k \mid k) = P(k-1 \mid k) \tag{6}$$

Case 2: For time instant $k \in \mathrm{k}_2$

$$\begin{aligned} \hat{x}(k \mid k) &= \hat{x}(k \mid k-1) + L(k) \\ &\quad (\bar{y}(k) - \bar{C}(k)\hat{x}(k \mid k-1) - \bar{D}(k)u(k)) \end{aligned} \tag{7}$$

$$\begin{aligned} L(k) &= P(k \mid k-1)\bar{C}^{\mathrm{T}}(k) \\ &\quad (\bar{R}(k) + \bar{C}(k)P(k \mid k-1)\bar{C}^{\mathrm{T}}(k))^{-1} \end{aligned} \tag{8}$$

$$P(k \mid k) = (I - L(k)\bar{C}(k))P(k \mid k-1) \tag{9}$$

where $L(k)$ is the filter gain.

The underlying idea of applying Kalman filter for fault detection lies in making using of the statistical property of innovation in the update step. Note that for $k \in \mathrm{k}_1$, no measurement means no innovation. Therefore, the residual signal can be defined as follows:

$$r(k) = \begin{cases} 0 & k \in k_1 \\ \bar{y}(k) - \bar{C}(k)\hat{x}(k \mid k-1) - \bar{D}(k)u(k), & k \in k_2 \end{cases} \tag{10}$$

The above equation (10) gives the form of multirate Kalman filter based residual generator. Next the available residual signal needs to be further evaluated to achieve successful fault detection.

B. Residual Evaluation

By further analysis of residual signal, one can find that for $k \in k_2$, the residual signal $r(k)$ is a zero-mean white Gaussian noise with variance $\sum(k) = \bar{R}(k) + \bar{C}(k)P(k \mid k-1)\bar{C}^{\mathrm{T}}(k)$ under the fault-free case. Define the following scalar function:

$$r_e(k) = \begin{cases} 0 & k \in k_1 \\ r^{\mathrm{T}}(k)\sum^{-1}(k)r(k) & k \in k_2 \end{cases} \tag{11}$$

Then $r_e(k), k \in k_2$ is a random variable satisfying χ^2 distribution under the fault-free case. When a fault occurs, the statistical property of $r_e(k)$ has changed. However, since the degree of freedom of $r_e(k)$ equals to the dimension of $\bar{y}(k)$, it is also time varying, which bring some difficulty to threshold setting. Recall that the system matrices are periodic time varying with period $T_s = NT$ and there are always Q measurements in one repetition period. Thus, we should evaluate the residual in a dynamic window with the length T_s:

$$r_f(k) = \sum_{k-N}^{k} r_e(k) \tag{12}$$

Obviously, $r_f(k)$ is a random variable satisfying χ^2 distribution with the degree of freedom equal to Q. Then the following decision rule can be made to determine the occurrence of the fault:

$$\begin{aligned} r_f(k) < \chi^2_\beta(Q) \quad &\Rightarrow \quad no\ fault \\ r_f(k) \geq \chi^2_\beta(Q) \quad &\Rightarrow fault\ occurs \end{aligned} \tag{13}$$

where β is the significance level. $\chi^2_\beta(Q)$ is the threshold that can be obtained using the table of χ^2 distribution.

Remark 3. It can be seen that $r_f(k)$ is generated at every time instant k, which means a fast rate fault detection scheme. Compared with the one proposed in [13], the method in this paper has advantages, such as simple structure, easy and intuitive to use, since it has no need to process the noises correlation caused by lifting technique.

IV. NUMERICAL SIMULATION

In this section, a numerical example is given to shown the effectiveness of the proposed method. The considered system is borrowed from [13], as follows:

$$A = \begin{bmatrix} 0.37 & 0.16 \\ 0 & 0.14 \end{bmatrix}, B = \begin{bmatrix} 1.00 \\ 0.43 \end{bmatrix}, B_f = \begin{bmatrix} 1.06 \\ 0.43 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$D = D_f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, W = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}, \bar{x} = [10 \quad 5],\ T = 1, T_1 = 2, T_2 = 3$$

Then we can get $N = 6$, $Q = 5$ and the system matrices in (3). It is assumed in simulation that $u(k) = 0$. Firstly, the state estimation result of multirate Kalman filtering algorithm (4)-(9) under the fault-free case is given in Fig.1. It can be seen that the estimation accuracy of State 1 is better than the one of State 2. This is mainly because that the sampling rate of Sensor 1 is faster than the one of Sensor 2.

