

PAPER

Brain Tumor Localization Using N-Cut

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ABSTRACT

A brain tumor is an abnormal collection of tissue in the brain. When tumors form, they are classified as either malignant or benign. It is critical to notice and identify the existence of tumors in brain images since they can be life threatening. This paper illustrates a novel segmentation method in which threshold technique is combined with normalized cut (Ncut) for the segregation of the tumors from brain magnetic resonance (MR) images. Image segmentation is a technique for grouping images. It is a method of splitting an image into sections with comparable attributes such as intensity, texture, colour, and so on. In thresholding, an object is distinguished from the background, and for the proposed segmentation methodology, the threshold value is determined by normalized graph cut. A weighted graph is divided into disjointed sets (groups) in which the similarity within a group is high and the similarity across groups is low. A graph-cut is a grouping approach in which the total weight of edges eliminated between these two pieces is used to calculate the degree of dissimilarity between these two groups. The normalized cut criterion is used to calculate the total likeness within the groups as well as the dissimilarity between the different groups.

KEYWORDS

Ncut, segmentation, MR image, graph cut, thresholding

1 INTRODUCTION

The gray levels of pixels that belong to an object and those that belong to the background differ significantly in many image processing algorithms. In order to separate the items from their background, thresholding techniques might be applied. Thresholding is an important procedure in many image-processing applications, including document processing, image compression, particle counting, cell motion estimates, and object recognition. When it comes to processing time and ease of application, thresholding techniques are an effective way to segment images.

A thresholding technique works by comparing each pixel in a picture with a predetermined threshold value. The crux of a thresholding problem is choosing an acceptable threshold value, which divides or segments a gray-level image into object and background sections. An ideal threshold value is a level of gray that separates

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an image into two sections, a background segment and an object segment, without impairing the object's integrity. The best threshold value, however, is frequently difficult to determine automatically [1]. Applying fresh ideas and concepts to image thresholding is still an exciting and difficult topic of research, despite the fact that numerous methods for automatically determining thresholds have been put forth over the past few decades.

The image subdivision is done by the segmentation method to further classify it into its objects or regions of components making it a crucial step in image processing for medical science [2]. Image segmentation is done for locating objects and distinguishing various lines, curves, shapes, etc. Pixels and voxels are identified from an object by the implementation of segmentation as it is an important tool to display, measure, and extract features of an image [3, 4]. The structure of the brain is segmented by magnetic resonance imaging (MRI), which has received major importance because it can be separated from other technologies and can be used for brain tissue volumetric analysis such as Alzheimer's disease [5], schizophrenia, brain tumor detection, epilepsy, multiple sclerosis, Parkinson's disease, cerebral atrophy, etc. [6–8]

Image thresholding is one of the image segmentation methods used to distinguish the object from the background using a proper threshold value for a particular image [6]. Thresholding using the graph cut method can exactly distinguish an object from the background by considering the minimum Ncut value where the threshold value is the optimum one [9]. In some cases, it's difficult to distinguish objects from the background as the histogram of the image is bimodal or multimodal. In those cases, it is difficult to distinguish an object from the background by thresholding the image using the optimum threshold value based on the minimum of normalized cut. So, in this case, it is needed to segment the thresholded image again using the generalized eigenvalue problem based on normalized cut to get our object of interest, which gives a good segmentation result without any bias.

The main motivation of this work is to get good segmentation results using the generalized eigenvalue problem based on the normalized cut and the thresholding method based on the minimum normalized cut. These methods help us to distinguish our object of interest from the background without any bias and are applicable for medical image processing such as tumor detection from MRI brain image, infrared image detection etc.

The early thresholding method and its reviews are given in [10] and [11,12]. Thresholding techniques and their global correlated performance research are addressed by S. U Lee [13], C. Glasbey [14], and N. Otsu [15], who proposed techniques to maximize the between-class variance for the thresholding method. Emerging interest in image segmentation using the graph cut method is shown in [16–17]. The image in the graph cut technique is represented as a graph that contains vertices and nodes where nodes are the representation of each pixel and the edges are the distance between them. Two pixels belong to the same segment when the weight on the edge reflects their likelihood. The method of thresholding is rooted in the minimal Ncut rate, which is used for determining the threshold rate for the brain's MR image. The thresholded image's segmentation is done using the $N \times N$ (N – number of pixels) symmetrical weight matrix, which is based on pixels.

