

Multi-agent Finite-time Ideal Convergence Algorithm Based on Bilinear Manifold

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Abstract—Based on the traditional belief updating model, a concept of belief distance is put forward in this paper. On the basis of this concept, a new belief distance updating model for the multi-agent system is proposed, and the rationality of the model is proved. In this model, the belief distance updating process of the multi-agent system is described, and the linear system is used to describe the belief convergence process of the multi-agent system, which has simplified the complexity of the belief reachability analysis for the multi-agent system. On the basis of this model, the belief reachability for the multi-agent systems in the network control is analyzed, and the necessary and sufficient conditions for judging the belief reachability of the multi-agent system are given.

Keywords—Network Control; Multi-agent; Reachability of Belief; Bilinear Manifold Network

1 Introduction

With the rapid development of network technology and application and the emergence of multi-terminal and the integration with the multi-service, the Internet has shown more and more complex, heterogeneous, ubiquitous and other characteristics, which makes the current network more and more complicated and difficult to control. In order to improve the controllability of the network, a lot of literatures at home and abroad put forward the architecture of new bilinear manifold network [1-2] at present. As the multi-agent system has the distribution, autonomy, sociality and other characteristics, the realization of network control by using the multi-agent system is conducive to reducing the complexity of the network control and improving the reliability and intelligence of the network control. And the use of multi-agent system to realize the network control has become an effective means to implement the bilinear manifold network [3].

In order to realize the controllable network, in the network control based on the multi-agent system, each agent makes synergic control on the network through negotiation. The multi-agent system has the characteristics of decision logic concentration and management behavior distribution [4], requiring that each agent should make distributed control on the network according to the current belief, and that the decision of each agent will not generate conflict. Therefore, to achieve the network con-

control by using the multi-agent system, it is necessary to ensure that each agent before making the decision should have the belief that is consistent with the current network status, that is, the multi-agent system should have the belief reachability [5]. In the bilinear manifold network, what people are concerned is whether the belief of the multi-agent system has the reachability under the current network conditions as well as the belief convergence rate of the system, while they are not concerned about the specific behavior of each agent. In this case, if the traditional belief updating model of the multi-agent system is used to study the multi-agent system belief updating model, a large volume of unnecessary calculations will be introduced [6].

Aiming at the above problem, a belief distance concept and a new belief distance updating model for the multi-agent system in the new bilinear manifold network is put forward on the basis of the traditional multi-agent belief updating model in this paper, and the rationality of the model is proved. The model describes the belief distance updating process of each agent in the multi-agent system, regardless of the specific behavior of each agent in the belief updating process. A linear system is used to describe the belief convergence process of the multi-agent system, which has simplified the belief updating process of the agent, and provides an abstract platform for the direct research of the belief reachability of the multi-agent system in the bilinear manifold network. On the basis of the model, the belief reachability of the multi-agent system with bilinear manifold network realized in the normal environment is analyzed, and the upper bound of the necessary and sufficient conditions for the belief reachability that the multi-agent system has is given.

2 Multi-agent Belief Reachability Algorithm

2.1 Multi-agent System Belief Distance Updating Model

As network control is a dynamic process, the belief updating process of each agent may be interrupted by the network events at any time, people are not concerned about the updating behavior of each agent in the process of the belief updating, but are concerned about whether the belief of each agent can reach its reachability rate under the current network conditions and the magnitude of the reachability rate. Therefore, in this section, a new model for the analysis of the belief reachability and the reachability rate of the multi-agent systems in the bilinear manifold network is provided.

In the bilinear manifold network, as the observability of each agent is different, they can only observe the local state changes in their own surroundings, and the other relevant knowledge must be obtained from the other agents, which requires that each agent should conduct constant interaction to achieve the consistency of their own belief with the network state. A complete agent belief interaction process is a closed-loop change - observation - updating - change again - observe again - updating again process, and it ultimately comes to the consistency of the belief and network status, as shown in Figure 1. For example, when the agent is making decision on the route, the routing changes within an AS will lead to the change of the related route within another AS, at the same time, the change process of the latter may in turn lead to the

further change of the former route. Through the repetition, it finally reaches the stability of the two AS routing. In the bilinear manifold network, the state of the network at the moment t can be expressed by an n dimensional vector $y(t)$, and the belief of $agent_i$ at the moment t can also be represented by an n dimensional vector $x^i(t)$.