The fault detection scheme is verified next. Let $f(t)$ be a constant fault with amplitude 0.5 occurring from 40 sec to 80 sec. Fig.2 shows the results of state estimation under faulty case. It can be seen that the performance of state estimation is poorer than the one under the fault-free case. Since $Q = 5$, by choosing $\beta = 0.05$ and inquiring the table of χ^2 distribution, one can get the threshold $\chi^2_{0.25}(5) = 11.071$. The result of residual evaluation is given in Fig.3. It shows that the proposed method can detect the occurrence and disappearance of the fault effectively.

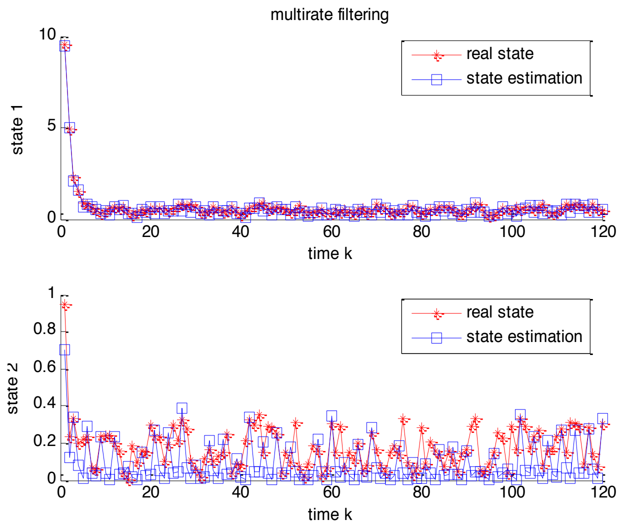

Figure 1. State estimation results under fault-free case

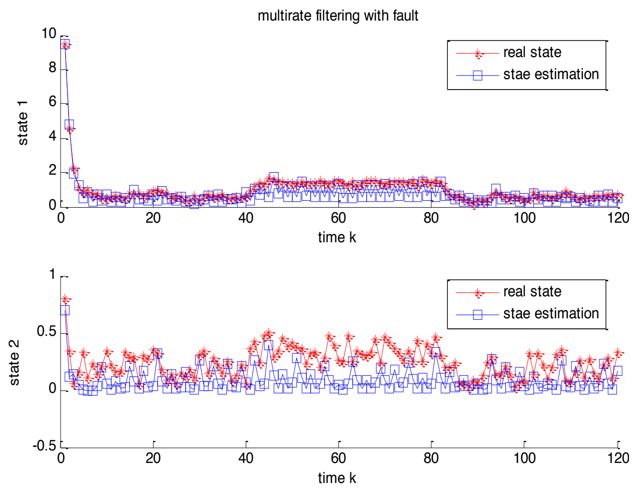

Figure 2. State estimation results under faulty case

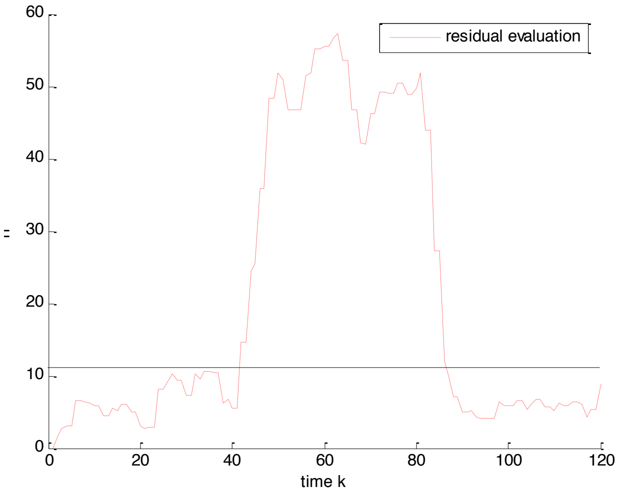

Figure 3. Residual evaluation

V. NUMERICAL SIMULATION

A multirate Kalman filtering based residual generation and evaluation scheme is developed for a class of stochastic multirate sampled systems. The proposed scheme can detect fault quickly and avoid the complicated problems of causality constraint and noises correlation, since it does not use the lifting technique to handle multirate sampled systems. However, the proposed scheme may have a high computation cost since it needs to calculate the inverse of covariance matrix on line. A feasible way to reduce the on line computation cost is to design a steady multirate Kalman filter based fault detection method, which will be investigated in our future work.

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AUTHORS

Aibing Qiu is with the School of Electrical Engineering, Nantong University, Nantong, China. His research interest covers fault diagnosis and sampled-data control systems. Corresponding author of this paper. (e-mail: aibqiu@ntu.edu.cn)

Junjie Shi is now a lecture of Mange Information Department, Nantong Shipping College, Nantong, China. (e-mail:sjj@ntsc.edu.cn)

Shengfeng Wang is now a lecture of School of Electrical Engineering, Nantong University, Nantong, China. (e-mail:wangsf@ntu.edu.cn)

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