The organization of this paper is as follows: In section 2, for effectively partitioning of the brain's MR image by graphical method, the thresholding method of the new image is proposed and then segmentation is done on the thresholded MR brain tumor image by Ncut. In section 3, a detailed elaboration of the method involved is given. In section 4, results are described and in section 5, the paper is concluded.

2 GRAPH CUT-BASED THRESHOLDING

In the graph cut technique, the image is represented in the form of graphs, which means having nodes and vertices like a graph. So, every pixel is represented as a node, and the edges are the distances between those nodes.

In graph theory, a cut is a partition of the nodes that divides the graph into two disjointed subsets. The set of cuts of the cut is the set of edges whose ending points are in different subsets of the divided region. If edges are in its cut-set then they are said to be crossing the cut. In an unweighted undirected graph, we can say that the weight or size of a cut is the number of edges that are crossing the cut in an image. And in the case of a weighted graph, it is defined as the sum of the weights of all the edges crossing the cut.

The basic cuts in the graph theory are minimum cut and maximum cut. In the minimum cut technique, the size of the cut is not larger than the size of any other cut. A cut is maximum if the size of the cut is not smaller than the size of any other cut in the image. Figure 1(i) shown below depicts a minimum cut with a cut size of 2 as the graph is bridgeless and Figure 1(ii) shows a maximum cut with a cut size of 5. In general, finding a maximum cut in an image is computationally hard. In the minimum cut technique, each and every pixel of an image must be segmented. That means in this technique, each pixel in an image, even if those pixels are similar with respect to color or intensity or texture, are also needed to be cut. So, the minimum cut technique doesn't give a better-segmented image as compared to the other techniques. So, this limitation of minimum and maximum cuts leads to the emergence of the normalized cut method to segment the image, which includes a new measure of disassociation between two groups to avoid unnatural bias for partitioning out small sets of points.



Fig. 1. (i) Minimum cut (ii) Maximum cut

2.1 Normalized Graph Cut (Ncut)

The normalized cut is a global criterion for segmenting graphs used in image data rather than focusing on local features and consistencies [18]. This algorithm is used as a criterion to measure total dissimilarity between various groups and total similarity within the groups. This technique can be applied to static general and medical imaging. Here each pixel is represented as a vertex or node and the distance between those nodes is the edges. Each edge in the model could contain a value (weight), which could be used as flow or importance of it. This kind of graph is called a “weighted” graph.

The term “cut” refers to eliminating a set of edges in an image to make the graph “unconnected”, and the value of the cut is the total weight on this set of edges. For example, if all the thick edges in the given Figure 2 will be eliminated, then the nodes with white color will be “unconnected” to the nodes with dark color, and thereafter

it can be said that the graph has been separated into two connected graphs. So, from the graph theory, the image segmentation problem is modeled as a graph cut problem. The weights on edges have a similar meaning between pixels. Thus, if we want the division of two pixels into two different regions, then their similarity is expected to be very small.