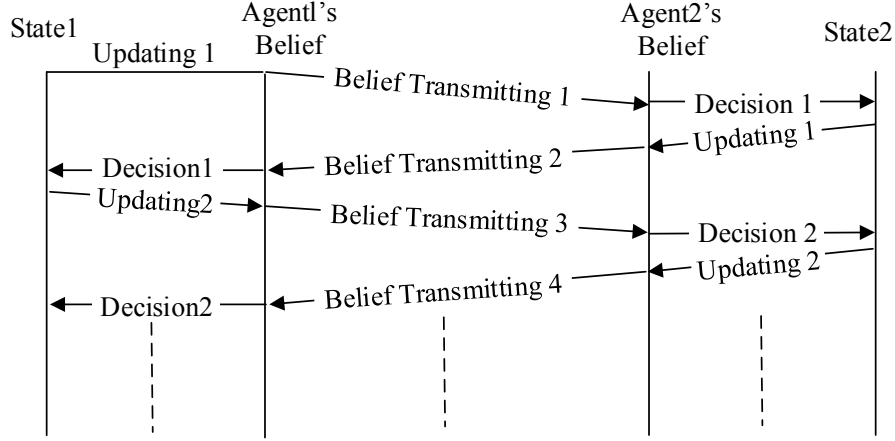


Fig. 1. Multi-agent system belief interaction process

Definition 1 (Belief distance). At the moment t , the belief distance $\tilde{x}^i(t)$ of $agent_i$ is defined as the Euclidean distance from the belief $x^i(t)$ of $agent_i$ to the network state $y(t)$, that is

$$\tilde{x}^i(t) = \|x^i(t) - y(t)\| \quad (1)$$

At the moment t , the belief distance of each agent can form an n dimensional vector $\tilde{X}(t)$. And it can be considered that when the belief distance of each agent is 0, the belief of the agent is consistent with the actual state of the network, that is, the SCE state. Therefore, SCE state can be defined as the following:

Definition 2 (SCE state). At the moment t , if the belief distance $\tilde{x}^i(t)$ of any of the agents in the multi-agent system is 0, that is

$$\tilde{x}^i(t) = 0 \quad (2)$$

It is referred to as that the system belief has reached the SCE state at the moment t .

Definition 3 (belief reachability). If the belief of the multi-agent system converges to the SCE state, that is

$$\lim_{t \rightarrow \infty} \tilde{X}(t) = 0 \quad (3)$$

It is referred to as that the belief of the multi-agent system has the reachability.

It can be obtained from the belief updating process of the agent that, the belief distance of each agent to the network state is constantly updated through the interaction between each other. In order to make it more convenient to study the belief reachability and the reachability of the multi-agent system in the bilinear manifold network, in this paper, a new belief distance updating model based on the updating process of the belief distance of each agent is put forward, as shown in the theorem a the following.

Theorem 1. In the bilinear manifold network, the dynamic updating process of the belief of each agent can be expressed by the linear differential equations as the following:

$$\dot{\tilde{X}}(t) = A\tilde{X}(t) \quad (4)$$

In which, $\tilde{X}(t)$ stands for n dimensional vector, which shows the difference of the belief distance of each agent at the moment t , and A is a coefficient matrix

Prove: The difference equation of *agent_i* from the belief to the network state is as the following

$$\tilde{x}^i(t+1) + \tilde{x}^i(t) = \|x^i(t+1) - y(t+1)\| - \tilde{x}^i(t) = \|\eta^i(t+1)\| - \tilde{x}^i(t)$$