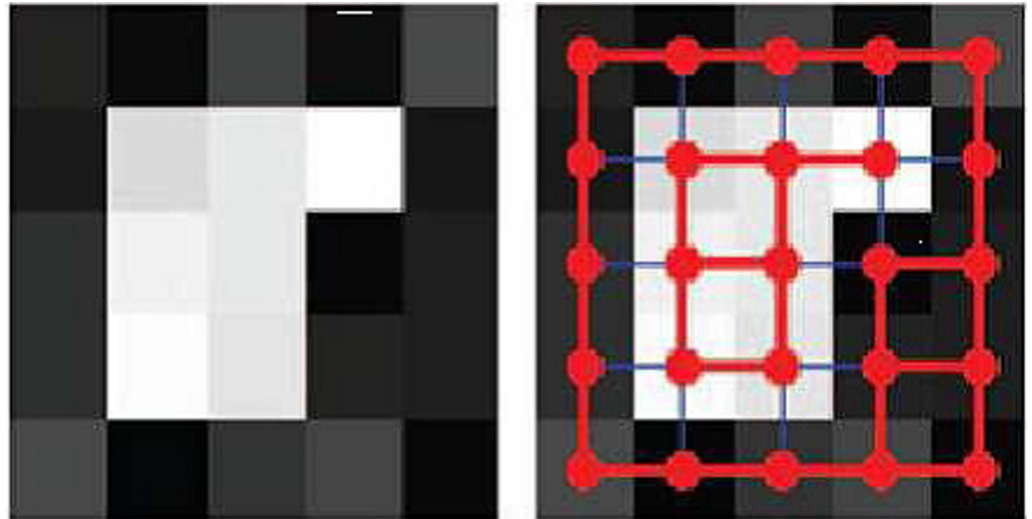


Fig. 2. Normalized cut framework

The threshold to segment the image can be determined using a graph cut. As per the algorithm, the set of points in an arbitrary feature space is represented as a weighted undirected graph $G = (V, E)$, where V is the set of vertices, E is the set of edges, and the cardinality of V is $N = |V|$. An edge is formed between each pair of nodes, and the weight on the edge, $z(s, t)$, is a function describing the similarity between two nodes s and t .

The complementary disjointed sets P and $Q = V - P$ are the two partitions of graph G . The total weight of the boundaries linking the two segments is responsible for the calculation of the degree of unlikeness among the two sets and this relates to the formulation of a cut [16, 18]. This formulation of a cut (expressed as C) is given in the following equation (1)

$$C(P, Q) = \sum_{s \in P, t \in Q} z(s, t) \quad (1)$$

The graph G is optimally bi-partitioned by this method. The proposal of Shi and Malik [18] described a new value termed normalized cuts or Ncut (NC), which is the disassociation between the two sets to halt abnormal prejudice of dividing into small sets of nodes, which are formulated as follows.

$$NC(P, Q) = \frac{C(P, Q)}{Ass(P, V)} + \frac{C(P, Q)}{Ass(Q, V)} \quad (2)$$

The relation between nodes of P to all other nodes in the graph is given by $Ass(P, V) = \sum_{s \in A, r \in V} z(s, r)$. Here, $Ass(Q, V)$ is the bi-partitioning process for minimizing N-cut, which is also the optimal bi-partitioning of G . Equation (2) can be converted according to the standard eigensystem as follows

$$R^{-\frac{1}{2}}(R-W)R^{-\frac{1}{2}}x = \lambda x \quad (3)$$

Here, matrix W is symmetrical with elements $w(i, j)$ and R is a diagonal matrix ($N \times N$) having diagonal elements $r_i = \sum_j w(i, j)$, where eigenvalues λ and x with their corresponding eigenvectors are shown. The entire graph is optimally sub-partitioned by the eigenvector corresponding to the second smallest eigenvalue, which is a real-valued solution [18,19].

Due to the fact that eigenvector elements might take on continuous values, a heuristic approach was chosen for the splitting-point search. Up until the value of N_{cut} exceeds a predetermined threshold, the method is repeatedly applied to each subgraph. When the size of the weight matrix is high, the normalized-cut approach is computationally expensive since the pixel-based weight matrix necessitates computing the $N \times N$ weight matrix and solving the eigensystem as mentioned in equation (3).

Although the approximate eigenvalue approach and the structure of the algorithm optimize the implementations, the computational complexity for an image of a moderate or large size remains unreasonably high. Additionally, the choice of parameters has a significant impact on the partitioning procedure's effectiveness and stability, although the ideal values are more or less data-dependent. Because the smallest nonzero eigenvalues for a given eigensystem with sparse matrices frequently have very small magnitudes, the arithmetic precision affects the choice of the splitting point in a heuristic manner. The choice of the splitting point, the accuracy of the eigenvalue calculations, and the relative segment position are only a few of the variables that affect stability. The practical use of normalized cuts may be constrained by all these facts.

2.2 Proposed approach for calculation of the optimal threshold using N_{cut}

Suppose, $V = \{(i, j): i = 0, 1, \dots, n_h - 1; j = 0, 1, \dots, n_w - 1\}$, $L = \{0, 1, \dots, 255\}$, in which the variables are n_h (height) and n_w (weight) of the image respectively. Let $g(u, v)$ be the gray-level value of the image at the pixel (u, v) . V and $g(u, v)$ satisfy

$$g(u, v) \in L, \forall (u, v) \in V \quad (4)$$

$$V_k = \{(u, v): g(u, v) = k, (u, v) \in V\}, \quad k \in L \quad (5)$$

$$\bigcup_{k=0}^{255} V_k = V, V_j \cap V_k = \emptyset, k \neq j, \quad k, j \in L \quad (6)$$