In which, η^{n+1} stands for the remaining distance of the network state fitting by the *agent_i*. In the the multi-agent system, the remaining distance of the network state fitting by the *agent_i* is caused by the difference of the belief from the network state of the agent in the process of interacting with the other agents. And the magnitude of the remaining distance is determined by the distance of the belief of each agent to the network state, that is, $\|\eta^i(t+1)\| = h(\tilde{x}^1(t), \dots, \tilde{x}^i(t), \dots, \tilde{x}^n(t))$. It can be simply considered that h is a linear function, that is, $\|\eta^i(t+1)\|$ can be expressed as a linear combination of $\tilde{x}^j(n)$, that is, $\|\eta^i(t+1)\| = \sum_{j=1}^n (b_{ij} \tilde{x}^j(t))$, in which, $0 \leq b_{ij} \leq 1$.

Let $a_{ij} = b_{ij}$ ($i \neq j$), then $a_{ii} = b_{ii} - 1$. Hence the difference equation of the belief of the *agent*_{*i*} to the network state can be written as the following:

$$\tilde{x}^i(t+1) - \tilde{x}^i(t) = \sum_{j=1}^n (a_{ij} \tilde{x}^j(t)) \quad (5)$$

Therefore, the belief updating process of the multi-agent system can be expressed as the difference equation form as $\dot{\tilde{X}}(t) = A\tilde{X}(T)$. Thus it is proved.

In the equation (5), a_{ij} stands for the coefficient of the ratio of the distance change of the belief of the related *agent*_{*i*} from the network state in the next moment caused by the change in the network state which is perceived by the *agent*_{*j*} at the moment t , and $0 \leq a_{ij} \leq 1$ ($i \neq j$), $-1 \leq a_{ii} \leq 0$, which is referred to as the belief coupling between the agents. The coefficient matrix A is referred to as the belief coupling matrix.

2.2 Analysis of the Multi-agent System Belief Reachability

In the bilinear manifold network, the belief reachability of the multi-agent system is the basis for the effective control of the network. Each agent can control the network only when it has the same belief, so as to avoid the ambiguity on the control of the network. In this section, based on the belief distance updating model of the multi-agent system, the related knowledge of modern cybernetics will be used to make analysis on the realization of the belief reachability of the multi-agent system of the bilinear manifold network in general network environment, and the corresponding examples will be given.

Theorem 2. The belief of the multi-agent system has the reachability, if and only if all the eigenvalues of the belief coupling matrix have the negative real part.

Proof: Assuming that A has the different eigenvalue $\lambda_1, \lambda_2, \dots, \lambda_n$, then the non-singular transform matrix must be constructed from n independent eigenvectors so as to achieve the diagonalization of A , that is, $A' = P^{-1}AP$, in which A' is the diagonal matrix with $\lambda_1, \lambda_2, \dots, \lambda_n$ as the diagonal elements.

$$e^{At} = Pe^{A't}P^{-1} = P \begin{bmatrix} e^{\lambda_1 t} & & & \\ & e^{\lambda_2 t} & & \\ & & \dots & \\ & & & e^{\lambda_n t} \end{bmatrix} P^{-1} = P \sum_{i=1}^n E_i e^{\lambda_i t} P^{-1} = \sum_{i=1}^n P E_i P^{-1} e^{\lambda_i t} = \sum_{i=1}^n R_i e^{\lambda_i t},$$

In which, E_i stands for a matrix, the i -th element on its diagonal is 1, and the remaining elements are 0; $R_i = PE_iP^{-1}$ stands for a constant matrix. Therefore, the solution for the equation (4) is $\tilde{X}(t) = e^{At} \tilde{X}(0) = \left(\sum_{i=1}^n R_i e^{\lambda_i t} \right) \tilde{X}(0)$, in which $\tilde{X}(0)$ is the initial value.

Sufficiency: If $R_e(\lambda_i) < 0$ is established, $\lim_{t \rightarrow \infty} \tilde{X}(t) = 0$. Therefore, belief of the multi-agent system has the reachability. Hence the sufficiency is proved.

Necessity: The reduction to absurdity method is adopted. $\lim_{t \rightarrow \infty} \tilde{X}(t) = 0$, but there are some λ_i existing which make $R_e(\lambda_i) > 0$ and, then when $\tilde{X}(0) \neq 0$, the corresponding items in $\tilde{X}(t)$ will be extended infinitely, which is contradictory to $\lim_{t \rightarrow \infty} \tilde{X}(t) = 0$. Therefore, $R_e(\lambda_i) < 0$. In addition, if $R_e(\lambda_i) = 0$ is existent, the corresponding item of $\tilde{X}(t)$ is constant, which is also contradictory to $\lim_{t \rightarrow \infty} \tilde{X}(t) = 0$ at this point. Hence the necessity is proved

Theorem 2 gives the necessary and sufficient conditions for the belief reachability of the multi-agent system under the new belief distance updating model. It can be seen that the problem of the belief reachability of the multi-agent system in the bilinear manifold network is equivalent to the problem of the asymptotic stability of the linear system constructed by the equation (4).

3 Experiment and Analysis

Example 1: A_1, A_2, A_3 stand for the belief coupling matrices of the three multi-agent systems MAS1, MAS2 and MAS3, respectively, and each row of the matrix indicates the influence of other agents on the belief distance after the agent corresponding to this row has an interaction with the other agents. A_1, A_2, A_3 are generated randomly according to the requirement on the coupling matrix in the belief distance updating model put forward according to Theorem 1, as shown in the following:

$$A_1 = \begin{pmatrix} -0.84878 & 0.075706 & 0.063819 & 0.12743 & 0.085986 & 0.066239 \\ 0.11169 & -0.99357 & 0.042132 & 0.062858 & 0.015051 & 0.12193 \\ 0.16031 & 0.093739 & -0.94285 & 0.15005 & 0.12255 & 0.11411 \\ 0.027163 & 0.062052 & 0.1613 & -0.96943 & 0.0007 & 0.16308 \\ 0.12477 & 0.13213 & 0.079968 & 0.061386 & -0.89948 & 0.033964 \\ 0.062344 & 0.13254 & 0.061388 & 0.15291 & 0.15948 & -0.90112 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -0.9207 & 0.062538 & 0.039798 & 0.064406 & 0.070308 & 0.01094 \\ 0.021689 & -0.98948 & 0.010758 & 0.024773 & 0.073458 & 0.060394 \\ 0.04289 & 0.00358 & -0.95968 & 0.014821 & 0.058335 & 0.07496 \\ 0.053028 & 0.03091 & 0.0788 & -0.94243 & 0.062974 & 0.014225 \\ 0.033413 & 0.057775 & 0.030645 & 0.021995 & -0.91878 & 0.003585 \\ 0.040551 & 0.077985 & 0.027374 & 0.038145 & 0.033513 & -0.96007 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} -0.95034 & 0.076475 & 0.053419 & 0.04989 & 0.000489 & 0.061663 \\ 0.065152 & -0.96766 & 0.082992 & 0.018435 & 0.06805 & 0.06826 \\ 0.002799 & 0.039332 & -0.95837 & 0.033223 & 0.057574 & 0.021737 \\ 0.071415 & 0.007338 & 0.035838 & -0.9749 & 0.081749 & 0.058668 \\ 0.056884 & 0.077092 & 0.069097 & 0.052795 & -0.99149 & 0.024706 \\ 0.066821 & 0.05654 & 0.009724 & 0.078749 & 0.003746 & -0.99314 \end{pmatrix}$$

And the corresponding eigenvalues of A_1, A_2, A_3 are as the following

$$\lambda(A_1) = \begin{pmatrix} -0.4853 \\ -1.1201 + 0.0248i \\ -1.1201 + 0.0248i \\ -1.0319 + 0.0777i \\ -1.0319 + 0.0777i \\ -0.9975 \end{pmatrix}, \lambda(A_2) = \begin{pmatrix} -0.7313 \\ -1.1035 \\ -0.9518 \\ -1.0227 - 0.0208i \\ -1.0227 - 0.0208i \\ -1.0066 \end{pmatrix},$$

$$\lambda(A_3) = \begin{pmatrix} 1.8834 \\ -0.6045 + 0.3004i \\ -0.6045 + 0.3004i \\ -1.0009 - 0.3919i \\ -1.0009 - 0.3919i \\ -1.1825 \end{pmatrix}$$