Taking each pixel as a node and linking each pair of them by an edge, the construction of $G = (V, E)$ graph is achieved. Two pixels belong to the same object when the weight on the edge reflects their likelihood. The weight of the graph edge linking the two nodes s and t may be determined just by the brightness of the pixels and their spatial locations by using the equation (7). $F(s)$ and $X(s)$ stand for the gray-scale and spatial location of node s , respectively. Additionally, d_i and d_x are positive scaling factors that specify the number of neighboring nodes included in the weight calculations, respectively, and r is a positive integer that indicates the sensitivity of $z(s, t)$ to the intensity difference and spatial location between the two nodes.

The number of nodes involved in computing the weight grows as r does, increasing the computational cost.

$$z(s,t) = \begin{cases} - \left[\frac{\|F(s) - F(t)\|_2^2}{di} + \frac{\|X(s) - X(t)\|_2^2}{dx} \right] & , \text{ if } \|X(s) - X(t)\|_2 < r \\ 0 & , \text{ otherwise} \end{cases} \quad (7)$$

For any $T (0 \leq T < 255)$, a unique bisection $V = \{P, Q\}$ was acquired of the associated graph $G = (V, E)$ with sets P and Q expressed as:

$$P = \bigcup_{k=0}^T V_k, Q = \bigcup_{k=T+1}^{255} V_k, k \in L \quad (8)$$

$$\begin{aligned} C(P, Q) &= \sum_{s \in P, t \in Q} z(s, t) = \sum_{s \in P} \left[\sum_{t \in Q} z(s, t) \right] \\ &= \sum_{i=0}^T \sum_{s \in V_i} \left[\sum_{j=T+1}^{255} \sum_{t \in V_j} z(s, t) \right] \\ &= \sum_{i=0}^T \sum_{j=T+1}^{255} \left[\sum_{s \in V_i, t \in V_j} z(s, t) \right] \\ &= \sum_{i=0}^T \sum_{j=T+1}^{255} C(V_i, V_j) \end{aligned} \quad (9)$$

$$\begin{aligned} Ass(P, P) &= \sum_{s \in P, t \in P} z(s, t) = \sum_{i=0}^T \sum_{j=i}^T \left[\sum_{s \in V_i, t \in V_j} z(s, t) \right] \\ &= \sum_{i=0}^T \sum_{j=1}^T C(V_i, V_j) \end{aligned} \quad (10)$$

$$\begin{aligned} Ass(Q, Q) &= \sum_{s \in Q, t \in Q} z(s, t) = \sum_{i=T+1}^{255} \sum_{j=i}^{255} \left[\sum_{s \in V_i, t \in V_j} z(s, t) \right] \\ &= \sum_{i=T+1}^{255} \sum_{j=i}^{255} C(V_i, V_j) \end{aligned} \quad (11)$$

$$\begin{aligned} Ass(P, V) &= Ass(P, P) + C(P, Q) \\ Ass(Q, V) &= Ass(Q, Q) + C(P, Q) \end{aligned} \quad (12)$$

Hence, (2) develops as

$$NC(P, Q) = \frac{C(P, Q)}{Ass(P, P) + C(P, Q)} + \frac{C(P, Q)}{Ass(Q, Q) + C(P, Q)} \quad (13)$$

Now, for every possible threshold, T normalized cut (NC) computation can be easily done. Threshold level T that provides minimal Ncut value among rest threshold levels within the dynamic range is taken as the image's thresholding point. The threshold value that is searched by this thresholding method minimizes the image's normalized cut.

3 ALGORITHM FOR TUMOR DETECTION USING NCUT AND THRESHOLD IN BRAIN MR IMAGES

1. To identify optimum threshold point in the MR brain image the thresholding method is implemented using equation (13) with minimal Ncut value. Then for the image, $Ass(P,P)$, $Ass(Q,Q)$ and $C(P, Q)$ are calculated.
2. The Ncut value is checked for every threshold T_h value ($0 \leq T_h \leq 255$). For minimal Ncut value, the threshold T is chosen as optimum threshold point.
3. Then a second smallest eigenvalue is taken for the image to be optimally bi-partitioned because optimally thresholded brain tumor image segmentation is done by normalized cut.
4. For the desired result, the image is recursively bi-partitioned.

The suggested thresholding method minimizes the associated normalized cuts of the image while locating the ideal threshold value. Figure 3 depicts the proposed approach, where T is the optimum value of threshold, T_h is a variable threshold, and $Ncut_{min}$ is the minimum value of the normalized cut.

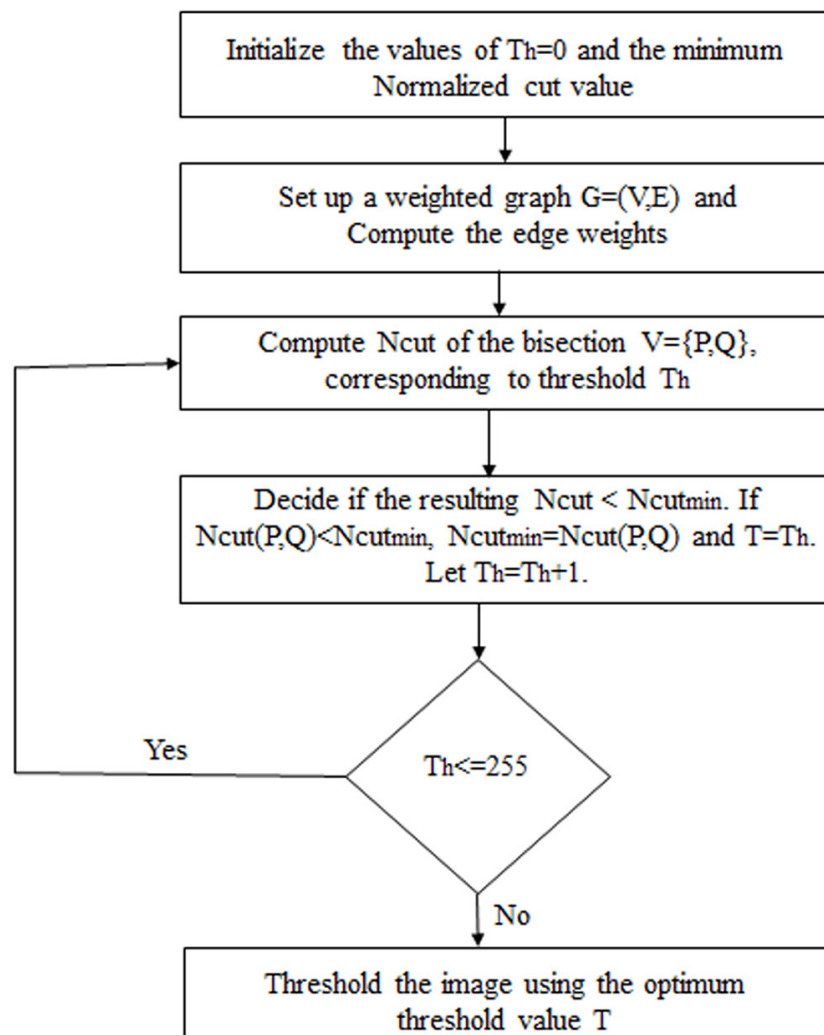


Fig. 3. Flowchart for implementation of the suggested algorithm

4 EXPERIMENTAL RESULTS

The correct values for the variables r , dx , and di are taken for the calculation of the T (threshold) for the MR brain image. An optimally thresholded image with a T value is shown in Figure 4.