Since the real parts of the eigenvalues $\lambda(A_1)$ and $\lambda(A_2)$ of A_1 and A_2 are both less than 0, the belief space of the two multi-agent systems is finally belief accessible. As shown in Fig. 2, each curve in the figure stands for the belief convergence process of an agent. In the figure, the curves are divided into two groups: The belief convergence process of each agent in MAS1 is represented by the dotted line in the figure; and the time convergence process of each agent in MAS2 is represented by the solid line in the figure. It can be seen from the figure that, the belief distance updating process of the two systems is convergent, and there is a non-negative real part existing in $\lambda(A_3)$. The belief updating between the agents will increase with the network time. And the belief of MAS3 is divergent, as shown in Figure 3.

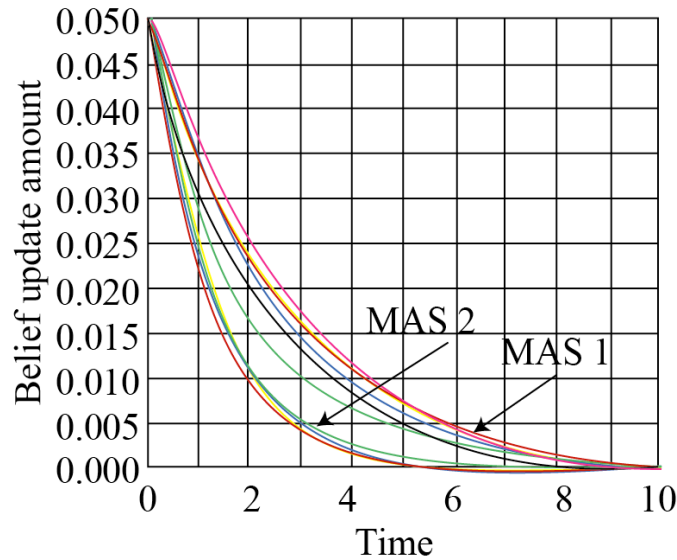


Fig. 2. MAS1 and MAS2 belief convergence process

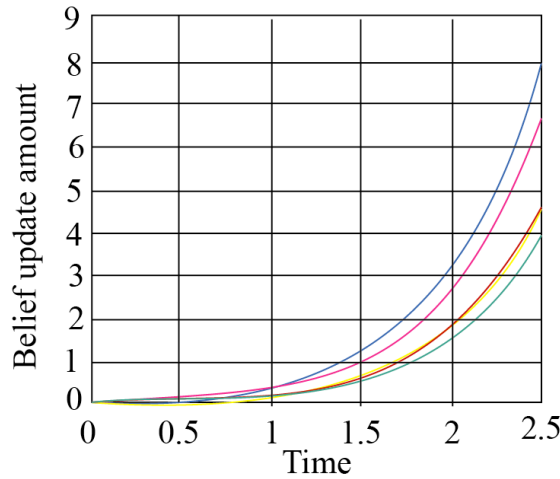


Fig. 3. MAS3 belief convergence process

4 Conclusions

In the bilinear manifold network, the use of multi-agent system is an important method of the network control, while the belief reachability of the multi-agent system is the basis for the effective control of the network. Therefore, the studies on the belief reachability of the multi-agent system are of very important significance. In this paper, a new belief distance updating model for the multi-agent system is put forward, which transforms the belief reachability problem of the multi-agent system in the bilinear manifold network into a linear system problem. This has simplified the complexity of the studies of the belief reachability of the multi-agent system in the bilinear manifold network. On the basis of the model, the belief reachability of the multi-agent system is studied accordingly.

5 References

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