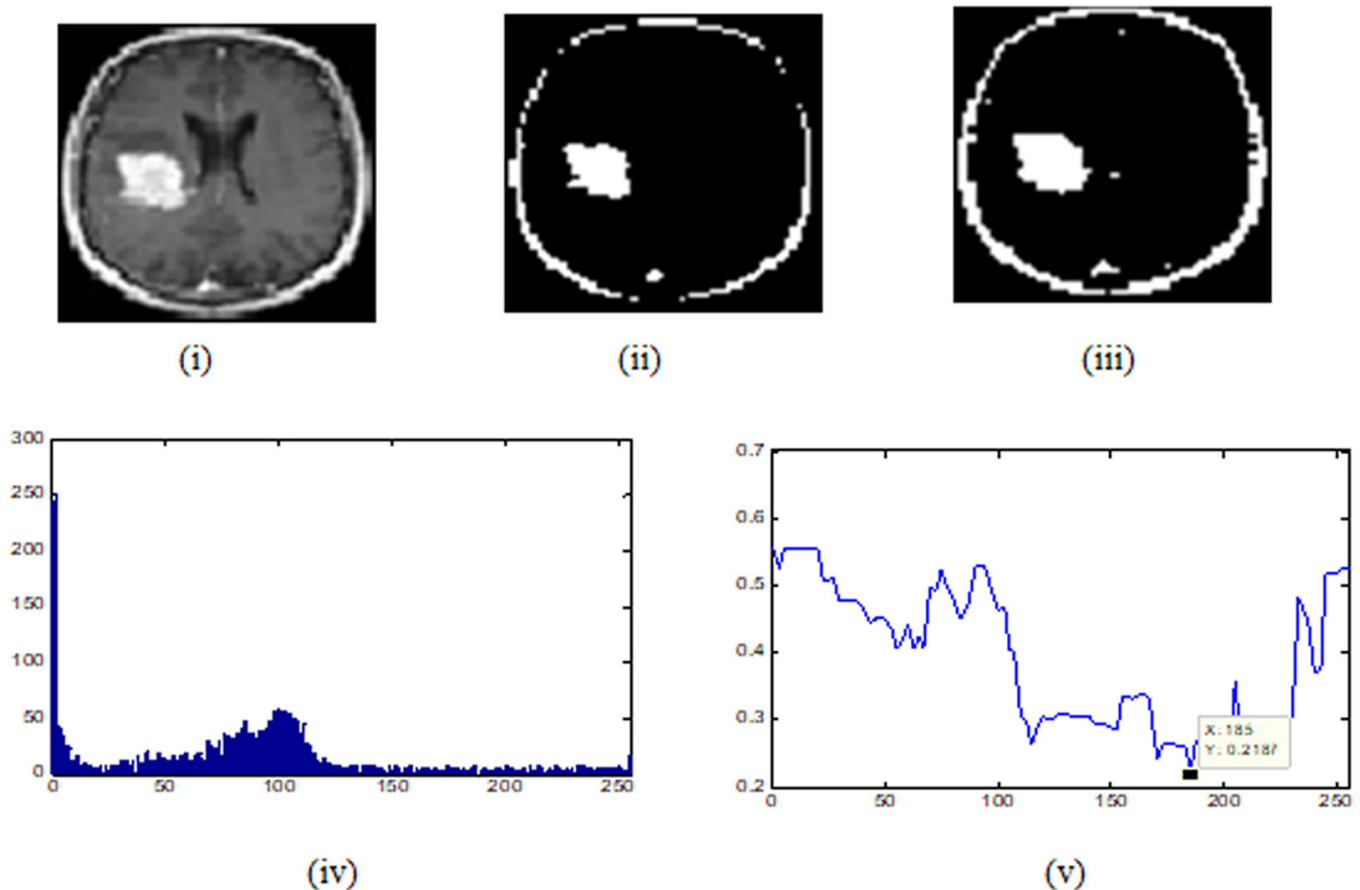


Fig. 4. (i) Original brain image (MR) (ii) thresholding result by the proposed method ($T = 185$) (iii) manually thresholding image ($T = 140$) (iv) histogram of the original image (v) value of N_{cut} versus threshold T

The performance of the proposed algorithm for threshold calculation can be quantified by calculating the absolute error ratio as expressed in equation 14. It is defined as a ratio of absolute difference in the number of object pixels in the optimal thresholded image and manual thresholded image

$$r_{err} = \frac{n_{diff}}{N} \times 100 \quad (14)$$

Taking threshold $T = 140$ (manually) and threshold $T = 185$ (thresholding method implemented result) we get the absolute error value as 3.97. The optimal threshold (for $0 \leq T_n \leq 255$) is determined by the method of thresholding with minimal N_{cut} value, which minimizes the image's normalized cuts. The obtained results have the same intensity values from the thresholding process that consisted of tumors with further unwanted pixels.

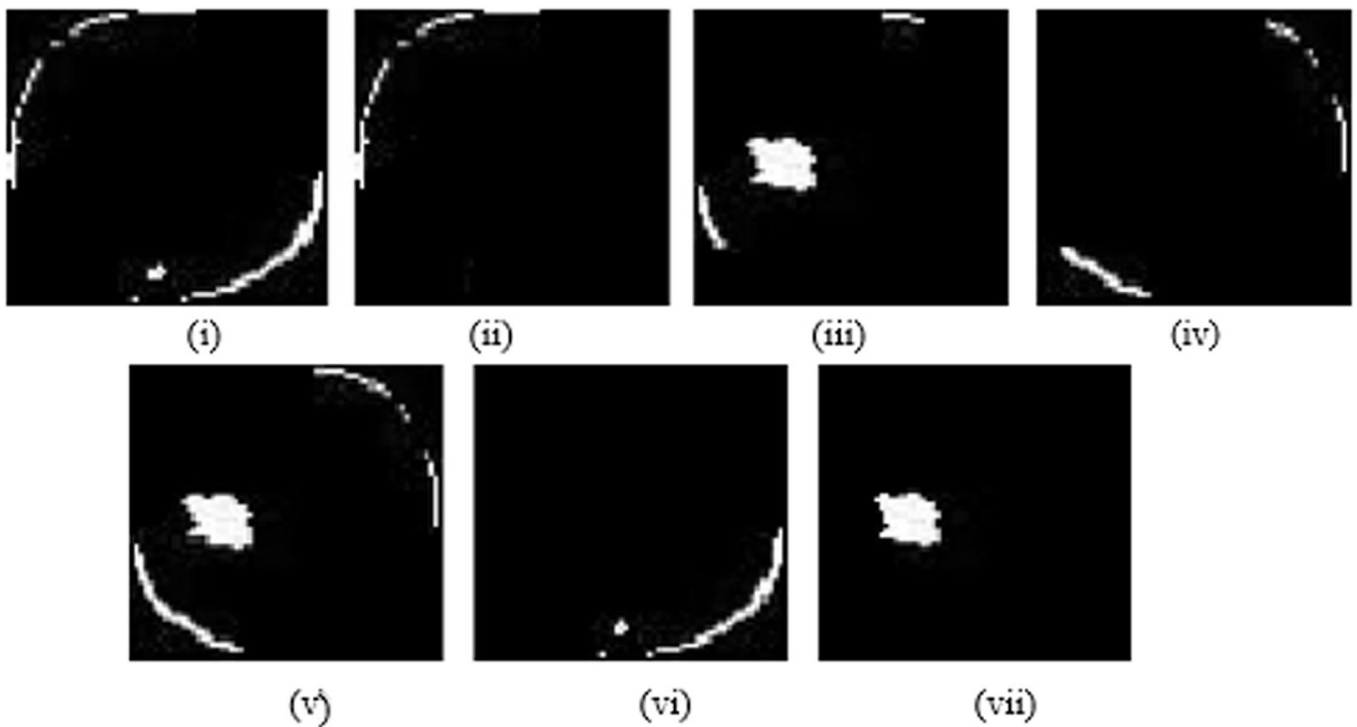


Fig. 5. (i)–(vii) displays partition components, (vii) displays location of the tumor

The thresholded image segmentation gives specific location of tumor and the thresholded result shows its boundaries. Then the optimally thresholded image from Figure 4(ii) along with the outcome obtained from the partitioning of image from Figure 5(vii) was considered and the absolute error ratio was found as 0.66, which is less in comparison to the existing absolute error ratio.

5 CONCLUSION

In this study, a thresholding algorithm based on the normalized-cut measure has been established. The proposed method requires significantly fewer computations than existing graph-cut-based image segmentation approaches, which makes them unsuitable for real-time vision applications like biomedical imaging because of their high computational complexity. The normalized cut avoids small sets and also assumes some degree of intensity homogeneity within the object and background as well as some degree of intensity discontinuity between the object and the background. Due to the great homogeneity inside the object, the approach appears to perform effectively even in circumstances when the background is not homogeneous. The creation of a new weight matrix based on gray levels rather than pixels allows for a significant reduction in computational expense and memory storage. Additionally, the thresholding principle of the normalized-cut measure allows for discriminating an object from the background without bias. The graph-cut value can easily be acquired for each potential thresholding value and identify the ideal threshold values as the weight matrix is compact and constant in size. The localization of a tumor in a brain MR image demonstrated the superiority of the normalized cut as a thresholding principle over alternative graph-cut measures and other conventional

methods for determining the threshold value for image segmentation. The usefulness of the suggested method is well-proven by resulting in less absolute error ratio.

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